Integration by substitution worksheet answers

 Integration U Substitution With Answers - Displays the top 8 spreadsheets found for this concept. Some spreadsheets for this concept are Integrated Over time on Replacement, Integrated by U Alternative, Integrated by Alternative, Integrated by Alternative, November 18 2014 Working 19 Integrated by, Math 34b Integrated Work Solution, Math 229 Work, 06.Found Spreadsheet You're Looking for? To download/print, click the pop-up icon or print icon on the worksheet to print or download. The worksheet opens in a new window. You can & download or print using your browser's document reader options. We promote this section with an example. Let \(f(x) = (x^2+3x-5)^{10}\). We can calculate \(f'(x)\) by string rule. These are: \[f'(x) = 10(x^2+3x-5)^9\cdot(2x+3) = (20x+30)(x^2)+3x-5)^9.\] Now consider this: \(\int (20x+30)(x^2+3x-5)^9\ dx\)? We have the answer before us; \[\int (20x+30)(x^2+3x-5)^9\ dx = (x^2+3x-5)^{10}+C.\] How are we going to evaluate this indefinitely analysis without starting with \(f(x)\) as we did? This section explores integration by alternative. It allows us to un-unmede String Rules. The alternative allows us to evaluate the above analysis without knowing the original function first. The basic principle is to rewwwwww a complex analysis of the form \(\int f(x)\dx\) as a non--to-complex analysis \(\int h(u)\ du\). We will officially establish later how this is done. First, let's reconsider our indefinitely introduced analysis, \(\int (20x +30)(x^2+3x-5)^9\ dx\). Believed to be the most complex part of the integrand is \((x^2+3x-5)^9\). We want to make this simpler; we do so through an alternative. Please \(u=x^2+3x-5\). so \[(x^2+3x-5)^9 = u^9.\] We've set \(u\) as a function of \(x\), so now consider the distinction of \(u\): \[du = (2x+3)dx.\] Remember that \(((((2x+3)\) and \(dx\) are \(dx\) is not just sitting there. Back original analysis and implementation of an alternative number through angth of numbers: \[\begin{align} \int (20x+30)(x^2+3x-5)^9\ dx &= \int 10 (2x +3)(x^2+3x-

5)^9\ dx \\ &==\int 10(\underbrace{x^2+3x-5}_u)^9\underbrace {(2x+3)\ dx}_{du} \\ &=\int 10u^9\ du \\ &= u^{10} + C \quad \text{(replace \(u\) with \(x^2 +3x-5\)}}\\\\\&= (x^2+3x-5)^{10} +C\end{align}\] One can also look at this and think I (kind of) did it, but I could never come up with it myself, but this process can be learned. This section contains many examples through which the reader will gain understanding and mathematical maturity that allows them to consider replacement as a natural tool when evaluating analytics. We outlined before that integration by replacing un-unsaled String Rules. Specifically, \(F(x)\) and \(g(x)\) are functions that can distinguish and review the iono of their components: \[\frac{d}{dx}\Big (F\big(g(x)\big)\Big) = F'(g(x)))g '(x).\] So \[\int F'(g(x))g'(x)\ dx = F(g(x)) + C.\] Integrated by replacing the operation by recognizing the internal function \(g(x)\) and replacing it with a variable. By setting \(u=g(x)\), we can rewwwww as \[\frac{d}{dx}\Big(F\big(u\big)\Big) = F'(u)u'.\] Because \(du = g'(x)dx\), we can rewriting the analysis above is \[\int F'(g(x))g'(x)\ dx = \int F'(u) du = F(u)+C = F(g(x))+ C.\] This concept is important so we reset it in the context of a theedoth. \(\PageIndex{1}\): Integrated by Replacing Make \(F\) and \(g\) are distinct functions, where the range \(g\) is a period of time \(I\) contained in the domain of \(F\). Then \[\int F'(g(x))g'(x)\ dx = F(g(x)) + C.\] If \(u = g(x)\), then \(du = g'(x)dx\) and \[[\int F'(g(x))g'(x)\ dx = \int F'(u)\ du = F(u)+C = F(g(x)+C.