Orthogonal transformation quadratic form

For information about using polynomial statistics in term 2 of all degrees, see secondary format (statistics). In mathematics, the secondary form is a term expression of 2 to all degrees. For example, 4 x 2 + 2 x y - 3 y 2 {\displaystyle 4x^{2} +2xy-3y^{2}} is a secondary format for variables x and y. Coefficients typically belong to a fixed field K, such as a real or complex number, and represent a secondary form for K. If K = R, and the secondary format only take zero if all variables are zero at the same time, it is a clear secondary format, otherwise a secondary form of ethnity. Secondary forms occupy the central location of various branches of mathematics, including mathematical, linear algebra, group theory (orthogonal groups), differential geometry (Lehman metric, second basic form), differential topology (intersectional form of four manibodies), and Lee theory (Killing form). The secondary form has only one variable and should not be confused with a quadr order equation containing terms of no more than 2 degrees. The secondary form is a case of one of the more general concepts of homogeneous polynomials. The introduction secondary form is a homogeneous secondary polynomial in the n variable. For 1, 2, and 3 variables, it is called a single term and has binary, and 3 terms and the following explicit form: q (x) = a x 2 (single term) q (x , y) = x 2 + x x x y + c y 2 (binary) q (x , y , z) = x 2 + b x y + c y 2 + y y + c y 2 + y z + f x z (turnernery) {\display style {\open Start{2}{Align}q(x)=ax{2}&{Align}q(x)=ax{2}&{\text,y)&=ax^{2}+bxy+cy^{2}&&&{textrm{(binary)}\\q(x,y,z)&=ax^{2}+{2}+{2}+{2}+dyz+ez^{2}+fxz&{\textrm}{(section 3) End {}a,.f[1] ⟨ notation is 1 ,.. ⟩) {\display style \langle a_{1},\ldots,a_{n}\langle } is often used for secondary q (x) = 1 x 1 2 + a 2 x 2 + a n x n 2. {\Display style q(x)=a_{1}x_{1}^{2}+a_{2}x_{2}^{2}+\cdots +a_{n}x_{n}{2}.} The theory of secondary forms and methods used in research largely depends on the nature of coefficients and can be real or complex, rational, or integer. For most linear al albra, analytical geometry, and secondary-form applications, coefficients are real or complex. In secondary theory of numbers, coefficients are elements of a particular field. In secondary arithmetic theory, the coefficient belongs to a fixed replacement ring and is often an integer Z or p-adic integer Zp. [2] Binomials have been extensively studied in number theory, especially in the theory of secondary, continuous fractions, and modular forms. The theory of integral secondary forms in n variables has important applications to altological topologies. Non-zero secondary format of n variables using like-for-like coordinates(n-1) A rectangle of (n-2) dimensions in the injection space of the dimension. This is the basic structure of the injection geometry. In this way, a three-dimensional real secondary form can be visualized as a cone cross section. For example, three-dimensional Euclidean space is shown by the squaring of euclidean norms, which represent the distance between coordinates (x, y, z) and points with origins ‖ ‖. {\Display style q(x,y,z)=d(x,y,z),(0,0,0))^{2}=\| (x,y,z)\| {2}=x^{2}+y^{2}+z^{2}.} A concept closely related to geometric oversoning is secondary space, which is a pair (V, q) with vector space on field K, and q : V →V. The question of whether a secondary form of study on V →V. history can be a secondary form of value, especially a secondary form in which a particular integer exceeds an integer, goes back centuries. One such case is Fermar's theorem on the sum of the two squares, which determines when integers are represented in the form of x2 + y2. This problem is related to the problem of .C pythagorean triplets that appeared in the second millennium B. [3]628, Indian mathematician Bramagupta wrote, among many others, a study of equations in the form x2 - ny2 = Brāhmasphuṭasiddhānta c. In particular, he considered what is now called Pell's equation, x2 - ny2 = 1, and found a way for its solution. In 1801 Gauss published the arithmetic of Disquiiones devoted to the complete theory of binary