Probability and stochastic processes yates 3rd edition pdf

5 coment-rios 0 gostaram Estatsticas Notas Seja a primeira pessoa a gostar disto 1. Probability and stochastic processes A friendly introduction for electrical and computer engineers Third Edition INSTRUCTOR'S SOLUTION MANUAL Roy D. Yates, David J. Goodman, David Famolari September 8, 2014 Comments on this solution manual • This solution manual is largely complete. Please send bug reports, suggestions and comments to ryates@winlab.rutgers.edu. • To create solution sets for your class, use the solution set constructor at the www.winlab.rutgers.edu/probsolns site. • Send an email to ryates@winlab.rutgers.edu to access the instructors' web site. • Matlab functions written as a solution to homework problems can be found in the matsoln3e.zip archive (available for instructors). For more Matlab features used in text or these homework solutions, see the matcode3e.zip archive. The .m files in matcode3e are available for download on the Wiley website. Two other documents of interest are also available for download: - A probmatlab3e manual.pdf describes the matcode3e .m features - The Quizsolutions manual quizsol.pdf. • This guide uses a page size that fits the screen of an iPad tablet. If you print on paper and have good vision, you can print two pages per sheet in landscape mode. On the other hand, a Fit to Paper print option produces the Large Print output. 1 Probability and stochastic processes A friendly introduction for Electric and Com Full Download: This is just example, download all chapters at: AlibabaDownload.com 2. Problem Solving – Chapter 1 Problem 1.1.1 Solution Based on the Venn diagram on the right is the complete Gerlandas Pizza Menu • Regularly without toppings • Regular with mushrooms • Regular with onions • Regular with mushrooms and onions • Tuscan without toppings • Tuscan with mushrooms M O T Problem 1.1.2 Solution Based on the Venn diagram on the right, the answers are mostly simple. The only trickiness is that a pizza is either Tuscan (T) or Neapolitan (N) so, T, T is a partition, but they are not represented as a partition. In particular, event N is the area of the Venn diagram outside the square block of event T. If this is clear, the questions are simple. M O T (a) Since N = Tc, N ∩ M = φ. N and M are therefore not mutually exclusive. b) Each pizza is either Neapolitan (N) or Tuscan (T). Therefore, N ∪ T = S, so that N and T are collectively exhaustive. So it is also (trivially) true that N ∪ T ∪ M = S. The R, T and M are also collectively exhaustive. c) From the Venn diagram, T and O are mutually exclusive. In words, this means that Tuscan pizzas never have onions or pizzas with onions, never Tuscan. By the way, by the way, is a fake pizza name; one should not conclude that the people of Tuscany don't really like onions. d) From the Venn diagram, M ∩ T and O are mutually exclusive. So Ger- landa's does not make a Tuscan pizza with mushrooms and onions. e) Yes. In the Venn diagram, these pizzas are in the set (T ∪ M ∪ O)c. 2 3. Problem 1.1.3 Solution With Ricardo, the pizza crust is either Roman (R) or Neapolitan (N). To draw the Venn diagram on the right, let's make the following observations: R N M OW • The set R, N is a partition so that we can draw the Venn diagram with that partition. • Only Roman pizzas can be white. Therefore W ⊂ R. • Only a Neapolitan pizza can have onions. Therefore, O ⊂ N. • Both Neapolitan and Roman pizzas can have mushrooms, so that event M spans the partition R, N. • Neapolitan pizza can have both mushrooms and onions, so M ∩O can't be empty. • The problem statement does not exclude the possibility of placing mushrooms on a white Roman pizza. Therefore, the junction W ∩ M should not be empty. Issue 1.2.1 Solution (a) A result indicates whether the connection speed is high (h), medium (m) or low (l) speed, and whether the signal is a mouse click (c) or a tweet (t). The sample space is S = s, hc, mt, mc, lt, lc. (1) b) The event that the Wi-Fi connection is medium rpm is A1 = 'mt, mc'. (c) The event that a signal is a mouse click is A2 = .hc, mc, lc. (d) The event that a connection is either high or at low speed is A3 = ht, hc, lt, lc. 3 4. (e) Since A1 is ∩ A2 = mc