\] The alternative point is to make the integration step easy. Indeed, the \(\int F'(u)\ du = F(u) + C\) step looks easy, since the cognitive anti-inflammatory of the \(F\) is only \(F\), plus a constant. The job involved is to make the appropriate alternative. There is not a step-by-step process that one can remember; instead, experience will be one of the guides. To get experience, we now embark on many examples. Example \(\PageIndex{1}\): Integration by replacing Rating \(\int x\sin(x^2+5)\ dx\). Solution Know that the replacement is related to the String Rule, we select to \(u\) the inner function of \(\sin(x^2+5)\). (This isn't always a good option, but it's usually the best place to start.) Please \(u = x^2+5\), hereby \(du = 2x\,dx\). Integrand has the term \(x\,dx\), but not the term \(2x\,dx\). (Recall that the cause is delivered, so \(x\) is not physical next to \(dx\) for a term \(x\,dx\) .) We can divide both sides of the expression \(du\) into 2: \[du = 2x\,dx \quad \Rightarrow \quad \frac12du = x\,dx.\] Now we can replace it. \[\start{align}\int x\sin(x^2+5)\ dx &= \int \int \underbrace{x\ dx}_{{\frac12du}\\\\\&= \int \frac12\sin u\ du\end{align}\] \[\begin{align} &= -\frac12\cos u + C \quad \text{ (now replace \(u\) with \(x^2+5\)\\\\\\&-frac12\cos(x^2+5) + C. \end{align}\] Therefore \(\int x\sin(x^2+5)\ dx = -\frac12\cos(x^2+5)+C\). We can check our work by evaluating the extracts of the right hand side. Exercise \(\PageIndex{1}\) Evaluation \(\int x\cos(x^2+5)\ dx\). Answer \frac12\sin(x^2+5)+C. Example \(\PageIndex{2}\): Integrated by replacing Rating \(\int \cos(5x)\ dx\). The solution again allows \(u\) to replace the internal functionality. Allow \(u = 5x\), we have \(du = 5dx\). Since our integrand doesn't have the term \(5dx\), we can divide the previous equation by \(5\) to get \(\frac15du = dx\). Now we can replace. \[\start{align} \int \cos(5x)\ dx &= \int \cos(\underbrace{5x}_u) \underbrace{dx}_{\frac{1}{5}du} \\&= \int \frac{1}{5}\cos u \du \\ &= \frac{1}{5} \sin u + C \\&= \frac{1}{5}\sin (5x)+C. \end{align}\] We can re-examine our work through differences. Exercise \(\PageIndex{2}\) Evaluation \(\int \sin(5x)\ dx\). Answer -\frac{1}{5}\cos(5x)+C. . Previous examples show a common and simple alternative type. The inner function is a linear function (in this case, \(y = 5x\)). When the internal function is linear, the result integration is very predictable, outlined here. Main idea 10: Replace with Linear Function Review \(\int F'(ax+b)\ dx\), where \(aeq 0\) and \(b\) are constants. Allow \(u = ax+b\) for \(du = a\cdot dx\), resulting in \[\int F'(ax+b)\ dx = \frac{1}{a}F(ax+b) + C.\] int \sin (7x-4)\ dx = -\frac17\cos(7x-4)+C\\ Our next example may be using Key Idea 10, but we will only use it after going through all the steps. Example \(\PageIndex{3}\): Integrated by replacing linear function Evaluation \(\int \frac{7}{-3x+1}\ dx\). Solution View this component of the \(f(g(x)\) functions, where \(f(x) = 7/x\) and \(g(x) = -3x+1\). Using our understanding of the alternative, we leave \(u = -3x+1\), the inner function. So \(du = -3dx\). The integrand is missing a \(-3\); thus dividing the previous equation by \(-3\) to obtain \(-du/3 = dx\). Now we can evaluate analytics through alternative. \[\start{align} \int \frac{7}{-3x+1}\ dx &= \int \frac{7}{u}\frac{du}{-3} \\ &= \frac{-7}{3}\int \frac {du}{u} \\ &= \frac{-7}{3}\ln |u| + C\\ &=-\frac{7}{3}\ln|-3x+1| + C. \end{align}\] Use Key Ideas 10 faster, realize that \(u\) is linear and \(a = -3\). One may want to continue writing down all the steps until they are comfortable with this particular shortcut. Exercise \(\PageIndex{3}\) Evaluation \(\int \frac{7}{3x+1}\ dx\). Answer \frac{7}{3|\ln|3x+1| + C. Not all the feces that benefit from the replacement all have clear internal functions. Some of the following examples will demonstrate how this happens. Example Integration by replacing Evaluation \(\int \sin x\cos x\dx\). The solution Does not have a component of functionality here to exploit; instead, just a product of functions. Don't be afraid to experiment; when giving an analysis to evaluate, it is often