Power tower puzzle math

Math Game or Puzzle This article is about the math disk game. For the card game, see Hanoy Tower. For other properties in the city such as Keangnam Hanoi Landmark Tower, it offers quality accommodation and great parking. A set of models of the Tower of Hanoi (with 8 discs) An animated solution of the Hanoi Tower puzzle for the T(4, 3) Tower of Hanoi interactive display at the Universum museum in Mexico City The Tower of Hanoi (also called Brahma Tower or Lucas Tower[1] and sometimes pluralized as Torres) is a mathematical game or puzzle. It consists of three rods and a series of discs of different sizes, which can slide on any rod. The puzzle begins with the discs in a polished pile in ascending order of size on a rod, the smallest at the top, thus making a conical shape. The goal of the puzzle is to move the entire pile to another rod, obeying the following simple rules: You can only move one disk at a time. Each move involves taking the top disk from one of the piles and placing it on top of another pile or an empty rod. A larger disk cannot be placed on top of a smaller disk. With 3 discs, the puzzle can be solved in 7 moves. The minimum number of moves needed to solve a Hanoi tower puzzle is 2nd − 1, where n is the number of disks. Origins The puzzle was invented by the French mathematician Édouard Lucas in 1883. Numerous myths about the ancient and mystical nature of the puzzle emerged almost immediately. [2] There is a story about an Indian temple in Kashi Vishwanath that contains a large room with three time-worn locations, surrounded by 64 golden discs. Brahmin's priests, acting in command of an ancient prophecy, have been moving these records according to Brahma's immutable rules ever since. The puzzle is therefore also known as the Tower of Brahma jigsaw puzzle. According to legend, when the last movement of the puzzle is completed, the world will end. [3] If the legend were true, and if priests were able to move discs at a rate of one per second, using the least number of moves it would take them 264 - 1 seconds or approximately 585 billion years to finish,[4] which is about 42 times the current age of the universe. There are many variations in this legend. For example, in some narratives the temple is a monastery, and priests are monks. It can be said that the temple or monastery is located in different parts of the world –including Hanoi, Vietnam- and can be associated with any religion. In some versions other elements are introduced, such as the fact that the tower was created at the beginning of the world, or that priests or monks can make only one move per day. you can play with any number of disks, although many versions of the dough have about 7 to 9 of them. The minimum number of moves needed to solve a Hanoi tower puzzle is 2nd − 1, where n is the number of disks. [5] This is precisely the enesia Mersenne Mersenne Iterative solution Animation of an iterative algorithm solving the problem of 6 discs A simple solution for the kitchen puzzle is to alternate movements between the smaller piece and a no smaller piece. When moving the smaller tile, always move it to the next position in the same direction (right if the starting number of pieces is even, left if the starting number of pieces is unknown). If there is no tower position in the chosen direction, move the piece to the opposite end, but then continue moving in the right direction. For example, if I started with three pieces, it would move the smaller piece to the opposite end, then it would continue in the left direction after that. When the turn is to move the piece no smaller, there is only one legal move. Doing so will complete the puzzle in the least moves. [6] Simplest declaration of iterative solution For an even number of records: make the legal move between pegs A and B (in any direction), make the legal move between pegs A and C (in any direction), make the legal move between pegs B and C (in any direction), repeat until complete. For an odd number of records: make the legal move between pegs A and C (in any direction), make the legal move between pegs A and B (in any direction), make the legal move between pegs B and C (in any direction), repeat until complete. In each case, a total of 2nd - 1 movements are made. Equivalent iterative solution Another way to generate the single optimal iterative solution: Numbering disks 1 to n (larger to smaller). If n is strange, the first movement is from clamp A to clamp C. If n is even, the first move is from peg A to peg B. Now, add these constraints: No strange disk can be placed directly on an odd disk. No uniform disk can be placed directly on a uniform disk. Sometimes there will be two possible pegs: one will have discs, and the other will be empty. Non-empty disk placements on the clamp. Never move a record twice in a row. Given these restrictions after the first move, there is only one legal move at every subsequent turn. The sequence of these unique movements is an optimal solution to the problem equivalent to the iterative solution described above. [7] Recursive Solution Illustration of a recursive solution for hanoi towers puzzles with 4 disks The key to solving a problem recursively is to recognize that it can be broken down into a collection of smaller subproblemes, to each of which applies this same general resolution procedure that we are looking for, and the total solution is somehow simple from the solutions of these subproblemas. Each of the subproblemes created to be smaller that the base cases will finally be reached. From there, for the Towers of Hanoi: label the pegs A, B, C, we will be the total number of discs, number the discs from 1 (smaller, higher) to n (larger, lower). Supposing all n disks are in valid arrangements between pegs; assuming there are m upper disks on a source clamp, and all other disks are larger than m, so they can be safely ignored; to move discs from a source clamp to a target clamp using a spare clamp, without violating the rules: Move m − 1 discs from the source to the spare clamp, by the same general resolution procedure. The rules are not violated, by supposition. This leaves the disk m as a disk higher than the source clipboard. Move the m disk from the source to the target clamp, which is guaranteed to be a valid move, by the assumptions - a simple step. Move the m − 1 disks that we just placed on the spare part, from the spare part to the target clamp by the same general resolution procedure, so that they were placed at the top of the disk m without violating the rules. The base case is to move 0 discs (in steps 1 and 3), i.e. do nothing – it obviously doesn't violate the rules. The complete Hanoi tower solution then involves moving n discs from the source of clamp A to the target clamp C, using B as the spare clamp. This approach can be given a rigorous mathematical test with mathematical induction and is often used as an example of recursion when teaching programming. Logical analysis of recursive solution As in many mathematical puzzles, finding a solution becomes easier by solving a slightly more general problem: how to move an h disk tower (height) from an initial clamp f = A (de) on a target clamp t =C(a), B being the third remaining clamp and assuming t ≠ f. First, note that the problem is symmetrical for peg name permutations (S3 symmetrical group). If a solution is known to go from clamp A to clamp C, then, by renaming the pegs, the same solution can be used for any other starting and target clamp option. If there is only one disk (or even none), the problem is trivial. If h = 1, then simply move the clip disk A to clamp C. If h > 1, then somewhere along the sequence of movements, the larger disk must be moved from clamp A to another clamp, preferably in clamp C. The only situation that allows such a movement is when all the smaller h − 1 discs are in clamp B. Therefore the only situation that allows this movement is when all the smaller h − 1 discs are in clamp B. , first, all smaller h − 1 discs must go from A to B. Then move the larger disk and finally move the smaller h − 1 disks from clamp B to clamp C. The presence of the larger disk does not prevent any movement of the smaller h − 1 disks and can be temporarily ignored. Now the problem is reduced to moving h − 1 discs from one clip to another, first from A to B and then from B to C, but the same method can be used both times by renaming the pegs. The strategy can be used to reduce the problem h − 1 to h − 2, h − 3, and so on until there is only one disk left. This is called recursion. This algorithm can be schematized as follows. Identify disks in order to increase the size by natural numbers from 0 to but not including h. h. Disk 0 is the smallest, and the disk h − 1 the largest. The following is a procedure to move an h-disc tower from a clamp A to a C clamp, with B being the third remaining clamp: If h > 1, then first use this procedure to move the h − 1 smaller discs from clamp A to clamp B. Now the larger disk, i.e. h disk can be moved from peg A to peg C. If h > 1, use this procedure again to move the h − 1 smaller disks