secondary forms on integers. Since then, the concept has become more generalized, and the connection with secondary fields, modular groups, and other areas of mathematics has been further elucidated. Actual secondary form See also: Sylvester inertia and clear secondary body law Any n×n real symmetric matrix A is equation q A (x 1 , .x n) = ∑ i = 1 n ∑ j = 1 n a i j x display style q_ {A}(x_{1} , \ldots, x_{n})=\sum _{{n}\sum _{j=1}a_}{ij}{x_{i}}x_{j}}}}}}}\mathbf{x}Conversely, given the secondary form of the n variable, the coefficient can be placed in the n×n symmetric matrix. An important problem in the theory of secondary forms is how to simplify the secondary form q by uniform linear changes in variables. The basic theorem by Yakobi argues that the actual secondary form q has orthogonal diagonalization. [5] λ 1 x to 1 2 + λ 2 x to 2 + λ n x to n 2 , {\display style \λ _{1}{\tilde {x}}{1}^{2}++ \lambda _{2}{\chilled{x}{2}^{2}+\cdots +\λ_{n}{childe{x}×××2×22%{2}}, corresponding symmetric matrixAchieved by the change of variables given by orthogonaed matrices – in this case, the coefficients λ1, λ2,..,λn are uniquely determined up to the permuting. There is always a change in the variable given by the invertable matrix that is not necessarily orthogoneable so that the coefficients λi are 0, 1, and −1. Sylvester's law of inertia states that the numbers 1 and -1 are secondary immutations, in the sense that the other diagonals each contain the same number. The signature of the secondary format is triple (n0, n+, n-), and n0 is the number of 0 and n ±± number of 1. Sylvester's law of inertia indicates that this is a well-defined amount attached to the secondary form. It is especially important if all λi have the same sign: in this case, the secondary form is called positive determination (all 1) or negative determination (all −1). If none of the terms are 0, the form is called de-degeneration. This includes positive firming, negative clarity, and indefinite (a combination of 1 and -1). Equivalently, the non-degenerate secondary form is a double linear form in which the associated symmetric form is non-degenerate. Real vector space (indicating p1 and q-1s) with indetermissed degenerate secondary forms of indexes (p, q) is sometimes indicated as Rp,q, especially in space-time physical theory. The second-form determination can also specifically define a matrix class of representation matrices at K/(K×)2 (up to non-zero rectangles), and the actual secondary format is more coarsely immutable than signatures that take only positive, zero, or negative values. Zero corresponds to degenerate, and in the case of a non-degenerate form, it is parity of the number of negative coefficients, but (-1) n−. {\Display Style (-1)^{n_{-}} } These results are re-formulaized in another way: Make q a secondary format defined in n-dimensional real vector space. Make A a matrix of secondary format q on a given basis. This means that A is a symmetrical n × n matrix, and q (v) = x T A x , {\display style q(v)=x{\mathrm {T} }Ax,x is a column vector with v coordinates for the selected criterion. Under the change in reference, column x is multiplied to the left by the n×n invertable matrix S, and symmetric matrix A is converted to another symmetric matrix B of the same size according to equation A→ of B=ST AS. {\Display Style A\to B=S^{\mathrm {T} }AS.} Any symmetric matrix A can be converted to diagonal matrix B = (λ 1 000 0 λ 2 } 0 ⋮ ⋮ ⋱. 0 0 0 λ λ) {\display style B={\start {pmatrix}\λ _{1}&0&\cdots &0\0&lambda _. {2}&\cdots &0\vdots &\vdots &\ddots &0\0&0&\cdots &\λ_{n}\end{pmatrix}} is determined by the appropriate selection of orthogonal matrix S, and the diagonal entry of B is determined at all - this is yabilf S is allowed to be any invertable matrix, B should have only 0, 1, and -1 diagonally, and the number of entries of each type (n0 for 0, n+ for 1, n+ for -1) depends only on A. This is one of the formulas of Sylvester's inertial law, and the number of n+ and n− is called the positive and negative exponents of inertia. Their definitions involve the selection of the basis and consideration of the corresponding real symmetric matrix A, but Sylvester's law of inertia means that they are the immutability of the secondary form q. If secondary format q is positive (resp., negative), q(v) > 0 (resp., q(v) < 0) all non-zero vectors v.