and is not empty, A1, A2, and A3 are not mutually exclusive. (f) Since A1 ∪ A2 ∪ A3 = ht, hc, mt, mc, lt, lc = S, (2) the collection A1, A2, A3 is collectively exhaustive. Problem 1.2.2 Solution (a) The sample space of the experiment is S = aaa, aaf, afa, faa, ffa, faf, aff, fff. (1) b) The event that the circuit of Z fails is ZF = aaf, aff, faf, fff. (2) The event that the circuit of X is acceptable is XA = aaa, aaf, afa, aff . (3) (c) Since ZF ∩ XA = aaf, aff = φ, ZF and XA are not mutually exclusive. (d) Since ZF ∪ XA = aaa, aaf, afa, aff, faf, fff = S, ZF and XA are not collectively exhaustive. e) The event that more than one circuit is acceptable is C = aaa, aaf, afa, faa. (4) The event that at least two circuits fail is D = sffa, faf, aff, fff. (5) (f) The test shows that C ∩ D = φ so c and D are mutually exclusive. (g) Since C ∪ D = S, C and D are collectively exhaustive. 4 5. Problem 1.2.3 Solution The sample room is S = A♣, . . , K♣, A♦, . . , K♦, A♥, . . , K♥, A♠, . . ♠. (1) Event H that the first card is a heart is the set H = A♥, . . .♥ . (2) Event H 13 results, which corresponds to the 13 hearts in one deck. Problem 1.2.4 Solution The sample room is S = ⎧ ⎨ ⎩ 1/1 . . . 1/31, 2/1 . . . 2/29, 3/1 . . . 3/31, 4/1 . . . 4/30, 5/1 5/1 . . 5/31, 6/1 . . . 6/30, 7/1 . . . 7/31, 8/1 . . . 8/31, 9/1 . . . 9/31, 10/1 . . . 10/31, 11/1 . . . 11/30, 12/1 . . . 12/31 ⎫ ⎬ ⎭ . (1) Event H, which is defined by the event of a July birthday, is given by the following et with 31 sample results: H = 7.1, 7,2, . . . (2) Problem 1.2.5 Solution Of course there are many answers to this problem. Here are four partitions. 1. We can divide students into engineers or non-engineers. Let A1 match the set of engineering students and A2 to non-engineers. The pair A1, A2 is a partition. 2. To separate students by GPA, let Bi designate the subset of students with GPAs G satisfying i 1 ≤ G < i. For Rutgers, partition B5 is a partition. Note that B5 is the set of all students with perfect 4.0 GPAs. Of course, other schools use different standards for GPA. 3. We can also divide students by age. Let Ci designate the subset of students in the age of i in years. At most universities, C10, C11, . . . C100 an event room. Since a university can have miracle children under 10 or over 100, we find that C0, C1, . . . is always a partition. 5 6. 4. Finally, we can categorize students by presence. Let D0 indicate the number of students who missed zero lectures, and let D1 designate all other students. Although it is likely that D0 is an empty set, it is a well-defined partition. Problem 1.2.6 Solution Let R1 and R2 denote the measured resistors. The pair (R1, R2) is a result of the experiment. Some partitions contain 1. If we need to check if neither of the two resistors is too high, a partition is A1 = R1 < 100, R2 < 100 , A2 = R1 ≥ 100 ∪ R2 ≥ 100 . (1) 2. If we need to check whether the first resistor exceeds the second resistor, a partition is B1 = R1 > R2 ≤. (2) 3. If we need to check whether each resistor does not fall below a minimum value (in this case 50 ohms for R1 and 100 ohms for R2), a partition is C1, C2, C3, C4 where C1 = R1 < 50, R2 < 100 , C2 = R1 < 50, R2 ≥ 100, (3) C3 = R1 ≥ 50, R2 < 100 , C4 = R1 ≥ 50, R2 ≥ 100 . (4) 4. If we want to check whether the resistors are in parallel in an acceptable range of 90 to 110 ohms, a partition is D1 = (1/R1 + 1/R2)-1 < 90 , (5) D2 = 90 ≤ (1/R1 + 1/R2)-1 ≤ 110 , (6) D2 = 110 < (1/R1 + 1/R2)-1. (7) 6 7. Problem 1.3.1 Solution (a) A and B, which are mutually exclusive and collectively exhaustive, imply P[A]+P[B] = 1. Since P[A] = 3 P[B] we have P[B] = 1/4. (b) Since P[A is ∪ B] = P[A], we see that B ⊆ A. This implies P[A ∩ B] = P[B]. Since P[A ∩ B] = 0, then P[B] = 0. (c) Since it is always true that P[A ∪ B] = P[A] + - P[AB], we have this P[A] + P[B] - P[AB] = P[A] - P[B]. (1) This implies 2 P[B] = P[AB]. However, since AB ⊂ B, we can conclude that 2 P[B] = P[AB] ≤ P[B]. This implies P[B] = 0. Problem 1.3.2 Solution The roll of red and white cubes assumed to be independent. For each die, all reels in 1, 2, . . . , 6 probably have 1/6. (a) Thus P[R3W2] = P[R3] P[W2] = 1 36 . (b) In fact, each pair of possible riwj roles has a probability of 1/36. To find P[S5], Let's add up the probabilities of all pairs that add up to 5: P[S5] = P[R1W4] + P[R2W3] + P[R3W2] + P[R4W1] = 4/36 = 1/9. Problem 1.3.3 Solution A result is a pair (i, j), where i is the value of the first cube and j is the value of the second cube. The sample space is the set S = s (1, 1), (1, 2), . . (1) with 36 results, each with probably 1/36 note that the event that the absolute value of the difference of the two reels is 3 is D3 = (1, 4), (2, 5), (3, 6), (4, 1), (5, 2), (6, 3) . (2) Since there are 6 results in D3, P[D3] = 6/36 = 1/6. 