from clamp B to clamp C. By mathematical induction, it is easily demonstrated that the above procedure requires the least number of possible movements, and that the solution produced is the only one with this minimum number of movements. Using recurrence relationships, the exact number of moves required by this solution can be calculated by: 2 h − 1 {\displaystyle 2^{h}-1} . This result is obtained by pointing out that steps 1 and 3 take T h − 1 {\displaystyle T_{h-1}} moves, and step 2 makes a move, giving T h = 2 T h − 1 + 1 {\displaystyle T_{h}=2T_{h-1}+1} . Recursive implementation The following Python code highlights an essential function of the recursive solution, which may otherwise be misunderstood or ignored. That is, with all recursion levels, the first recursive call reverses the target and auxiliary stacks, while in the second recursive call the source and auxiliary stacks are reversed. A= [3, 2, 1] B = [] C = [] def move(n, source, target, wizard): if n > 0:# Move n - 1 source disks to wizard, so they are out of the way move(n - 1, source, wizard, target) # Move the disk first from source code to target.append(source.pop()) # Show our progress printing (A, B, C, '##################', sep=')# Move n - 1 discs we leave auxiliary destination moving(n - 1, auxiliary, target, source) # Start the call from source A to target C with auxiliary movement B (3, A C, B) The following code implements more recursive functions for a text-based animation: import time A = [and for and in range(5 , 0,-1)] height = len(A) - 1 # Stable height value for animation B = [] C = [] def move(n , font, goal, wizard): if n > 0: # Move n - 1 source disks to wizard, so they are out of the way move(n - 1, source, wizard, target) # Move the source nth disk to target.append(source.pop()) # Show our progress using a recursive function to draw it draw_disks(A, B, C, height) print() # Provide spacing time.sleep(0.3) # Pause for a moment to cheer # Move n - 1 discs we left in auxiliary in destination motion(n - 1, auxiliary, target, font) def draw_disks(A, B, C, position, width=2* int(max(A))): # default width parameter to duplicate the largest size of the starting disk tower. if the position >= 0: # If A has a value in the list at the given position, create disc a la seva posició (alçada) valueInA = si la posició >= len(A) else create_disk(A[position]) # Same for B and C valueInB = if position position len(B) else create_disk(B[position]) valueInC = if the position >= len(C) else create_disk(C[position]) # Print each row letter ({0:^{width}}{1:^{width}}{2:^{width}}.format(valueInA, valueInB, valueInC, width=width)) # Recursively re-call this method to the next position (height) draw_disks (A, B, C, position - 1, width) plus: # When done recursively, print print column labels ({0:^{width}}{1:^{width}}{2:^{width}}.format(A, B, C, width=width)) def create_disk(size): Simple recursive method to create a disk tilt. /\\else: return/ + create_disk(size - 1) + \\ # Start call from source A to target C with auxiliary movement B (len(A), A, C, B) Non-recursive solution The list of moves for a tower that is carried from one clamp to another , as the recursive algorithm produces, has many regularities. When counting the movements from 1, the ordinal of the disk to move during movement m is the number of times that m can be divided by 2. Therefore, each strange move involves the smaller disk. It can also be observed that the smaller disk crosses the pegs f, t, r, f, t, r, etc. for the strange height of the tower and crosses the pegs f, r, t, f, r, t, etc. for even the height of the tower. This provides the following algorithm, which is easier, carried out by hand, than the recursive algorithm. In alternate moves: Move the smaller disk to the clamp it didn't come from recently. Move another disk legally (there will only be one possibility). For the first move, the smaller disk goes to peg t if h is weird and to peg r if h is even. Also note that: The discs whose ordinals have even parity move in the same sense as the smaller disk. The disks whose ordinals have a strange parity move in the opposite direction. If h is even, the third remaining clamp during successive movements is t, r, f, t, r, f, etc. If h is strange, the third remaining clamp during successive movements is r, t, f, r, t, f, etc. With this knowledge, you can recover a set of disks in the middle of an optimal solution without more status information than the positions of each disk: Call the detailed movements on the natural movement of a disk. Examine the smaller upper disk