[6] q(v) assume both positive and negative values, q is an indeterndable secondary format. Yacobi and Sylvester's theorems show that the positive clear secondary form of the n variable can be bringed into the sum of the squares of n by a proper invertable linear transformation: geometrically, there is only one positive deterministic secondary form of all dimensions. The iso-sex group is the compact orthogonal group O(n). This is in contrast to the case of the indetermoid, when the corresponding group, the indetermoted orthogonal group O (p, q), is non-compact. In addition, the iso-sex group of Q and -Q is the same (O(p, q) ≈O(q, p)), and the associated Clifford alal (and therefore the pin group) is different. Definition The secondary format of field K is map q: V →K {\displaystyle q:V\to K} for K from finite dimension K vector space to K q (v) = 2 q (v) {\display style q(av)=a^{2}q(v) } all } for ∈ K, v ∈ V {\displaystyle a\in K,v\in V} and function q (u + v) - q (u) - q (v) {\display style q(u+v)-q(u)-q(v)} are double linear. Specifically, the n-term secondary form on field K is an all-like polynomial in the n variables of K:q(x1、..、x n)=∑i = 1 n∑j = 1 n a i x i x x j j, a i j ∈ K. {\Display style q(x_{1},\ldots,x_{n})=\Total _{i=1}} {n}\Total_{j=1}}{{{x_\x_ a_ n}{{{i}{{{i}{i}}{x_{j}},\Quad a_{ij}\K.} This formula can be rewritn using a matrix, with x as the component x1、..、xn, A = (aij) as the n×n matrix for K, who has a coefficient of q. Then q (x) = x T A x . {\Display style q(x)=x^{\mathrm {T} }Ax.} Vector v = (x 1 , ., x n) {\display style v=(x_{1},\ldots,x_{n}) is a null vector for q(v) = 0. The two n-term secondary shapes φ and ψ for k are equivalent when there is a single linear transformation C, such as gl(n, K) ∈ψ (x) = φ (C x). {\Display style \psi (x)=\varphi (Cx).} Keep the characteristics of K different from 2. [7] The coefficient matrix A of q may be replaced by a symmetric matrix (A + AT)/2 of the same secondary form, or may be assumed from the beginning that A is symmetric. In addition, symmetric matrix A is uniquelyBy the corresponding secondary form. In equivalence C, φ symmetric matrix A and ψ symmetric matrix B are related as follows: {\Display style B=C^{\mathrm {T} }AC.} The associated double linear form of the secondary form q is defined by b q(x,y)=1 2(q(x+y)-q(x) - q - q(y))=x T A y = yT A x. {\Display style b_ {q}(x,y)={\tfrac {1}{2}}(q(x+y)-q(x)-q(y))=x{\mathrm{T} }Ay=y^{\mathrm {T}Therefore, bq is a symmetrical two-wire type for K with the mother matrix A, and symmetric bilinelineal format b defines the secondary form q (x) = b (x , x) {\displaystyle q(x)=b(x,x)} , these two processes are inverses of each other. As a result, in the field of characteristics not equal to 2, the theory of symmetric double-ray and secondary forms in n variables is essentially the same. The secondary form q of the n variable on the secondary space K directs the map from the N-dimensional coordinate space Kn to K: Q (V) = q ∈ (v). {\Display style Q(v)=q(v), \quad v=[v_{1},\ldots,v_{n}][{\mathrm {T} }\In K^{n}} Map Q is a similar function of the number 2, which has the property of V: Q (a v) = 2 Q (v) for all of K's a and V. {\Display style Q(av)=a^{2}Q(v)} If the characteristics of K are not 2, the following bilinear maps B:V×V V →K or less are defined: B(v,w)=1 2 (Q(v+w)-Q(v) - Q({\Display Style B(v,) w)={\tfrac {1}{2}}(Q(v+w)-Q(v)-Q(w)) This two-linear form B is symmetric, i.e. B(x,y)=B(y,x) for all x and y in V. Determine q:Q(x)=B(x,x) for all x in V. If K has a characteristic of 2 and 2 is not a unit, you can use the secondary format to define symmetric secondary forms B′(x, y) = Q(x+y) - Q(x) - Q(y). However, because B(x, x) = 0 is for all xs (and therefore alternately [8]), Q(x) can no longer recover from this B′ in the same way. Instead, there is always a double-wire B (generally neither unique nor symmetrical) such as B(x, x) = Q(x). The pair (V, Q) consisting of a finite dimensional vector space V on K and a secondary map Q from V to K is called a secondary space, and the B defined here is an associated symmetric double-ray type of Q. The concept of secondary space is a coordinate-free version of the concept of the secondary form. Q is also known as a secondary form. The two n-dimensional secondary spaces (V, Q) and (V′, Q′) are ethno-values when Q(v)=Q)(Tv) has an invertable linear transformation T:V→V′ (isotivity) such as all v∈V for Vvvv. {\Display style Q(v)=Q′(TV){\text{All}}v\in V.