7 8. Problem 1.3.4 Solution (a) FALSE. Because P[A] = 1 x P[Ac] = 2 P[Ac] implies P[Ac] = 1/3.b) FALSE. Suppose A = B and P[A] = 1/2. In this case P [AB] = P [A] = 1/2 > 1/4 = P [A] P [B] . (1) (c) TRUE. Since AB ⊆ A, P[AB] ≤ P[A], this means P [AB] ≤ P [A] < P [B] . (2) (d) FALSE: For a counterexample, leave A = φ and P[B] > 0, so that A = A∩B = φ and P[A] = P[A ∩ B] = 0 = P[A] < P[B]. Problem 1.3.5 Solution The sample space of the experiment is S = LF, BF, LW, BW . (1) From the problem instruction we know that P[LF] = 0.5, P[BF] = 0.2 and P[BW] = 0.2. This implies P[LW] = 1 x 0.5 - 0.2 - 0.2 = 0.1. The questions can be answered with Theorem 1.5. (a) The probability that a program is slow is P [W] = P [LW] + P [BW] = 0.1 + 0.2 = 0.3. (2) b) The probability that a program is large is P [B] = P [BF] + P [BW] = 0.2 + 0.2 = 0.4. (3) (c) The probability that a program is slow or large is P [W ∪ B] = P [W] + P [B] - P [BW] = 0.3 + 0.4 - 0.2 = 0.5. (4) 8 9. Problem 1.3.6 Solution An example result indicates whether the mobile phone is handheld (H) or mobile (M) and whether the speed is fast (F) or slow (W). The sample room is S = HF, HW, MF, MW . (1) The problem instruction tells us that P[HF] = 0.2, P[MW] = 0.1 and P[F] = 0.5. We can use these facts to determine the probabilities of the other results. In particular, P [F] = P [HF] + P [MF] . (2) This implies P [MF] = P [F] - P [HF] = 0.5 x 0.2 = 0.3. (3) Since the probabilities must be sumred to 1, P [HW] = 1 - P [HF] - P [MF] - P [MW] = 1 - 0.2 - 0.3 - 0.1 = 0.4 must be sumred. (4) Now that we have found the probabilities of the results, it is easy to find a different probability allowance. (a) The probability that a mobile phone is slow is P [W] = P [HW] + P [MW] = 0.4 + 0.1 = 0.5. (5) b) The probability that a cell hpon is mobile and fast is P[MF] = 0.3.c) The probability that a cell hpon is mobile and fast. mobile phone is held by hand, P [H] = P [HF] + P [HW] = 0.2 + 0.4 = 0.6. (6) Problem 1.3.7 Solution A reasonable probability model that matches the concept of a mixed deck is that each card in the deck the first card. Let Hi denote the event that the first drawn card is the ith heart, where the first heart is the ace, 9 10. the second heart is the deuce and so on. In this case p[Hi] = 1/52 for 1 ≤ i ≤ 13. The event H, that the first card is a heart, can be written as the mutually exclusive union H = H1 ∪ H2 ∪. ∪ H13. (1) With Theorem 1.1 we have P [H] = 13 i=1 P [Hi] = 13/52. (2) This is the answer you would expect, since 13 out of 52 cards are hearts. The point to keep in mind is that this is not just the answer to common sense, but the result of a probability model for a mixed deck and the axioms of probability. Problem 1.3.8 Solution Let si denote the result that the down face has i points. The sample space is S = s1, ... s6. The probability of each sample result is P[si] = 1/6. As of Theorem 1.1, the probability of event E is that the role is straight, P [E] = P [s2] + P [s4] + P [s6] = 3/6. (1) Problem 1.3.9 Solution Let si match the result of the student quiz. The rehearsal room is then composed of all sorts of notes that it can receive. S = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 . (1) Since each of the 11 possible results is equally likely, the probability of getting a grade of i is for each i = 0, 1, . . . 