that is not disk 0 and consider what would be your only (legal) move : if there is no such disk, then we are in the first or last move. If that move is the natural disk movement, then the disk has not been moved since the last disk move 0, and that move should be taken. If this move is not the natural disk movement, move disk 0. Binary solution The disk positions can be determined more directly from the binary representation (base-2) of the movement number (the initial state is moving #0, with digits 0, and the final state is with all digits 1), using the following rules: There is a binary digit (bit) for each disk. The most significant bit (further to the left) represents the largest disk. A value of 0 indicates that the disk is in the initial clamp, while a 1 indicates that it is in the final clamp (right peg if number of discs is strange and medium clamp otherwise). Bitstring is read from left to right, and each bit can be used to determine the location of the corresponding disk. Somewhat with the same value as the previous one means that the corresponding disk is stacked at the top of the disk above the same clamp. (That is: a straight sequence of 1 or 0s means that the corresponding disks are all in the same clamp.) Somewhat with a value other than the previous one means that the corresponding disk is a position to the left or right of the previous one. If left or right is determined by this rule: Let's say the initial clamp is on the left. Also assume wrapping – so that the right clamp counts as a left clamp of the left clamp, and vice versa. Let n be the number of larger disks found in the same clamp as your first largest disk and add 1 if the larger disk is on the left clamp. If n is even, the disk is a clamp on the right, if n is strange, the disk located a clamp on the left (in case of even number of disks and vice versa otherwise). For example, in an 8-disk Hanoi: Move 0 = 000000000. The largest disk is 0, so it is on the left (initial) clamp. All other disks are 0 too, so they are stacked on top of it. Therefore, all disks are in the initial clipboard. Movement 28 − 1 = 11111111. The largest disk is 1, so it's in the middle (end) clamp. All other disks are 1 too, so they are stacked on top of it. Therefore, all disks are in the final clip and the puzzle is complete. Move 21610 = 11011000. The largest disk is 1, so it's in the middle (end) clamp. The second disc is also 1, so they are stacked on top of it, on the center clamp. Disk three is 0, so it's on another clamp. Since n is strange (n = 1), it is a clamp on the left, i.e. on the left clamp. Disk four is 1, so it's on another clamp. Since n is strange (n = 1), it is a clamp on the left, that is, in the right clamp. Disk five is also 1, so they are stacked on top of it, on the right clamp. Disk six is 0, so it's in another clamp. Since n is even (n = 2), the disk is a clamp on the right, that is, on the left clamp. Disks seven and eight are also 0, so they are stacked on top of it, on the left clamp. The source and target pegs for the mth movement can also be found elegantly from the binary representation of m using bitwise operations. To use the C programming language syntax, move m is peg (m & m - 1) %3 to peg ((m | m - 1) + 1) %3, where disks start with clamp 0 and end in clamp 1 or 2 seconds if the number of disks is even or Another formulation is peg (m - (m and -m)) % 3 in peg (m + (m and -m)) % 3. In addition, the disk to move is determined by the number of times the movement count (m) can be divided by 2 (i.e. the number of zero bits on the right), counting the first movement as 1 and identifying the numbers 0, 1, 2 etc. in order of increase in size. This allows very fast non-recursive computer support to find disk positions after M moves without reference to any previous movement or disk distribution. The operation, which counts the number of consecutive zeros at the end of a binary number, gives a simple solution to the problem: the disks are numbered from scratch, and when moving m, the count of disk numbers that follows zeros moves the minimum possible distance to the right (turning left as needed). [8] Gray code solution The gray code binary numeral system gives an alternative way to solve the puzzle. In the Gray system, numbers are expressed in a binary combination of 0s and 1s, but instead of being a standard positional numbering system, the Gray code operates under the premise that each value differs from its predecessor by only one (and exactly one) bit changed. If one counts gray code of a bit size equal to the