} The isomerability class of the n-dimensional secondary space on K replaces K. generalization R, and corresponds to the equivalent class of the n-term secondary form for the R module, b:M×M.R is an R-secondary form. [9] Mapping q: M → R : v ↦ b(v, v) is the associated secondary format of b, and B : M × M → R : (u, v) ↦ q(u +v) - q(u) - q(v) is the polar form of q. The secondary form q:M→R may be characterized in the following equivalent way: there is an R hyperlineable form of R in which R→M×M is present, such as Q(v) being the relevant secondary form. q(av) = a2q(v) is ∈ R and v ∈ M, and the polar shape of q is R-2 linear. Related concept reference: The two elements of the ethnolytic secondary form V and the w of v are called orthogonal if B(v, w) = 0. A dual-wire B kernel consists of elements that are orthogonal to all elements of V.Q {0} the associated dual linear form of the kernel is available. If there is a non-zero v in V, such as Q(v) = 0, the secondary form Q is ishoidal, otherwise an anthodic. The term also applies to vectors and partial spaces in secondary space. If the Q limit for V's partial space U is zero, U is completely single. The orthogonal group of non-single secondary form Q is a group of linear self-type bodies of V that hold Q, i.e. a group of isofers of (V, Q). The secondary space (A, Q) contains the product of which A is an alms on the field, and the whole secondary form of the n variable on the field with characteristics not equal to ∀x, y∈A Q(x y)=Q(x) form equivalence 2 is equivalent to diagonal q (x) = 1 x 1 2 + a 2 x 2 + + n x n 2. {\Display style q(x)=a_{1}x_{1}^{2}+a_{2}x_{2}^{2}+\cdots +a_{n}x_{n}{2}.} These diagonal shapes are often indicated by 、..、)(n. {\Display Style \Langle a_{1},\ldots,a_{n}\Langle .} Thus, the classification of all secondary forms up to equivalence can be reduced in the case of diagonal forms. Geometric meaning Using three-dimensional orthogonal coordinates, use x = (x , y , z) T {\display style \mathbf {x} = (x,y,z){\text{T}} to make {\displaystyle A} a symmetric matrix of 3×3. Next, it depends on the unique value of equation x T A x + b T x = 1 {\display style \mathbf {x} ^{\text{T}}A\mathbf {x} +\mathbf b} ^{\text{T}\mathbf {x} =1} matrix A {\display style A}. If all the unique values for A {\display style A} are non-0, the solution set will be ellipsoidal or hyperbolic[citation required]. If all unique values are positive, it is an ellipsoid. If all the unique values are negative, it is an ellipsoid of imaginary numbers (we get an ellipsoidal equation, but with an imaginary radius). If the unique value is positive and there is a negative value, it is hyperbolic. If one or more unique values λ i = 0 {\display style \lambda_{i}=0}, the shape depends on the corresponding b i.b_ the corresponding b i ≠0 {\displaystyle b_{i}eq 0}, the solution set is a runa (ellipse or hyperbolic). For the corresponding b i = 0 {b_ i}i}=0}, the dimension i {\display style i} does not degenerate and play back, and the geometric meaning is determined by other unique values and other components of the b {\displaystyle mathbf {b}. If the solution set is a normal line, whether it is an ellipse or a hyperbola is determined by whether all other non-zero unique values are the same sign: if so, it is an ellipse. Otherwise, it is hyperbolic. The integral secondary form secondary form on an integer ring is called the integral secondary form, and the corresponding module is a secondary lattice (sometimes simply a lattice). They play an important role in mathematical and topology. The secondary form of an integer