beneficial to think If I let \(u\) be this, then \(du\) must be that ... and see if this simplifies integration at all. In this example, let's set \(u = \sin x\). Then \(du = \cos x\ dx\), which we have as part of the integrand! The alternative becomes very simple: \[start{align} \int \sin x\cos x\dx &= \int u\ du \\ &= \frac{1}{2}u^2+ C \\ &= \frac{1}{2}\sin^2 x + C. \end{align}\] People would do well to ask What if we let \(u = \cos x\)? The results are also easy to find, but look very different. The challenge for readers is to evaluate the permission to analyze \(u = \cos x\) and discover why the answer is the same, but looks different. Exercise \(\PageIndex{4}\) Evaluation \(\int \sin^3 x\cos x\dx\).. Answer \frac{1}{3}\sin^4 x + C. Our example has so far required a basic replacement. The next example shows how replacements can be made that often attack new learns as non-standard. Example \(\PageIndex{5}\): Integration by replacing Rating \(\int x\sqrt{x+3}\ dx\). Solution Recognizes the composition of the function, set \(u = x+3\). Then \(du = dx\), giving what originally seemed to be a simple alternative. But at this stage, we have: \[\int x\sqrt{x+3}\ dx = \int x\sqrt{u}\ du.\] We cannot evaluate an analysis that has both a \(x\) and a \(u\) in it. We need to convert \(x\) to an expression related only to \(u\). Since we set \(u = x+3\), we can also say that \(u-3 = x\). So we can replace \(x\) in integrand with \(u-3\). It will also be useful to rewwwww \(\sqrt{u}\) as \(u^\frac12\). \[\start{align} \int x\sqrt{x+3} \ dx &= \int (u-3)u^\frac12\ du \\ &= \int \big(u^\frac32 - 3u^\frac12\big) \ du \\ frac25u^\frac52 - 2u^\frac32 + C \\ &= \frac25(x+3)^\frac52 - 2(x+3)^\frac32 + C.\end{align}\] Checking your work is always a good idea. In this particular case, an ang number will be needed to make a person's answer match the integrand in the original problem. Example \(\PageIndex{6}\): Integration by replacing Rating \(\int \frac{1}{x\ln x}\dx\). This solution is another example that does not seem to have clear components of the function. The line of thought used in Example \(\PageIndex{5}\) is useful here: choose something for \(u\) and consider what this implies \(du\) must be. If \(u\) can be selected so that \(du\) also appears in the integrand, then we have selected well. Select \(u = 1/x\) make \(du = -1/x^2\ dx\); that doesn't seem to be helpful. However, the \(u = \ln x\) setting makes \(du = dx\), is part of the integration. As follows: \[\begin{align} \int \frac1{x\ln x}\ dx &= \int \frac{1}{\underbrace{\ln \frac{1}{\underbrace{\ln dx}_{du} \\ &= \int \frac1u\ du \\ &= \ln |u| + C \\\&= \ln | \ln x| + C.\end{align}\] The final answer is very interesting; natural diary of natural diary. Get a lead to confirm that this answer is actually correct. Part 6.3 delves deeper into the analysis of a variety of hegonogon functions; here we use the alternative to establish a foundation that we will build. The next three examples will help fill in some missing parts of our antiderivative knowledge. We know the antiderivatives of sine and cosine functions; what about other standard tangar, cotanng, secant and cosecant functions? We discover these next. Example \(\PageIndex{7}\): Integration by Alternative: antiderivatives of \(\tan x\) Evaluation \(\int \tan x\ dx.\) The Previous Paragraph Solution is established that we do not know the antiderivatives of tangerines, so we must assume that we have learned something in this section that can help us evaluate this indefinitely analysis. Rewwwwwww as \(\sin x/\cos x\). While the presence of a component of the function may not be immediately apparent, realize that \(\cos x\) is inside the \(1/x\) function. So we see if setting \(u = \cos x\) returns usable results. We have that \(du = -\sin x\dx\), hereby \(-du = \sin x\dx\). We can integrate: \[\begin{align}\int \tan x \dx &= \int \frac{\sin x}{\cos x}\ dx \\ &= \int \frac1{\underbrace{\cos x}_u}\underbrace{\\\\sin x\dx}_{-du} \\ &= \int \frac {-1}u\du\\\\&= -\ln |u| + C \\ &= -\ln |\cos x| + C.