number of disks in a particular Hanoi Tower, starts at zero, and counts up, then the changed bit each move corresponds to the disk to move, where the least significant bit is the smallest disk, and the most significant bit is the largest. Counting the movements of 1 and identifying the disks by numbers from 0 in order to increase the size, the ordinal of the disk that will move during movement m is the number of times that can be divided by 2. This technique identifies which disk to move to, but not where to move it. For the smaller disk there are always two possibilities. For the other disks there is always a possibility, except when all the disks are in the same clamp, but in this case it is either the smallest disk to be moved or the target has already been achieved. Fortunately, there is a rule that says where to move the smallest disk to. Let f be the output clamp, t the target clamp, and are the third remaining clamp. If the number of disks is unknown, the smallest disk cycles along the pegs in the order f → t → r → f → t → r, etc. If the number of disks is uniform, this must be reversed: f → r → t → f → r → t, etc.[9] The position of the bit change in the Gray code solution gives the size of the moved disk at each step: 1, 2, 1, 3, 1, 2, 1, 4, 1, 2, 1, 3, 1, 2, 1, ... (sequence A001511 in the OEIS),[10] a sequence also known as the ruler function, or one more than the power of 2 within the number of movements. In the wolfram language, IntegerExponent[Range[2^8 - 1], 2] + 1 gives moves for the 8-disk puzzle. Graphical representation Main article: Hanoi graphic The game can be represented by an undirected graph, nodes representing disk distributions, and borders representing movements. For a disk, the graph is a triangle: the two-disk chart is three connected triangles to form the of a larger triangle. A second letter is added to represent the largest disk. It is clear that initially move. The highest small triangle now represents the possibilities of a single movement with two disks: The nodes in the vertices of the outer triangle represent distributions with all disks on the same clamp. For h + 1 discs, take the chart from h disks and replace each small triangle with the chart with two disks. For three disks the chart is: name the tweezers a, b and c lists the disk positions from left to right in order to increase the size The sides of the outer triangle represent the shortest ways to move a tower from one clip to another. The border in the center of the sides of the largest triangle represents a larger disk movement. The edge in the middle of the sides of each next smaller triangle represents a movement of each next smaller disk. The sides of the smaller triangles represent movements of the smaller disk. The level 7 game graph shows the relationship with the Sierpiński triangle. In general, for a puzzle with n disks, there are 3rd nodes in the chart; each node has three edges to other nodes, except for three corner nodes, which have two: it is always possible to move the smaller disk to one of the other two pegs, and it is possible to move a disk between these two pegs except in the situation where all disks are stacked on a single clamp. Corner nodes represent the three cases in which all disks are stacked on a single clipboard. The n+ 1 disk chart is obtained by taking three copies of the disk diagram n —each representing all the states and movements of the smaller disks for a particular position on the new larger disk— and joining them to the corners with three new borders, representing the only three opportunities to move the larger disk. The resulting figure thus has 3n+1 nodes and still has three corners remaining with only two edges. As more disks are added, the game's graphical representation will resemble a fractal figure, the Sierpiński triangle. Clearly, the vast majority of positions in the puzzle will never be reached when using the shortest possible solution; in fact, if the priests of legend are using the longest possible solution (without revisiting any position), it will take them 364 − 1 movements, or more than 1023 years. The longest non-repetitive mode for three disks can be displayed by clearing unfuled borders: By the way, this longer non-repetitive path can be obtained by prohibiting all movements from one to b. The Hamiltonian cycle for three discs is: The graphics clearly show that: From each arbitrary distribution of disks, there is exactly one shorter way to move all disks into one of the three pegs. Between each pair of arbitrary disk distributions there are one or