10 P[si] = 1/11. The probability that the student will receive an A is the probability that she will receive a score of 9 or higher. This is P [Class of A] = P [9] + P [10] = 1/11 + 1/11 = 2/11. (2) The probability of failure presupposes that the student receives a grade below 4. P [Failing] = P [3] + P [2] + P [1] + P [0] = 1/11 + 1/11 + 1/11 + 1/11 = 4/11. (3) 10 11. Problem 1.3.10 Solution Each instruction is a sequence of part 4 of Theorem 1.4. a) Since A ⊂ A ∪ B, P[A] ≤ P[A ∪ B]. (b) Since B ⊂ A ∪ B, P[B] ≤ P[A ∪ B]. (c) Since A ∩ B ⊂ A, P[A ∩ B] ≤ P[A]. (d) Since A ∩ B ⊂ B, P[A ∩ B] ≤ P[B]. Problem 1.3.11 Solution Specifically we use Theorem 1.4(c) stating that for all events A and B, P [A ∪ B] = P [A] + P [B] - P [A ∩ B] . (1) In order to prove the union bound by induction, we first prove the proposition in the case of n = 2 events. In this case by theorem 1.4(c), P [A1 ∪ A2] = P [A1] + P [A2] - P [A1 ∩ A2] . (2) With the first axiom of probability, P[A1 ∩ A2] ≥ 0. Thus, P [A1 ∪ A2] is ≤ P [A1] + P [A2] . (3) as evidenced by the union bound for case n = 2. Now we are putting forward our induction hypothesis that the union-linked one applies to each collection of n - 1 subsets. In this case, the given subsets define A1, . . . , To, we a = A1 ∪ A2 ∪ . ∪ To 1, B = On. (4) By our induction hypothesis, P [A] = P [A1 ∪ A2 ∪ ∪ to 1] ≤ P [A1] + . + P [An-1] . (5) This allows us to write P [A1 ∪ - ∪ An] = P [A ∪ B] ≤ P [A] + P [B] (by the union bound for n = 2) = P [A1 ∪ - ∪ To 1] + P [An] ≤ P [A1] + P [An-1] [An-1] P [An] (6), which completes the inductive evidence. 11 12. Problem 1.3.12 Solution It is tempting to use the following proof: since S and φ mutually exclusive, and since S = S ∪ φ, 1 = P [S ∪ φ] = P [S] + P [φ] . Since P[S] = 1, we must have P[φ] = 0. The above evidence used the property, the mutually exclusive sentences A1 and A2, P [A1 ∪ A2] = P [A1] + P [A2] . (1) The problem is that this property is a consequence of the three axioms and must therefore be proven. For proof that uses only the three axioms, let A1 be any set and for n = 2, 3, ... let To = φ. Since A1 = ∪∞ i=1Ai, we can use Axiom 3 to write P [A1] = P [∪∞ i=1Ai] = P [A1] + P [A2] + ∞ i=3 P [Ai]. (2) By subtracting P[A1] from both sides allows us to write the fact that A2 = φ P [φ] + ∞ n=3 P [Ai] = 0. (3) By Axiom 1, P[Ai] ≥ 0 for all i. So ∞ n=3 P[Ai] ≥ 0. This implies P[φ] ≤ 0. Because Axiom requires 1 P[φ] ≥ 0, we must have P[φ] = 0. Problem 1.3.13 Solution After the note, we define the set of events Ai|i = 1, 2, . . . so that i = 1, . . . , m, Ai = Bi and for i > m, Ai = φ. By construction ∪m i=1Bi = ∪∞ i=1Ai. Axiom 3 then implies P [∪m i=1Bi] = P [∪∞ i=1Ai] = ∞ i=1 P [Ai] . (1) 12 13. For i > m, P[Ai] = P[φ] = 0, which gives the claim P[∪m i=1Bi] = m i=1 P[Ai] = m i=1 P[Bi]. Note that the fact that P[φ] = 0 follows from axioms 1 and 2. This problem is more difficult if you are using axiom 3 only. We start by observing P [∪m i=1Bi] = m'1 i=1 P [Bi] + ∞ i=m P [Ai] . (2) Now we use Axiom 3 again in the countable infinite sequence Am, Am+1, . . . ∞ i=m P [Ai] = P [Write on ∪ Am+1 ∪. (3) Therefore, we only used Axiom 3 to prove Theorem 1.3: P [∪m i=1Bi] = m i=1 P [Bi]. (4) Problem 1.3.14 Solution Any claim in Theorem 1.4 requires proof that we can use to verify which axioms are being used. However, the problem is somewhat difficult because there is still a simpler proof that uses fewer axioms. Nevertheless, the proof for each part will need Theorem 1.3, which we are now proving. For the mutually exclusive events B1, . . . , Bm, leave Ai = Bi for i = 1, . . . , m and leave Ai = φ for i > m. In this case by Axiom 3, P [B1 ∪ B2 ∪ - ∪ Bm] = P [A1 ∪ A2 ∪ n] = m'1 i=1 P [Ai] + ∞ i=m P [Ai] = m'1 i=1 P [Bi] + ∞ i=m P [Ai] . (1) 13 14. Now we use Axiom 3 again on Am, Am+1, . . . ∞ i=m P [Ai] = P [Write on ∪ Am+1 ∪. (2) So we used only Axiom 3 to prove Theorem 1.3: P [B1 ∪ B2 ∪ . . ∪ Bm] = m i=1 P [Bi]. (3) (a) To display P[φ] = 0, leave B1 = S and Leave B2 = φ. Thus by sentence 1.3, P [S] = P [B1 ∪ B2] = P [B1] + P [B2] = P [S] + P [φ] . (4) Thus, = 0. Note that this proof uses only Theorem 1.3, which uses only Axiom 3. b) With Theorem 1.3 with B1 = A and B2 = Ac we have P [S] = P [A ∪ Ac] = P [A] + P [Ac] . . Axiom says 2 P[S] = 1, P[Ac] = 1 x P[A]. This proof uses axioms 2 and 3. (c) With Theorem 1.8 we can write both A and B as associations of mutually exclusive events: A = (AB) ∪ (ABc), B = (AB) ∪ (Ac B). (6) Now we use Theorem 1.3 to write P [A] = P [AB] + P [ABc] , P [B] = P [AB] + P [Ac B]. (7) We can describe these facts as P[ABc] = P[A] , P[AB], P[Ac