\end{align}\] Some texts prefer to include \((> -1\) inside the log as a power of \(\cos x\), as in: \[\begin{align} -\ln |\cos x| + C &= \ln |(\cos x)^{-1}| + C\\ &= \ln \left| \frac{1}{\cos x}\right| + C\\&= \ln |\sec x| + C.\end{align}\] Two answers are equivalent. Example \(\PageIndex{8}\): Integrated by alternative: antiderivatives of \(\sec x\) Rating \(\int \sec x\dx\). This example solution uses a great tip: integrand by 1 to see how to integrate more clearly. In this case, we write 1 as \[1 = \frac{\sec x + \tan x}{\sec x + \tan x}.\] This seems like it comes from the left wing, but it works beautifully. Review: \[\begin{align} \int \sec x\dx &= \int \sec x\cdot \frac{\sec x + \tan x}{\sec x + \tan x}\ dx \\ &= \\ int \frac{\sec^2 x + \sec x\tan x}{\sec x + \tan x}\ dx.\end{align}\] Now let\(u = \sec x+\tan x\); this means \(du = (\sec x\tan x+ \sec^2 x)\dx\), which is our number. Therefore: \[\begin{align} &= \int \frac{du}{u} \\ &= \ln |u| + C \\ &= \ln |\sec x+\tan x| + C.\end{align}\] We can use the same techniques as those used in Example \(\PageIndex{6}\) and \(\PageIndex{7}\) to find antiderivatives of \(\cot x\) and \(\csc x\) (which readers can explore during exercises.) We summarize our results here. \(\PageIndex{1}\): \(\PageIndex{1}\): trigonogonm function \(\int \sin x \ dx = -\cos x +C\) \(\int \cos x\ dx = \sine x + C\) \(\int \tan x\ dx = -\ln|\cos x|+C\) \(\int \csc x \ dx = -\\ln|\csc x+\cot x| +C\) \(\int \sec x\ dx = \ln|\sec x+\tan x| + C\) \(\int \cot x\ dx = \ln|\sin x|+C\) We explore a more common trigonogoniscoth analysis. Example \(\PageIndex{9}\): Integration by alternative: powers of \(\cos x\) and \(\sin x\) Evaluation \(\int \cos^2x\dx\). Solution We have a component of functions such as \(\cos^2x = \big(\cos x\big)^2\). However, the setting \(u = \cos x\) means \(du = -\sin x\dx\), which we do not have in the analysis. Another technique is necessary. The process we'll use is to use the Power Reduction formula for \(\cos^2x\) (perhaps referencing the back of this text for this formula), saying \[\cos ^2x = \frac{1+\cos(2x)}{2}.\] The right-hand side of this equation is not difficult to integrate. We have: \[\begin{align} \int \cos^2x\ dx &= \int \frac{1+\cos(2x)}2\dx \\ &= \int \left(\frac12 + \frac12\cos(2x)\right)\ dx. \end{align} \]10: \[\begin{align} &= \frac12x + \frac12\frac{\sin(2x)}{2} + C\\&= \frac12x + \frac{\sin(2x)}4 + C.\end{align}\] It is often reluctant to manipulate the integration of an analysis; At first, our grasp of integration is fragile and one might think that working with integrand will not properly change results. Integration by alternative works using a different logic: as long as equality is maintained, integration can be manipulated to make its form easier to deal with. The next two examples demonstrate common ways in which using the first number of numbers makes integration easier to implement. Example \(\PageIndex{10}\): Integration by alternative: simplify first rating \(\int \frac{x^3+4x^2+8x+5}{x^2+2x+1}\ dx\). Solution One can try to start by setting \(u\) with either a number or a number sample; in each case, the result is not feasible. When dealing with functions that are reasonable (i.e. numbered merchants made up of polythythy functions), it is an almost universal rule that everything works better when the level of the number is smaller than the number sample level. We therefore use polyathy division. We ignore the specifics of the steps, but note that when \(x^2+2x+1\) is divided into \(x^3+4x^2+8x+5\), it enters \(x+2\) times with the rest of \(3x+3\). So \[\frac{x^3+4x^2+8x+5}{x^2+2x+1} = x+2 + \frac{3x+3}{x^2+2x+1}.