two different shorter paths. From each arbitrary disk distribution, there are one or two more non-self-crossed paths to move all disks to one of the three pegs. Between each pair of arbitrary disks there are one or two different paths without longer self-crossing. Let Nh be the number of non-self-crossing paths to move an h-disk tower from one clamp to another. Then: N1 = 2 Nh +1 = (Nh)2 + (Nh)3 This gives Nh to be 2, 12, 1872, 6563711232, ... (sequence A125295 in the OEIS) Adjacent peg variations If all movements must be between adjacent pegs (i.e. given the pegs A, B, C, it cannot move directly between pegs A and C), then move a stack of n discs from clamp A to clamp C takes 3rd − 1 movement. The solution uses all valid 3rd positions, always taking the unique move that does not undo the previous move. The position with all the discs in clamp B is reached in half, that is, after (3rd − 1) / 2 movements. [citation needed] Cyclic Hanoi A Cyclic Hanoi, we are given three pegs (A, B, C), which are arranged as a circle with clockwise and counterclockwise directions that are defined as A – B – C – A and A – C – B – A respectively. The moving direction of the disk must be clockwise. [11] It is enough to represent the sequence of disks to move. The solution can be found using two recursive procedures each other: To move n disks counterclockwise to the neighboring destination clamp: move n − 1 counterclockwise disk to the target clamp motion disk #n a clockwise step move n − 1 clockwise discs at the beginning peg move disk #n step clockwise move n − 1 clock counter clock clockwise to the pinça de punt To move n disks clockwise to the neighbouring destination clamp: move n − 1 counterclockwise discs to #n spare clamp motion disk #n a clockwise step move n − 1 counterclockwise discs to the target clamp Let C(n) and A(n) represent moving n disks clockwise and counterclockwise , then we can type the two formulas: C(n) =A(n-1) n A(n-1) and A(n) =A(n-1) n C(n-1) n A(n-1). So C(1) = 1 and A(1) = 1 1, C(2) = 1 1 2 1 1 and A(2) = 1 1 1 2 1 2 1 1. The solution for Hanoi Cyclical has some interesting properties: 1)The motion patterns of transferring a disc tower from one clamp to another clamp are symmetrical in terms of the central points. 2) The smallest disk is the first and last disk to move. 3) Smaller disk groups move toggle with individual movements from other disks. 4) The number of disk movements specified by C(n) and A(n) are minimal. With four pegs and beyond Although the three-peg version has a simple recursive solution long has been known, the optimal solution to the Hanoi Tower problem with four pegs (called reve puzzle) was not verified until 2014, by Bousch. [12] However, in case of four or more pegs, the Frame-Stewart algorithm is known without an optimum test 1941. [13] For the formal derivation of the exact number of minimum movements needed to solve the problem by applying the Frame-Stewart algorithm (and other equivalent methods), see the following document. [14] For other variants of the four-peg Hanoi Tower problem, see baked paper. [15] The Frame-Stewart algorithm Frame-Stewart is described below: Let n {\displaystyle n} be the number of disks. Let r {\displaystyle r} be the number of pegs. Set T (n , r) {\displaystyle T(n,r)} as the minimum number of moves needed to transfer n disks using r pegs. The algorithm can be recursively described: For some k {\displaystyle k} , 1 ≤ k < n {\displaystyle 1\leq k<n} , transfer the upper {\displaystyle k} disks to a single clamp other than the start or destination pegs, taking T (k , r) {\displaystyle T(k,r)} movements. Without disturbing the clamp that now contains the top k {\displaystyle k} disks, transfer the remaining n − k {\displaystyle n-k} disks to the target clamp, using only the remaining moves r − 1 {\displaystyle r-1}, taking T (n − k , r − 1) {\displaystyle T(n-k,r-1)} Finally, transfer the top k {\displaystyle k} disks to the target clipboard, taking T (k , r) moves {\displaystyle T(k,r)}. The whole process takes 2 T (k , r) + T (n − k , r − 1) {\displaystyle 2T (k, r) + T (n-k, r-1)} moves. Therefore, you must choose the count k {\displaystyle k} for which this amount is minimum. In the case of 4 pegs, the optimum k {\displaystyle k} equals n − ⌊ 2 n + 1 ⌉ + 1 {\displaystyle n-\left\lfloor {\sqrt {2n+1}}\right\rceil +1} , where ⌊ · ⌉ {\displaystyle \left\lfloor \cdot \right\rceil } is the closest integer function. [16] For example, in the UPenn CIS 194 course in Haskell, the first page of allocation[17] lists the optimal solution for the