B] = P[B] - P[AB]. (8) Note that we have only used Axiom 3 so far. Finally, we find that A ∪ B can be written as a union of mutually exclusive events A ∪ B = (AB) ∪ (ABc) ∪ (Ac B). (9) 14 15. Once again, with Theorem 1.3 P[A ∪ B] = P[AB] + P[ABc] + P[Ac B] (10) Substituting the results of equation (8) in equation (10) results P [A ∪ B] = P [AB] + P [A] - P [AB] + P [B] Note that this claim requires only axiom 3. (d) Note that since A ⊂ B B we can write as mutually exclusive association B = A ∪ (AcB). Set 1.3 (used axiom 3), P [B] = P [A] + P [Ac B] . (12) From Axiom 1, P[AcB] ≥ 0, hich implies P[A] ≤ P[B]. This proof uses Axiome 1 and 3. Problem 1.4.1 Solution Each question requires a conditional probability. (a) Note that the probability that a call is short is P [B] = P [H0B] + P [H1B] + P [H2B] = 0.6. (1) The probability that a short call has no donations is P [H0| B] = P [H0B] P [B] = 0.4 0.6 = 2 3 . (2) b) The probability of a transfer is P[H1] = P[H1B] + P[H1L] = 0.2. The probability that a call with a handoff is long is P [L|H1] = P [H1L] P [H1] = 0.1 0.2 = 1 2 . (3) (c) The probability that a call is long is P[L] = 1 x P[B] = 0.4. The probability that a long call has one or more handovers is P

[H1 ∪ H2| L] = P [H1L ∪ H2L] P [L] = P [H1L] + P [H2L] P [L] = 0.1 + 0.2 0.4 = 3 4 . (4) 15 16. Problem 1.4.2 Solution Let si denote the result that the role is i. So, for 1 ≤ i ≤ 6, Ri = si. Similarly, gj = sj+1, ... (a) Since G1 = s2, s3, s4, s5, s6 and all results have the probability of 1/6, p[G1] = 5/6. The event R3G1 = s3 and P[R3G1] = 1/6, so that P [R3| G1] = P [R3G1] P [G1] = 1 5 . (1) (b) The conditional probability of 6 being rolled because the roller is greater than 3 is P [R6| G3] = P [R6G3] P [G3] = P [s6] P [s4, s5, s6] = 1/6 3/6 . (2) (c) Event E that the role is straight is E = s2, s4, s6 and has a probability of 3/6. The common probability of G3 and E is P [G3E] = P [s4, s6] = 1/3. (3) The conditional probabilities of G3 in E are P [G3| E] = P [G3E] P [E] = 1/3 1/2 = 2 3 . (4) d) The conditional probability that the role is even given that it is greater than 3 is P [E| G3] = P P [G3] = 1/3 1/2 = 2 3 . (5) Problem 1.4.3 Solution Since the 2 of the clubs is a straight numbered card, C2 is ⊂ E, so P[C2E] = P[C2] = 1/3. Because P[E] = 2/3, P [C2| E] = P [C2E] P [E] = 1/3 2/3 = 1/2. (1) The probability that a straight numbered card will be selected, since the 2 2 picked p [E| C2] = P [C2E] P [C2] = 1/3 1/3 = 1. (2) 16 17. Problem 1.4.4 Solution Let Ai and Bi indicate the events that the sold Ith phone is an apricot or a banana. Our goal is to find P[B1B2], but since it is not clear where to start, we should plan to fill in table A2 B2 A1 B1. This table has four unknowns: P[A1A2], P[A1B2], P[B1A2], and P[B1B2]. We begin to know that P [A1A2] + P [A1B2] + P [B1A2] + P [B1B2] = 1. (1) We need to solve three more equations for the four unknowns. From sales of apricots and bananas are equally likely, we know that P[Ai] = P[Bi] = 1/2 for i = 1, 2. This implies P [A1] = P [A1A2] + P [A1B2] = 1/2, (2) P [A2] = P [A1A2] + P [B1A2] = 1/2. (3) The final equation is that the first phone sold is a banana, the second phone is twice as likely to be a banana, which is P[B2| B1] = 2 P[A2| B1]. With the bayes set we have P [B1B2] P [B1] = 2 P [B1A2] P [B1] =⇒ P [B1A2] = 1 2 P [B1B2] . (4) Replacing P[B1A2] with P[B1B2]/2 in the first three equations gives P [A1A2] + P [A1B2] + 3 2 P [B1B2] = 1, (5) P [A1A2] + P [A1B2] = 1/2, (6) P [A1A2] + 1 2 P [B1B2] = 1/2. (7) Subtract (6) from (5) yields (3/2) P[B1B2] = 1/2 or P[B1B2] = 1/3, which is the answer we are looking for. 17 18. At this point, if you are curious, we can solve for the rest of the probability table. From (4) we have P[B1A2] = 1/6 and from (7) we get P[A1A2] = 1/3. It follows from (6) that P[A1B2] = 1/6. The probability table is A2 B2 A1 1/3 1/6 B1 1/6 1/3 . Problem 1.4.5 Solution The first generation consists of two plants each with genotype yg or gy. They are crossed to produce the following second-generation genotypes, S = ,yy, yg, gy, gg' . Each genotype is as likely as any other, so the probability of each genotype is therefore 1/4. A pea plant has yellow seeds if it has at least one dominant y-gene. The set