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For all , vertical asymptotes occur in where the integer is. Use the base period for point to find vertical asymptotes for . Set the inside of the secan function to equal to see where the vertical asymptote occurs . cos values at the appropriate theta values from M. Bourne Charts 'tan x', 'cot x', 'sec x' and 'csc x' are not as common as the sine and cosine curves that we encountered earlier in this chapter. However, they occur in engineering and scientific problems. They are interesting curves because they have discontinuities. Dosing, correction, mowed, and coseant curves are not defined for certain x values, and therefore there is a gap in the curve. [For more information about this topic, see the previous chapter under Continuous and disjoint features.] Recall from the trigonometric function that 'tan x' is defined as: 'tan x =(sin x)/(cos x)' Consider the denominator (bottom) of this fraction. For some x values, the 'cos x' function is 0. For example, if 'x=pi/2', the value 'cos {:π/2:}' is '0', and when 'x=(3pi)/2', we have 'cos{:(3π)/2:}=0'. When this happens, we have 0 in the denominator of the fraction, and that means that a fraction is not defined. So there will be a gap in function at this point. This gap is called discontinuity. The same thing happens with 'cot x', 'sec x' and 'csc x' for different values of 'x'. For each of them, the denominator will be 0 for certain x values. Graph y = tan x Sketch y = tan x. Solution As we saw above, 'tan x=(sin x)/(cos x)' That is, the function will have discontinuity where cos x = 0. This means that when x takes any of the values: 'x = ..., -(5π)/2, -(3π)/2, -π/2,' ' π/2,' ' (3π)/2,' ' (5π)/2, ...' It is very important to keep these values in mind when sketching this chart. When setting up a table of values, make sure that they include 'x'-values on both sides of the discontinuity. Recall that '-(3pi)/2=-4.7124' and '-pi/2= -1.5708'. So we take values pretty close to these discontinuities. x-4.7-4.5-4-3-3.5-3.14-1.58-1.56-100.511.51.57 tan x-80.57 57-4.6-1.2-0.370108-92-1.600.551.614.11,256 Warning on both sides '-pi/2', (our values -1.58 and -1.56 in the table above), we jump from a large positive number (108) to a small negative number (-92). If we continue with our table, we will get similar values (because it is a periodic graph). So we're ready to sketch our curve. Graph y = tan x: Note that there are vertical asymptotes (gray dotted lines) where the denominator 'tan x' has a zero value. (Asymptote is a straight line that the curve gets closer and closer to without actually touching. You can see more examples of asymptotes in the later chapter sketching curves using differentiation.) Also note that the chart 'y = tan x' is periodic with the period π. This means that it repeats after each step π we go from left to right on the chart. Interactive Animation In this interactive application you can see the animation tangential function. What to do Using the sliders below the chart, you can change: The amount of energy in the wave by changing the amplitude, and the frequency of the wave by changing the b phase shift of the wave by changing c Vertical wave shift by changing d Units on the horizontal x-axis are radians (in decimal form). Recall that: π radians = 3.14 radians = 180°. The chart shown is therefore from '-pi/2' to '(7pi)/2. Vertical dashed lines are asymptotes. The pink triangle that appears when the animation starts has a base length = 1. The height of this triangle is the tan ratio of the current angle. You may notice the triangle buckles are almost vertical when the graph goes off at ±∞. Copyright © www.intmath.com Frame rate: 0 (For more information about periodic functions and to see 'y=tan x' using degrees rather than radians, see trigonometric functions of any angle.) Graph y = cot x Recall from trigonometric functions that: 'cot x= 1/tanx = (cos x)/(sin x)' Now we need to consider when 'sin x' has zero value, because it will determine where our asymptotes should go. The function will have discontinuity, where 'sin x = 0', that is, when ' x = ..., -3π, -2π, -π, 0,' ' π,' ' 2π,' ' 3π,' ' 4π,' ' 5π, ...' Due to the values of cos x and sin x for different x values (or, more simply, find values of '1/tanx'), we can set a table of values. Then we can sketch the chart 'y = cot x' as follows. Graph y = sec x We could laboriously compile a table