case of 15 disks and 4-peg as 129 steps, which is obtained for the previous value of k. This algorithm is presumed to be optimal for any number of pegs; number of movements is 2Θ(n1/(r−2)) (per fixed r). Shorter general paths and number 466/885 A curious generalization of the original goal of the puzzle is to start from a certain configuration of disks where all disks are not necessarily in the same clamp, and arrive in a minimum number of moves in another given configuration. In general, it can be quite difficult to calculate a shorter sequence of moves to solve this problem. Andreas Hinz proposed a solution, and it is based on the observation that in a shorter sequence of moves, the larger disk to be moved (obviously you can ignore all the larger disks that will occupy the same clamp in both the initial and final configurations) will move exactly once or exactly twice. Mathematics related to this widespread problem is further interesting when considering the average number of movements in a shorter sequence of movements between two initial and end-of-disk configurations that are chosen at random. Hinz and Chan Tat-Hung discovered from independent[18][19] (see also [20]:Chapter 1, p. 14) that the average number of movements in a disk tower n is given by the following exact formula: 466 885 · 2 n − − 3 − 3 5 · (1 3) n + (12 59 + 18 1003 17) (5 + 17 18) n + (12 59 − 18 1003 17) (5 − 17 18) n . {\displaystyle {\frac {466}{885}}\cdot 2^{n}-{\frac {1}{3}}-{\frac {3}{5}}\cdot \left({\frac {1}{3}}\right)^{n}+\left({\frac {12}{59}}+{\frac {18}{1003}}{\sqrt {17}}\right)\left({{ \frac {12}{59}}\{\frac {18}{1003}}{\sqrt {17}}\right)\left(left({\frac {12}{59}}\{\frac {18}{1003}}{\sqrt {17}}\right)\left(left({\frac {12}{59}}\{\frac {18}{1003}}{\sqrt {17}}\right)\left(left({\frac {12}{59}}\{\frac {18}{1003}}{\sqrt {17}}\right)\left(left({\frac {12}{59}}\{\frac {18}{1003}}{\sqrt {17}}\right)\left(left({\frac {12}{59}}\{\frac {18}{1003}}\{\sq {\frac {5+{\sqrt {17}}}{18}}\right)^{n}+\left({\frac {12}{59}}-{\frac {18}{1003}}{\sqrt {17}}\right)\left({\frac {5-{\sqrt {17}}}{18}}\right)^n}.} For n big enough, only the first and second terms do not converge to zero, so we get an asymptotic expression: 466 / 885 · 2 n − 1 / 3 + or (1) {\displaystyle 466 / 885 \ cdot 2^{n}-1/3 +o(1)} , as n → ∞ \\displaystyle n\to \infty } So, intuitively, we could interpret the fraction of 466 / 885 ≈ 52.6 % {\displaystyle 466/885\approx 52.6\%} as a representative of the relationship of the work to be carried out when moving from a chosen configuration randomly to another randomly chosen configuration, in relation to the difficulty of having to cross the most difficult path of length 2 n − 1 {\displaystyle 2^{n}-1} which involves moving all disks from one clamp to another. An alternative explanation for the emergence of the constant 466/885, as well as a new and somewhat improved algorithm to calculate the shorter path, was given by Romik. [21] Magnetic Hanoi Main article: Hanoi Magnetic Tower in Hanoi Magnetic Tower, each disk has two different north and south sides (typically red and blue). Disks should not be placed with similar poles together: magnets on each disk prevent this illegal movement. In addition, each disk must be rotated as it moves. Hanoi's initial two-color tower configuration (n =4) Hanoi Bicolor Towers This variation of the famous Hanoi Tower puzzle was offered to undergraduate students 3-6 to 2ème Championnat de France des Jeux Mathématiques et Logiques held in July 1988. [22] Final configuration of Hanoi's two-color towers (n =4) Puzzle rules are essentially the same: disks are transferred between pegs one at a time. At no time can a larger disk be placed on top of a smaller disk. The difference is that now for each size there are two discs: one black and one white. In addition, there are now two towers of alternating coloured discs. The goal of the puzzle is to make the towers monochrome (same color). Larger disks at the bottom of the towers are supposed to exchange positions. Hanoy Tower A variation of the puzzle adapted as a solitaire game with nine game cards under the name Hanoy Tower. [23] It is not known whether the altered spelling of the original name is deliberate or accidental. [25] AFM 3D topographic image applications of multilayer palladium nanotalic on silicon wafer, with structure similar to the Tower of Hanoi. [26] Hanoi Tower is frequently used in psychological research on problem solving. There is also a variant of this task called Torre de neuropsychological diagnosis and treatment of executive