has an integer coefficient, such as x2 + xy + y2. Equally, when the lattice Λ of the vector space V (on the field of characteristics 0 such as Q and R) is given, the secondary form Q is indispensable for Λ only when it is an integer value on Λ, and in the case of X and y∈, it means Q (x, y) ∈Z. This is the current use of the term. In the past, it was sometimes used differently, as detailed below. Historical use Historically, there has been some confusion and controversy as to what the concept of integral secondary forms means: two two of the secondary forms associated with symmetric matrices with integer coefficients put out two polynomials with integer coefficients (hence therefore related) Symmetry Matrices may have diagonal to semi-integer coefficients) This argument is due to confusion between secondary forms (represented by polynomials) and symmetric double linearity (represented by matrices), and two outs is an accepted practice. Inside 2 is instead the theory of integral symmetric double-ray (integral symmetric matrix). In two, the binary secondary format is in the form of symmetric matrix (b b c){\display {2} {2}style{\start{pmatrix a&b\b\c\end{pmatrix}}. In put out two, the binary secondary form is in the form of x2 + b x y + c 2 {\display style ax^{2}+bxy+cy^{2}}, represented by a symmetric matrix (b / 2 b / 2 c). {\Display style {\start{pmatrix}a&b/2\b/2&c\end{pmatrix}}} Some perspectives mean that the two outs are adopted as standard treaties. These include: a better understanding of the 2-adic theory of secondary forms, a local source of difficulty; a lattice perspective commonly adopted by experts in secondary form arithmetic calculations in the 1950s. The actual needs of integral secondary morphology theory in the topology of cross theory;Secondary form Integral secondary formats with images consisting of all positive integers are also called universal. Lagrange's four-square theorem shows that w 2 + x 2 + y 2 + z 2 {\display style w^{2}+x^{2}+y^{2}+z^{2}} is universal. Ramanujan generalized this to w 2 + b x 2 + c y 2 + d z 2 {\display style aw^{2}+bx^{2}+cy^{2}+dz^{2}} and found ≤ ≤ ≤ ≤ 54 multiset {, b, c, d} , 3, d}, 3 ≤ ≤ d 6 {1, 2, 2, d}, 2, 2, d}, 2 ≤ d } 2 ≤ 7, Some formats contain all images of 2, 3, d}, 3 ≤ d ≤ 10 ≤ 10,1, 2, 4, d}, 4 ≤ d ≤ 14 {1, 2, 5, d}, 6 ≤ d ≤ 10 images. For example, {1,2,5,5} contains 15 exceptions. These days, the theorems of 15 and 290 characterize a completely universal integral secondary format: if all coefficients are integers, it represents all positive integers only if it represents all integers up to 290. If there is an integer matrix, it represents all positive integers only if it represents integers up to 15. 2nd Form ε Cube Form Hasse Minkosky Theorem Quadrance Secondary Scuadric Ramanujan's Teral Secondary Square Class Whit Group Wit Theorem Notes ^ Back to Gauss Tradition dictates the use of apparently uniform coefficients for the product of different variables, the determination of the cubic form of the secondary form, b, d, in the form of b, d, 3 in bb2b2b2b f. Both rules occur in the literature. Apart from ^ 2, that is, if 2 can be inverted in a ring, the secondary form is equivalent to a symmetrical double linear form (due to polarization equality), but in 2 it is a different concept. This distinction is especially important for secondary forms for integers. ^ Pythagorean biography of Babylonian Pythagoreas ^ Biography of Bagu gupta ^ Maxim Borcher (E.P.R. DuVal) (1907) Introduction to high alums, § 45 When reducing secondary forms to square sums via Hatty Trust ^ When non-strict inequality (with ≥ or ≤) holds ^ There is an important difference in secondary forms of theory for characteristic 2 fields Many definitions and theorems must be modified. ^ This alternating form is interesting associated with Arf immutability, associating the second form with feature 2 - Irving Kaplansky (1974), Linear Alcacath and Geometry, p. 27. ^ The secondary form with which the secondary form is associated is not limited to symmetry, and is important when 2 is not a unit of R. References

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