\] Integration \(x+2\) is simple. The segment can be integrated by setting \(u = x^2+2x+1\), for \(du = (2x+2)\ dx\). This is very similar to the number. Note that \(du/2 = (x+1)\ dx\) and then consider the following: \[\begin{align}\int \frac{x^3+4x^2+8x+5}{x^2+2x+1}\ dx & = \int \left(x+22 + dx \\ &= \int (x+2)\ dx + \int \frac{3(x+1)}{x^2+2x+1}\ dx \\ & = = + \int \frac{3}{u}\frac{du}{2} \\ &= \frac12x^2+2x+C_1 + \frac32\ln|u| + C_2 \\&= \frac12x^2+2x+\frac32\ln|x^2+2x+1| + C.\end{align}\] In some ways, we are lucky in that after division, replacement can already be done. In the following sections, we will develop techniques to handle reasonable functions in which replacement is not directly feasible. Example \(\PageIndex{11}\): Built-in methods of replacing Evaluation \(\int \frac{x^2+2x+3}{\sqrt{x}}\dx\) with and without replacement. Solution We already know how to integrate this particular example. Rewwwwwww as \(x^\frac12\) and simplify the percent: \[\[\frac{x^2+2x+3}{x^{1/2}} = x ^\frac32 + 2x^\frac12 + 3x^{-\frac12}.\] We can now integrate with Power Rules: \[[start{align} \int \frac{x^ 2+2x+3}{x^{1/2}}\ dx &= \int\left(x^\frac32 + 2x^\frac12 + 3x^{-\frac12}\right \\dx\\&= \frac25x^\frac52 + \frac43x^\frac32 + 6x^\frac12 + C\end{align}\] This is a perfect approach. We demonstrate how this can also be solved using alternative as its implementation is quite smart. Let\(u = \sqrt{x} = x^\frac12\); so \[du= \frac12x^{-\frac12}dx = \frac{1}{2\sqrt{x}}\ dx \quad \Rightarrow \quad 2du = \frac{1}{\sqrt{x }} }\ dx.\[This gives us \(\int \frac{x^2+2x+3}{\sqrt{x}}\ dx = \int (x^2+2x+3)\cdot2\du\). What do we do with other terms \(x\)? Because \(u = x^\frac12\), \(u^2 = x\), etc. We can then replace \(x^2\) and \(x\) with the appropriate power of \(u\). So we have \[\begin{align*}\int \frac{x^2+2x+3}{\sqrt{x}}\ dx &= \int (x^2+2x+3)\cdot2\ du\\&= \int 2(u^4 + 2u^2 +3)\ du \\ \ \ &= \frac25u^5 + \frac43u^3 + 6u + C \\&= \frac25x^\frac52 + \frac43x^\frac32 + 6x^\frac12+C,\end{align*}\] is clearly the same answer we've received before. In this situation, the alternative is said to work more than our other methods. The great thing is that it works. It demonstrates how flexible integration is. When we study the inverse functions, we learn that \[\frac{d}{dx}\big(\tan^{-1}x\big) = \frac{1}{1+x^2}.\] Applying String Rules to this is not difficult; for example, \[\frac{d}{dx}\big(\tan^{-1}5x\big) = \frac{5}{1+25x^2}.\] We now discover an alternative that can be used to un unsaft certain derivatives as a result of string rules applied to inverse Trigonous functions. Let's start with an example. Example \(\PageIndex{12}\): Integrated by alternative: Inverse inverse inverse function Evaluate \(\int \frac{1}{25+x^2}\ dx\). The Integrated Solution looks similar to the export of the arctang function. Comment: \[\begin{align}\frac{1}{25+x^2} &= \frac{1}{25(1+\frac{x^2}{25})}\\\frac{1}{25(1+\left(\frac{x}{5}\right)^2)} \\ &= \frac{1}{25}\frac{1} .\end{align}\] So \[\int\frac{1}{25+x^2}\ dx = \frac{1}{25}\int \frac{1}{1+\left(\frac{x}{5}\right)^2}\ dx.\] This can be integrated by using Replace. Set \(u = x/5\), so \(du\(du dx/5\) or \(dx=5du\). So \[\begin{align}\int\frac{1}{25+x^2}\ dx &= \frac{1}{25}\int \frac{1}{1+\left(\frac{x}x{5}\right)^2}\ dx \\ & \frac15\int \frac{1}{1+u 2 }\ du \\ &= \frac15\tan^{-1}u + C \\ &= \frac15\tan^{-1}\left(\frac x5\right)+C\end{align}\] Example \(\PageIndex{12}\) represents a common technique that can be applied to other integrations that lead to inverse functions. The results are summarized here. \(\PageIndex{2}\: Analysis related to inverse inverse quantible functions Make \(a>0\). \(\int \frac{1}{a^2+x^2}\ dx = \frac1a\tan^{-1}\left(\frac{x}{a}\right) + C\) \(\{ int \frac{1}{\sqrt{a^2-x^2}}} \dx = \sin^{-1}\left(\frac{x}{a}\right)\C\) \(\int \frac{1}{x\sqrt{x^2-a^2}}\ dx = \frac1a\sec^{-1}\left(\frac{|x|} {a}\right)+C\) Practice using the \(\PageIndex{2}\). Example \(\PageIndex{13}\): Integration by alternative: inverse inverse inverses Evaluate certain indefinitely analyzed analyses. \[\int \frac{1}{9+x^2}\ dx,\quad \int \frac{1}{x\sqrt{x^2-\frac{1}{100}}}} \ dx\quad \quad \text{ and }\quad \int \frac{1}{\sqrt{5-x^2}}\\dx.