with millions of values, or we could work smart and remind that 'sec =1/(cos x)' we know the sketch for y=cos x and we can easily derive a sketch for y=sec x, by finding a reciprocal of each y-value. (That is, search '1/y' for each y value on the curve 'y = cos x'.) For example (angles are in radians): x y = cos x 1/y = sec x 0 1 1 1 0.54 1.85 1.55 0.02 48.09 2 −0 42 (2.4) 3 (0.99) (1.01) 4 -0.65 (1.53) I included a value only below π/2=1.57 to get an idea what's going on in there. When 'cos x' is very small, 'sec x' will be very big. After using this concept in the entire range of x values, we can continue to sketch the chart 'y = sec x'. First the graph 'y = cos x' and then 'y = sec x' immediately below it. Compare the y values in each of the 2 charts to make sure they are mutual. y = cos x Chart 'y=cos(x)' for '0 ≤ x < (5pi)/2' y = sec x We draw vertical asymptotes (dashed lines) to values where 'y = sec x' is not defined. That is, when 'x = ..., -(5π)/2, -(3π)/2,' ' -π/2,' ' π/2,' ' '(3π)/2,' ' (5π)/2, ...' You will notice that these are the same asymptotes that we drew for y = tan x, which is not surprising as they both have the bottom of the bottom Exercise Download graph paperSketch y = csc x Answer We remind you that 'csc x = 1 / (sin x)' So we will have asymptotes where 'sin x' has zero value, that is: x = ..., -3π, -2π, -π, 0, π, 2π, 3π, 4π, ... First, we draw a graph y = sin x and mark it with dashed lines where the chart has a value of '0': Chart 'y=sin x'. Next, we consider the reciprocal values of all y values in the chart above (similar to what we did with table y = x, which we created above). 'x' 'y' '= sin x' 'csc x' '= 1/(sin x)' 0.01 0.01 100 0.5 0.48 2.09 'pi/2' 1 1 1 11 2 0.91 1.10 3 0.14 7.09 3.1 0.04 24.05 I chose values close to 0 and pi and some values between them. The pattern will be similar for the region from pi to '2pi' except that it will be on the negative side of the wasp. We continue on both sides and realize that the pattern will be repeated. Now for graph y = csc x: π2π−π-2π246810-2-4-6-8-10xyGraph y = csc x. You may also be interested in: The next section of this chapter lists some trigonometric chart applications. Sketch chart y =sec x. For which values x, 0 < =x < = 2 p is s x not defined ? pi / 4 , 5pi/4question_answerquestion_answerQ: Instructions: Determine if the relationships listed below are functions. Explain why not. A: 5.1.1,2, 3,2.5, 5,7, 10.65,12 This relationship has all x values representing only one value y. ∴This... question_answerquestion_answerquestion_answerquestion_answerquestion_answerquestion_answer in this section you will: Analyze graph y = tan x. Variation of graph y=tan x. Analyze graphs y=sec x and y=csc x. Chart variants y=sec x and y=csc x. Analyze graph y=cot x. Chart variants y=cot x. We know that tangential function can be used to find distances such as the height of a building, mountain or mast. But what if we want to measure repeated occurrences of distance? For example, imagine a police car parked next to a warehouse. The swivel light from the police car would move along the wall of the warehouse at regular intervals. If the input is time, the output would be the distance to which the light beam moves. A beam of light would repeat the distance at regular intervals. This distance can be used to approximate the tangent. Asymptotes would be needed to illustrate repeated cycles when the beam runs parallel to the wall, as it would seem that the beam of light would expand forever. A graph of the dot function would clearly illustrate the repeated intervals. In this section, we will examine tangent graphs and other trigonometric functions. We begin with a graph of tangential function, plotting points, as we did for sinus and cosine functions. Recall that tanx= sinx cosx= sinx cosx The tangential function time is π π because the graph itself at intervals kπ kπ, where k k k is a constant. If the graph of the dot function on − π 2 − π 2 π 2 , π 2 , we can see the behavior of the graph on one complete cycle. If we look at a larger interval, we will see that the properties of the chart are repeated. We can determine whether tangent is an odd or even function by using the tangent definition. tan(−x)= sin(−x) cos(−x) Tangent definition. = −sin x cosx Sine is a special function, the cosine is aligned. =− sinx cosx Quotient odd and even function is odd. =−tanx Tangent definition. tan(−x)= sin(−x) cos(−x) Tangent