of pea plants with yellow seeds is Y = yy, yg, gy . (1) The probability of a pea plant with yellow seeds is thus P [Y] = P [yy] + P [yg] + P [gy] = 3/4. (2) Problem 1.4.6 Solution Define D as the event that a pea plant has two dominant y genes. To find the conditional probability of D when the event Y , corresponding to a plant with yellow seeds, P [D|Y] = P [DY] P [Y] is evaluated. (1) Note that P[DY] is only the probability of the genotype yy. From Problem 1.4.5 we found that in terms of the color of the peas, the genotypes yy, yg, gy and gg were all equally likely. This implies P [DY] = P [yy] = 1/4 P [Y] = P [yy, gy, yg] = 3/4. (2) Thus, the conditional probability can be considered P] = P [DY] P [Y] = 1/4 3/4 = 1/3. (3) 18 19. Problem 1.4.7 Solution The sample results can be written ijk, where the first drawn card is i, the second j and the third k. The sample space is S = 234, 243, 324, 342, 423, 432 . (1) (1) each of the six results has a probability of 1/6. Events E1, E2, E3, O1, O2, O3 are E1 = 234, 243, 423, 432, O1 = 324, 342 , (2) E2 = 243, 324, 342, 423 , O2 = 234, 432 , (3) E3 = 234, 324, 342, 432 , O3 = 243, 423 . (4) (a) The conditional probability that the second card is even present is P [E2| E1] = P [E2E1] P [E1] = P [243, 423] P [234, 243, 423, 432] = 2/6 4/6 = 1/2. (5) b) The conditional probability that the first card is even given is even P [E1| E2] = P [E1E2] P [E2] = P [243, 423] P [243, 324, 342, 423] = 2/6 4/6 = 1/2. (6) c) The probability that the first two cards will even receive the third card is even P [E1E2| E3] = P [E1E2E3] P [E3] = 0. (7) d) The conditional probabilities of the second card are even given if the first card is odd P [E2| O1] = P [O1E2] P [O1] = P [O1] P [O1] = 1. (8) (e) The conditional probability that the second card is odd because the first card is odd is P [O2| O1] = P [O1O2] P [O1] = 0. (9) 19 20. Problem 1.4.8 Solution The problem statement provides the obvious facts that P[L] = 0.16 and P[H] = 0.10. The words 10% of ticks that carried either Lyme disease or HGE both diseases can be written as P [LH| L ∪ H] = 0.10. (1) a) Since LH ⊂ L ∪ H, P [LH| L ∪ H] = P [LH ∩ (L ∪ H)] P [L ∪ H] = P [LH] P [L ∪ H] = 0.10. (2) Thus p [LH] = 0.10 P [L ∪ H] = 0.10 (P [L] + P [H] - P [LH]) . (3) Since P[L] = 0.16 and P[H] = 0.10, P [LH] = 0.10 (0.16 + 0.10) 1.1 = 0.0236. (4) b) The conditional probability that a tick has HGE because it has Lyme disease P [H| L] = P [LH] P [L] = 0.0236 0.16 = 0.1475. (5) Problem 1.5.1 Solution From the table we search to add all mutually exclusive events to find each prob ability. (a) The probability of that a caller does not cancel is P [H0] = P [LH0] + P [BH0] = 0.1 + 0.4 = 0.5. (1) b) The probability that a call is short is P [B] = P [BH0] + P [BH1] + P [BH2] = 0.4 + 0.1 + 0.1 = 0.6. (2) c) The probability, that a call is long or makes at least two branches, P [L ∪ H2] = P [LH0] + P [LH1] + P [LH2] + P [BH2] = 0.1 + 0.1 + 0.2 + 0.1 = 0.5. (3) 20 21. Problem 1.5.2 Solution (a) From the given probability distribution of the Minutes M is the probability that a call will be billed for more than 3 minutes, P [L] = 1 - P [3 or less billed minutes] = 1 - P [B1] - P [B2] - P [B3] = 1 - α - α(1 - α) α(1 - α)2 = (1 α)3 = 0.57 . (1) (b) The probability that a call will be billed for 9 minutes or less is P [9 minutes or less] = 9 i=1 α(1 - α)i-1 = 1 - (0.57)3 . (2) Problem 1.5.3 Solution (a) For convenience pi = P[FHi] and qi = P[V Hi]. With this shorthand, the six unknowns fill p0, p1, p2, q0, q1, the table as H0 H1 H2 F p0 p1 p2 V q0 q1 q2 . (1) However, we are given a number of facts: p0 + q0 = 1/3, p1 + q1 = 1/3, (2) p2 + q2 = 1/3, p0 + p1 + p2 = 5/12. 