\] Solution Each solution can be answered using a simple application of the \(\PageIndex{2}\). \(\int \frac{1}{9+x^2}\ dx = \frac13\tan^{-1} \frac x3 + C\), as \(a = 3\). \(\int \frac{1}{x\sqrt{x^2-\frac{1}{100}}}\ dx = 10\sec^{-1}10x + C\), such as \(a = \frac1{10}\). \(\int \frac{1}{\sqrt{5-x^2}} = \sin^{-1}\frac{x}{\sqrt{5}}+C\), such as \(a = \sqrt{5}\). Most applications of the \(\PageIndex{2}\) the the best are not so simple. Further examples show that some common analytics can still be approached with this theo theo. Example \(\PageIndex{14}\): Integration by alternative: complete square Evaluation \(\int\frac{1}{x^2-4x+13}\ dx\). Original solution, this analysis seems to have nothing in common with the analyses in the \(\PageIndex{2}\). Since it lacks a second-tier base, it is almost certainly not related to arcsine or arcsecant. However, it is related to the arctang function. We see this by completing the square in the numbered form. We give a brief reminder of the process here. Start with a second tier with a leading 1. It will take the form \(x^2 + bx + c\). Take 1/2 of \(b\), square it, and add/subtract it back into the expression. That is, \[\begin{align} x^2+bx+ c&= \underbrace{x^2 + bx + \frac{b^2}4}_{(x+b/2)^2 } - \frac{b^2}4 + c\\&align}\] we take half \(-4\) and square it, get \(4\). We add/subtract it to the number template as follows: \]\begin{align}\frac{1}{x^2-4x+13} &= \frac{1}{\underbrace{x^2-4x+4}_{(x-2)^2}-4+13}\\\\frac{1}{(x-2)^2 + 9}\end{align}\] We can now integrate this using arctangent rules. Technically, we need to replace first with \(u=x-2\), but we can key idea 10 instead. So we have \[\int \frac{1}{x^2-4x+13}\ dx = \int \frac{1}{(x-2)^2+9}\ dx = Example \(\PageIndex{15}\): Analysis requires multiple Evaluation methods \(\int \frac{4-x}{\sqrt{16-x^2}}\dx\). This Analysis solution requires two different methods to evaluate it. We get to those methods by separating integrals: \[\int \frac{4-x}{sqrt{16-x^2}}\ dx = \int \frac{4}{\sqrt{16-x^2}}\ dx - \int \frac{x}{\sqrt{16-x^2}}\dx.\] The first analysis is processed using a simple application of the \(\PageIndex{2}\); The second analysis is processed by replacement, with \(u = 16-x^2\). We handle each way separately. \(\int \frac{4}{\sqrt{16-x^2}}\ dx = 4\sin^{-1}\frac{x}{4} + C.\) \(\int\frac{{ x}{\sqrt{16-x^2}}\ dx\): Set \(u = 16-x^2\), so \(du = -2xdx\) and \(xdx = -du/2\). We have \[\start{align} \int\frac{x}{\sqrt{16-x^2}}\ dx &= \int\frac{-du/2}{\sqrt{u}}\\\\\\&= -\frac12 \int \frac{1}{\sqrt{u}}\ du \\ &= - \sqrt{u} + C\\ &= -\sqrt{16-x^2} + C.\end{align}\] Combine them together, we have \[\int \frac{4-x}{\sqrt{16-x^2}}\ dx = 4\sin^{{{\sqrt{16-x^2}}\ dx = 4\sin^{{-1}\frac x4 + \sqrt{16-x^2}+C.\] Exercise \(\PageIndex{1}\) Used instead to find counterinflexes of \(∫3x^2(x^3−3)^2dx.\) Suggestions \(u=x^) 3−3.\) Reply \[∫3x^2(x^3−3)^2dx=\dfrac{1}{3}(x^3−3)^3+C\] Sometimes we need to adjust the constants in our analysis if they don't exactly match the expressions we're replacing. Example \(\PageIndex{2}\): Use an alternative use alternative to find the antiderivative of \[∫z\sqrt{z^2−5}\,dz.\] Rewover Solution analysis such as \(∫z(z^2−5)^{1/2}\,dz.\) Make \(u=z^2−5\) and \(du=2z\,dz\). Now we have a problem because \(du=2z\,dz\) and the original expression is only \(z\, dz\). We have to change our expression for du or the analysis in you will be twice as big as it should be. If we cause both sides of the equation to travel with \(\dfrac{1}{2}\). we can solve this problem. Vì vậy, \[u=z^2−5\] \[du=2z\, dz\] \[\dfrac{1}{2}du=\dfrac{1}{2}(2z)\,dz=z\,dz.