functions. Zhang and Norman[27] used various isomorphic representations (equivalents) of the game to study the impact of the representative effect on task design. They demonstrated an impact on user performance by changing the way the rules of the game are represented, using variations in the physical design of the game's components. This knowledge has impacted the development of the TURF framework[28] for the representation of human-computer interaction. Hanoi Tower is also used as a backup rotation scheme when backing up computer data where various tapes/media participate. As mentioned above, Hanoi Tower is popular for teaching recursive algorithms to start programming students. A pictorial version of this puzzle is programmed in the emacs editor, which is accessed by typing M-x hanoi. There is also a sample algorithm written in Prolog. Hanoi Tower is also used as a test by neuropsychologists trying to assess frontal lobe deficits. [29] In 2010, researchers published the results of an experiment that found that the Linepithema humile ant species was able to successfully solve the 3-disk version of the Hanoi Tower problem through nonlinear dynamics and ionone signals. [30] In 2014, scientists synthesized multilayer palladium nanoslanmines with a Hanoi tower as a structure. [26] In popular culture In the science fiction story Now Inhale, by Eric Frank Russell,[31] a human is a prisoner on a planet where local custom is to make the prisoner play a game until it is won or lost before its execution. The protagonist knows that a rescue ship may take a year or more to arrive, so he chooses to play Hanoi towers with 64 discs. (This story refers to the legend about Buddhist monks playing the game to the end of the world.) In Doctor Who's 1966 story The Celestial Toymaker, the self-titled villain forces the Doctor to perform a ten-piece game of 1,023 Hanoi Tower moves titled The Trilogic Game with the pieces forming a pyramid shape when stacked. In 2007, the concept of the Towers of Hanoi problem was used in Professor Layton and the diabolical box in puzzles 6, 83 and 84, but the discs had been changed to pancakes. The puzzle was based on a dilemma where the chef of a restaurant had to move a stack of pancakes from one plate to the other with the basic principles of the original puzzle (i.e. three dishes that pancakes could move, not being able to put a larger pancake on a smaller one, etc.) In the film Rise of the Planet of the Apes (2011), this puzzle, in the film Lucas Tower, is used as evidence to study the intelligence of apes. The puzzle is regularly presented in adventure games and puzzles. Since it is easy to implement, and easily recognized, it is suitable for use as a puzzle in a (e.g. Star Wars: Knights of the Old Republic and Mass Effect). [32] Some implementations use straight disks, but others disguise the puzzle in some other way. There is an arcade version of Sega. [33] A 15-disc version of the puzzle appears in the Sunless Sea game as a lock to a tomb. The player has the option to click through each puzzle move in order to solve it, but the game points out that it will take 32767 moves to complete. If a specially dedicated player clicks to the end of the puzzle, it is revealed that completing the puzzle does not open the door. A Yu-Gi-Oh! VRAINS, a hacking group called Knight of Hanoi create a structure called Hanoi Tower within the VRAINS virtual reality network of the same name. This was first used as a challenge to survivor Thailand in 2002, but instead of rings, the pieces were made to resemble a temple. Sook Jai launched the challenge of getting rid of Jed even though Shii-Ann knew full well how to complete the puzzle. The issue appears as part of a reward challenge in a 2011 episode of the US version of the Survivor TV series. Both players (Ozzy Lusth and Benjamin Coach Wade) struggled to understand how to solve the puzzle and are helped by their fellow members of the tribe. See also part of a series onPuzzles Types Guessing Riddle Situation Logic Dissection Induction Logic Grid Self-reference Mechanical Combination Construction Disentanglement Lock Go Problems Folding Stick Tiling Tour Sliding Chess Maze Mazes Metapuzzles Themes Brain Teaser Dilemma Optical Illusion Problems Packing Paradox Problem solve syllogism Puzzlehunt Thinking outside the box Lists Impossible Puzzle video games Nikoli puzzle type puzzle games Puzzle themes vte ABACABA pattern Rotation scheme a TOH Baguenaudier Recursion application (computer science) Notes

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