5/12. From these facts, other facts such as q0+q1+q2 = 7/12 can be derived. So we have four equations and six unknowns, the selection p0 and p1 will specify the other unknowns. Unfortunately, arbitrary decisions for p0 or p1 lead to 21 22. negative values for the other probabilities. With respect to p0 and p1, the other unknowns are q0 = 1/3 x p0, p2 = 5/12 (p0 + p1), (4) q1 = 1/3 , p1, q2 = p0 + p1 . (5) Since the probabilities do not have to be negative, we find that 0 ≤ p0 ≤ 1/3, (6) 0 ≤ p1 ≤ 1/3, (7) 1/12 ≤ p0 + p1 ≤ 5/12. (8) Although there are infinitely many solutions, three possible solutions are: p0 = 1/3, p1 = 1/12, p2 = 0, (9) q0 = 0, q1 = 1/4, q2 = 1/3. (10) and p0 = 1/4, p1 = 1/12, p2 = 1/12, (11) q0 = 1/12, q1 = 3/12, q2 = 3/12. (12) and p0 = 0, p1 = 1/12, p2 = 1/3 (13) q0 = 1/3, q1 = 3/12, q2 = 0. (14) b) With respect to pi, qi notation, the new facts are p0 = 1/4 and q1 = 1/6. These additional facts clearly indicate the probabilities. In this case p0 = 1/4, p1 = 1/6, p2 = 0, (15) q0 = 1/12, q1 = 1/6, q2 = 1/3. (16) 22 23. Solution This problem poses the question whether A and B can be independent events, but meet A = B? By definition, events A and B are independent if p[AB] = P[A] P[B]. We can see that if A = B, i.e. they are the same set, P [AB] = P [AA] = P [A] = P [B] . (1) For A and B so p [A] = P [AB] = P [A] P [B] = (P [A])2 . (2) There are two ways in which this requirement can be met: • P[A] = 1 implies A = B = S. • P[A] = 0 implies A = B = φ. Problem 1.6.2 Solution From the problem statement, we learn three facts: P [AB] = 0 (because A and B are mutually exclusive) (1) P [AB] = P [A] P [B] (because A and B are independent) (2) P [A] = P [B] (because A and B are equal) (3) If we apply these facts in the given order, we see that 0 = P [AB] = P [A] (4) It follows that P[A] = 0. Problem 1.6.3 Solution Let Ai and Bi denote the events that the sold Ith phone is an apricot or a banana. The works of each phone sold is twice as likely to be an apricot as a banana tells us that P [Ai] = 2 P [Bi] . (1) 23 24. However, since each phone sold is either an apricot or a banana, Ai and Bi are a partition and P [Ai] + P [Bi] = 1. (2) If we combine these equations, we have P[Ai] = 2/3 and P[Bi] = 1/3. The probability that two sold phones are the same is P [A1A2 ∪ B1B2] = P [A1A2] + P [B1B2] . (3) Since every telephone sale is independent, P [A1A2] = P [A1] P [A2] = 4 9 , P [B1B2] = P [B1] P [B2] = 1 9 . (4) Thus, the probability that two sold telephones are the same, P [A1A2 ∪ B1B2] = P [A1A2] + P [B1B2] = 4 9 + 1 9 = 5 9 . (5) Problem 1.6.4 Solution A B In the Venn diagram goes that the sample space has area 1 corresponding to the probability 1. As drawn, both A and B have the surface 1/4, so that P[A] = P[B] = 1/4. 1/4. the junction AB has the area 1/16 and comprises 1/4 of A and 1/4 of B. That is, A and B are independent because P [AB] = P [A] P [B] . (1) Problem 1.6.5 Solution a) Since A and B are mutually exclusive, P[A ∩ B] = 0 are mutually exclusive. Because P[A ∩ B] = 0, P [A ∪ B] = P [A] + P [B] - P [A ∩ B] = 3/8. (1) 24 25. A Venn diagram should convince you that A ⊂ Bc, so that A ∩ Bc = A. This implies P [A ∩ Bc] = P [A] = 1/4. (2) It also follows that P[A ∪ Bc] = P[Bc] = 1 x 1/8 = 7/8.b) Events A and B are dependent because P[AB] = P[A] P[B]. Problem 1.6.6 Solution (a) Since C and D are independent, P [C ∩ D] = P [C] P [D] = 15/64. (1) The next points are a bit more difficult. From Venn diagrams we see P [C ∩ Dc] = P [C] - P [C ∩ D] = 5/8 x 15/64 = 25/64. (2) It follows that P [C ∪ Dc] = P [C] + P [Dc] - P [C ∩ Dc] (3) = 5/8 + (1 x 3/8) - 25/64 = 55/64. (4) According to the DeMorgan Act, we have P [Cc ∩ Dc] = P [(C ∪ D)c] = 1 x P [C ∪ D] = 15/64. (5) b) Because P[CcDc] = P[Cc] P[Dc], Cc and Dc are independent. Problem 1.6.7 Solution (a) Because A ∩ B = ∅, P[A ∩ B] = 0. To find P[B], we can write P [A ∪ B] = P [A] + P [B] - P [A ∩ B] (1) or 5/8 = 3/8 + P [B] - 0. (2) Thus p[B] = 1/4. Since A is a subset of Bc, P[A is ∩ Bc] = P[A] = 3/8. Since A is a subset of Bc, P[A is ∪ Bc] = P[Bc] = 3/4. 25 26.b) Events A and B are dependent because P [AB] = 0 = 3/32 = P [A] P [B] . (3) Problem 1.6.8 Solution (a) Since C and D are independent P[CD] = P[C] P[D]. So P [D] = P [CD] P [C] = 1/3 1/2 = 2/3. (1) In addition, P[C ∩ Dc] = P[C] - P[C ∩ D] = 1/2 x 1/3 = 1/6. To find P[Cc ∩ Dc], we first find that P [C ∪ D] = P [C] + P [D] - P [C ∩ D] = 1/2 + 2/3 - 1/3 = 5/6. (2) According to De Morgan's law, Cc ∩ Dc = (C ∪ D)c. This implies P [Cc ∩ Dc] = P [(C ∪ D)c] = 1 x P [C ∪ D] = 1/6. (3) Note that a second way to find P ∩ Dc] is to use the fact that if C and D are independent, then Cc and Dc are independent. So P [Cc ∩ Dc] = P [Cc] P [Dc] = (1 x P [C])(1 - P [D]) = 1/6. (4) Since C and D are independent events, p[C|D] = P[C] = 