\] Viết tích phân về bạn, nhưng kéo \(\dfrac{1}{2}\) bên ngoài ký hiệu tích hợp: \[∫z(z^2−5)^{1/2}\,dz=\dfrac{1}{2}∫u^{1/2}\,du.\] Tích hợp biểu thức trong \(u\): \(\dfrac{1}{2}∫u^{1/2}\,du=(\dfrac{1}{2})\dfrac{u^{3/2}}{\dfrac{3}{2}}+C\) \(=(\dfrac{1}{2})(\dfrac{2}{3})u^{3/2}+C\) \(=\dfrac{1}{3}u^{3/2}+C\) \(=\dfrac{1}{3}(z^2−5)^{3/2}+C\) Bài tập \(\PageIndex{2}\) Dùng sự thay thế để tìm chất chống derivative của \(∫x^2(x^3+5)^9\,dx.\) Gợi ý Nhân phương trình du bằng \(\dfrac{1}{3}\). Answer \[\dfrac{(x^3+5)^{10}}{30}+C\] Sometimes we need to manipulate an analysis in more complex ways than just by bying or dividing by a constant. We need to remove all expressions in the integrand that is about the original variable. When we're done, \(u\) will be the only variable in the integrand. In some cases, this means solved for the initial variable on \(u\). This technique will in the next example. Example \(\PageIndex{4}\): Find an anti-derivative solution Use u-Substitution Use to find \[∫\dfrac{x}{\sqrt{x−1}}\,dx.\] Solution If we allow \(u=x−1,\) then \(du=dx\). But this does not account for the x in the integrand's number. We need to show x about you. If \(u=x−1\), then \(x=u+1.\) We can now rewwwer your analysis: \[∫\dfrac{x}{\sqrt{x−1}},dx=∫\dfrac{u+1}{sqrt{{ u}}\,du=∫\sqrt{u}+\dfrac{1}{\sqrt{u}}\,du=∫(u^{1/2}+u^{−1/2})\,du.\] replaces you with the original expression, and factors and simplify the results. Therefore, \(∫(u^{1/2}+u^{−1/2})\,du=\dfrac{2}{3}u^{3/2}+2u^{1/2}+C\) \(=\dfrac{2}{3}(x−1)^{3/2}+2 (x −1)^{1/2}+C\) \(=(x−1)^{1/2}[\dfrac{2}{3}(x−1)+2]+C\) \(=(x−1)^{1/2}(\dfrac{2}{3}x−\dfrac{2}{3} +\1dfrac{6}{3}))\) \(=(x−1)^{1/2}(\dfrac{2}{3}x+\dfrac{4}{3}))\) \(=\dfrac{2}{3}(x−1)^{1/2}(x+2)+C.\) Example \(\PageIndex{4}{2}{3} 2\ Use Replace with Trigonothothm Function Instead to evaluate analysis \(∫\dfrac{\sin t}{\cos^3t}\,dt.\) The solution We know the result of \(\cos t\) is \(−\sin t\), so we set \(u=\cos t\). Then \(du=−\sin t\,dt.\) Replace it in the analysis, we have \[∫\dfrac{\sin t}{\cos^3t}\,dt=−∫\dfrac{du}{u^3}.\] Evaluating the analysis, we received \[[∫\dfrac{du}{u^3}=−∫u^{−3}\,du=-(−\dfrac{1}{2})u^{−2}+C.\] Put the answer back in t, we received \[∫\dfrac{\sin t}{{\cos^3t}\,dt=\dfrac{1}{2u^2}+C=\dfrac{1}{2\cos^2t}+C.. \[PageIndex{4}\) Using An Alternative to Evaluate Analysis \[∫\dfrac{\cos t}{\sin^2t}\,dt.\] Suggests Using the process from example to resolve the issue. Answer \(-\dfrac{1}{\sin t}+C\) Exercise \(\PageIndex{4}\) Use alternatively to evaluate indefinitely analysis \[∫\cos^3t\sin t\,dt.\] Suggests Using process from example to solve the problem. Answer \[-\dfrac{cos^4t}{4}+C\] Example \(\PageIndex{5}\): \[∫\sec(t)\,dt.\] Exercise \[4t}/d.\] Exercise \[PageIndex{5}\): \[∫\sec(t)\,dt.\] Exercise \[∫ (\PageIndex{5}\) \[∫\csc(t)\,dt.\] Reply More text replies here and it will automatically be hidden if you have an Active AutoNum template on the page. A common mistake when dealing with exponential expressions is to handle exponents the same way we handle exponents in polycrysal expressions. We can't use power rules for exponents. This can be especially confusing when we have both exponential and polycrysal functions in the same expression, as in the previous checkpoint. In these cases, we should always check carefully to make sure that we are using the appropriate rules for the functions that we are integrating. Example \(\PageIndex{5}\): Align second level of The Exponential Find Countersaced Exponential Function \(e^x\sqrt{1+e^x}\). Solution First rewrets the problem by using a reasonable exponent: \(∫e^x\sqrt{1+e^x}dx=∫e^x(1+e^x)^{1/2}dx.\) Use select \(u=1+e^x.u=1+e^x\)Then \(du=e^xdx\). We have (Figure) \(∫e^x(1+e^x)^{1/2}dx=∫u^{1/2}du.\) Then \(∫u^{1/2}du=\dfrac{u^{3/2}}{{23/2}+C=\dfrac{2}{3}u^{3/2}+C=\dfrac{2}{3}(1+e^x)^{3/2}+C\) Figure \(\PageIndex{1}\): Chart showing exponential function with exponential align. Exercise \(\PageIndex{6}\) Find the anti-derivative of \(e^x(3e^x−2)^2\). Hint Make \(u=3e^x−2u=3e^x−2.\) Reply \(∫e^x(3e^x−2)^2dx=\dfrac{1}{9}(3e^x−2)^3\) Wallet example \(\PageIndex{7}\): Use Replace with Exponential Alternative to evaluate indefinitely analysis \(∫3x^2e^{2x^3}dx.\) Solution Here we choose to u by expression in exponents on e. Make \(u=2x^3\) and \(du=6x^2dx\).. Again, travel is reduced by a constant cymathm; the root function contains a non-\(3x^2,\) \(6x^2\) meast. Cause both sides of the equation with \(\dfrac{1}{2}\) to integrate in u equals integration in x. Therefore, \(∫3x^2e^{2x^3}dx=\dfrac{1}{2}∫e^udu\). Integrate the expression in you and then replace the original expression in x back into the u-analysis: \(\dfrac{1}{2}∫e^udu=\dfrac{1}{2}e^u+C=\dfrac {1}{2}e^2x^3+C.\) Exercise \(\PageIndex{8}\) Indefinitely Analysis Evaluation \(∫2x^3ex^4dx\). Hint Please \(u=x^4.\) Reply \(∫2x^3ex^4dx=\dfrac{1}{2}e^{x^4}\) Example \(\PageIndex{9} \): Find an antiderivative Related \(\ln x\) Find the antiderivative function of the \[\dfrac{3}{x−10}.\] First Factor 3 solution outside the analysis icon. Then use the \(u^{−1}\) rule. Therefore, \[∫\dfrac{3}{x−10}dx=3∫\dfrac{1}{x−10}dx=3∫\dfrac{du}{u}=3\ln |u|+C=3\ln |x−10|+C,x≠10.\] View Picture. Figure \(\PageIndex{3}\): The domain of this function is \(x eq 10.\) Exercise \(\PageIndex{9}\) Find the antiderivative of \[\dfrac{1}{x+2}.\] Sample Suggestions from example to solve the problem. Answer \[\ln |x+2|+C\] Example \(\PageIndex{10}\): Find an Antiderivative of a rational function Find the antiderivative of \[\dfrac{2x^3+3x}{x^4 +3x^2}.\] This solution can be rewritten as \(∫(2x^3+3x)(x^4+3x^2)^{−1}dx.\) Use instead. Make \(u=x^4+3x^2\), then \(du=4x^3+6x.\) Change the travel by calculating 2. Therefore, \[du=(4x^3+6x)dx=2(2x^3+3x)dx\] \[\dfrac{1}{2}du=(2x^3+3x)dx.\] Write back integration in u \\[∫(2x^3+3x)(x^4+3x^2)^{−1}dx=\dfrac{1}{2}∫u^{−1}du.\] Then, we have \[\dfrac{1}{2}∫{1}{2}∫ <9>u^{−1}du=\dfrac{1}{2}\ln |u|+C=\dfrac{1}{2}\ln |x^4+3x^2|+C.\] Example \(\PageIndex {11}\): Find an Antiderivative of a Logarithmic Function Find the antiderivative of the login function \(\dfrac{\ln(2x)}{x}.\) Solution Make \(u= \ln(2x)\). Then \(du=\dfrac{2}{x} dx\). Now, \[∫\dfrac{\ln(2x)}{x}.dx= \int \dfrac{1}{2} {u}du= \dfrac{1}{2} u^2+C= \dfrac{1}{2}(\ln x)^2+C.\] Replacing key concepts is a simplified technique for integrating functions as a result of string rules The term 'alternative' refers to changing variables or replacing your and du variables for the appropriate expressions in integration. Formulas for inverse inverse inverse inverse inverse inverse inverses developed in Exponential and Logarithm lead directly to integrated formulas associated with inverse inverse inverse inverse functions. Using formulas listed in the built-in formula rule leads to inverse inverse inverse inverse functions to match the correct format and make changes as needed to resolve the issue. Replacements are often required to put integrand in the correct form. Replace it with indefinitely analysis \(∫f[g(x)]g′(x)dx=∫f(u)du=F(u)+C=F(g(x))+C\) Analysis that produces inverse Trigonogones \(∫ ∫f(u)du=F(u)+C=F(g(x))+C\) <2>\dfrac{du}{\sqrt{a^2−u^2}}=sin^{−1}(\dfrac{u}{a})+C\) \(∫\dfrac{du}{a^2+u^2}=\dfrac{1}{a}tan^{−1}(\dfrac{u}{a})+C\) \(∫\dfrac{du}{u\sqrt{u^2−a^ 22}}=\dfrac{1}{a}sec^{{−1}(\dfrac{u}{a})+C\) Contributors Gilbert Strang (MIT) and Edwin Jed Herman (Harvey Mudd) with many contributing authors. This content of OpenStax is licensed with license CC-BY-SA-NC 4.0. Download for free at . Gregory Hartman (Virginia Military Institute). The contributions were made by Troy Siemers and Dimplekumar Chalishajar of VMI and Brian Heinold of Mount Saint Mary's University. This content is copyrighted by creative commons attribution license - Noncommercial (BY-NC). Pamini Thangarajah (Mount Royal University, Calgary, Alberta, Canada) Canada)