1/2. (b) Note that we found P[C ∪ D] = 5/6. We can also use the previous results to display P [C ∪ Dc] = P [C] + P [D] - P [C ∩ Dc] (5) = 1/2 + (1 x 2/3) - 1/6 = 2/3. (6) (c) According to definition 1.6, events C and Dc are independent because P [C ∩ Dc] = 1/6 = (1/2)(1/3) = P [C] P [Dc] . (7) 26 27. Problem 1.6.9 Solution For a sample space S = 1, 2, 3, 4 with equiprobable results, consider the events A1 = 1, 2, A2 = 2, 3 , A3 = 3, 1 . (1) Each Event Ai has the probability of 1/2. In addition, each event pair since P [A1A2] = P [A2A3] = P [A3A1] = 1/4 is independent. (2) However, the three events A1, A2, A3 are not independent because P [A1A2A3] = 0 = P [A1] P [A2] P [A3] . (3) 1.6.10 Solution There are 16 different, equally probable probable for the second generation of pea plants based on a first generation of rwyg, rwgy, wryg, wrgy. These are: rryy rryg rrgy rrgg rwyy rwyg rwgy rwgy rwgg wryy wryg wrgy wryg wrgy wwyg wwyg wwgy wwgg A plant has yellow seeds, i.e. Event Y occurs when a plant has at least one dominant y-gene. Apart from the four results with a pair of recessive g-genes, the remaining 12 results have yellow seeds. From the above we see that P [Y] = 12/16 = 3/4 (1) and P [R] = 12/16 = 3/4. (2) To find the conditional probabilities P[R| Y] and P[Y | R] we have to find P[RY] first. Note that RY , the event that a plant has rounded yellow seeds, is the set of results RY = rryy, rryg, rrgy, rwyy, rwyg, rwgy, wryy, wryg, wryg, wrgy . (3) 27 28. Da P[RY] = 9/16, P [Y | R] = P [RY] P [R] = 9/16 3/4 = 3/4 (4) and P [R | Y] = P [RY] P [Y] = 9/16 3/4 = 3/4. (5) Thus P[R| Y] = P[R] and P[Y | R] = P[Y] and R and Y are independent events. There are four visibly different pea plants that correspond to the question whether the peas are round (R) or not (Rc) or yellow (Y) or not (Y c). These four visible events have probable P [RY] = 9/16 P [RY c] = 3/16, (6) P [Rc Y] = 3/16 P [Rc Y c] = 1/16. (7) Problem 1.6.11 Solution (a) For all events A and B we can write the law of total probability in the form of P [A] = P [AB] + P [ABc]. (1) Since A and B are independent, P[AB] = P[A] P[B]. This implies P [ABc] = P [A] - P [A] P [B] = P [A] (1 x P [B]) = P [A] P [Bc] . (2) Thus, A and Bc are independent. b) Proof that Ac and B are independent is not really necessary. Since A and B are arbitrary terms, it is really the same claim as in part (a). In other words, simply reversing the labels of A and B proves the claim. Alternatively, you can construct exactly the same proof as in part (a) with inverted labels A and B. (c) To prove that Ac and Bc are independent, we apply the result of Part a) to sentences A and Bc. Since we know from part (a) that A and Bc are independent, Part b says that Ac and Bc are independent. 28 29. Problem 1.6.12 Solution A AC AB ABC C BCB In the Venn diagram on the right, it is assumed that the sample space has area 1 corresponding to the probability 1. As drawn, a, B, and C each have the area 1/2 and thus the probability of 1/2. In addition, the three-way junction ABC probably has 1/8. Thus, A, B and C are independent of each other because P [ABC] = P [A] P [B] P [C] . (1) Problem 1.6.13 Solution A AB B AC C BC In the Venn diagram on the right, it is assumed that the sample space has area 1 corresponding to the probability 1. As drawn, A, B, and C each have the area 1/3, and thus the probability of 1/3. The ABC has zero probability, which means that A, B, and C are not independent of each other because P [ABC] = 0 = P [A] P [B] P [C] . (1) Ab, BC and AC each have area 1/9. Therefore, each event pair is independent because P [AB] = P [A] P [B] , P [BC] = P [B] P [C] , P [AC] = P [A] P [C] . (2) Problem Solution We can use the 200 × 1 vector T, t denoted in Matlab, via the command T=50+ceil(50*rand(200,1)) Remember that 50*rand(200,1) generates a 200×1 vector of random numbers, each at the interval (0, 50). Applying the ceiling function converts these random numbers to rndom integers in the set of 1, 2, ... , 50 . Finally, we add 50 to generate random numbers between 51 and 100. 29 Probability and Stochastic Processes A friendly introduction for Electric and Com Full Download: This is just example, download all chapters at: AlibabaDownload.com AlibabaDownload.com