definition. = −sin x cosx Sine is a special function, the cosine is aligned. =− sinx cosx Quotient odd and even function is odd. =−tanx Tangent definition. Tangent is therefore an odd function. We can further analyze the graphical behavior of tangential functions by looking at values for some special angles, as shown in Table 1. x x − π 2 - π 2 - π 3 - π 3 - π 4 - π 4 - π 6 - π 6 0 π 6 π 6 π 4 π 4 π 3 π 3 π 2 π and 2 tans(x) tan(x) undefined - 3 - 3 -1 - 3 3 - 3 3 - 3 0 3 3 3 3 3 3 undefined table 1 These points will help us draw our chart, but we need to determine how the chart behaves where it is not defined. If we look closely at the values at π 3 <x< π 2 , π 3 <x< π 2 , we can use the table to look for the trend. Since 3 π 3 ≈1.05 π 3 ≈1.05 and π 2 ≈1.57, π 2 ≈1.57, we evaluate x x on radian dimensions 1.05<x<1.57 1.05<x<1.57, as shown in Table 2. x 1.3 1.5 1.55 1.56 tanx tanx 3.6 14.1 48.1 92.6 Table 2 How x x x approaches π 2 , π 2 , function outputs increase and increase. Because y=tanx y=tanx is an odd function, we see the corresponding table of negative values in Table 3. x x -1.3 (1.5) (1.55) (1.56) tanx tanx -3.6 (14.1) (48.1) (92.6) Table 3 We see that with x x approaches - π 2 , - π 2 , outputs decrease and decrease. Note that there are some x x values for which cosx = 0. cosx=0. For example, cos(π 2)=0 cos(π 2)=0 and cos(3π 2)=0. cos(3π 2)=0. At these values, the tangential function is undefined, so the graph y=tanx y=tanx has discontinuities at x= π 2 and 3π 2 . x = π 2 and 3π 2 . At these values, the tangent graph has vertical asymptotes. Figure 1 represents the graph y=tanx. y=tanx. The tangent is positive from 0 to π 2 π 2 and from π π to 3π 2 , 3π 2 corresponding to quadrants I and III of the unit circle. Figure 1 Tangential function Graph As with sinus and cosine functions, tangential function can be described by a general equation. y=Atan(Bx) y=Atan(Bx) can identify horizontal and vertical segments and compresses using A A and B. B. Horizontal stretching can usually be determined from the graph period. For tangent charts, you often need to specify a vertical segment using a point in the chart. Because there are no maximum or minimum tangential function values, the term amplitude cannot be interpreted as it is for sinus and cosine functions. Instead, we use the phrase stretching/compression factor when referring to constant A.A. Stretching is | And |. | A |. Period is P = π | B | . P = π | B | . The domain is all real numbers x, x, where x≠ π 2| B | + π | B | k x≠ π 2| B | + π | B | k so that k k is an integer. The range is (−∞.∞). (−∞,∞). Asymptotes occur at x= π 2| B | + π | B | k, x= π 2| B | + π | B | k, where k k is an integer. y=Atan(Bx) y=Atan(Bx) is an odd function. We can use what we know about the properties of tangential function to quickly sketch a graph of any stretched and/or compressed tangential function of form f(x)=Atan(Bx). f(x)=Atan(Bx). We focus on one period of function including origin, because the periodic property allows us to extend the graph to the rest of the function domain if we so wish. Our limited domain is then the interval (− P 2 , P 2) (− P 2, P 2) and the graph has vertical asymptotes to ± P 2 ± P 2, where P = π B . P = π B . At (− π 2 , π 2), (− π 2 , π 2), the graph appears from the left asymptote to x=− π 2 , x=− π 2 , intersecing the beginning and will continue to increase as it approaches the right asymptote at x= π 2 . x = π 2 . If you want the function to approach asymptotes at the correct speed, we also need to set a vertical scale by actually evaluating the function for at least one point through which the chart passes. For example, we can use f(P 4)=Atan(B P 4)=Atan(B π 4B)=A f(P 4)=Atan(B P 4)=Atan(B π 4B)=A, because tan(π 4)=1. tan(π 4)=1. Due to function f(x)=Atan(Bx), f(x)=Atan(Bx), chart one dot. Identify stretching factor| And |. | A |. Identify B B and specify period, P = π | B | . P = π | B | . Draw vertical asymptotes at x=− P 2 x=− P 2 and x= P 2 . x = P 2 . For A>0, A>0, the graph approaches the left asymptote at negative output values and the right asymptote at positive output values (reverse for A<0 A<0). Plot the reference points in points a) - P 4 , A (0.0), (0.0) and (− P 4 ,−A), (− P 4 ,−A), and draw a graph with these points. Sketch a graph of one period of function y=0.5tan(π 2 x). y=0.5 tem(π 2 x). First we identify A and B. B. Because A = 0.5 A = 0.5 and B = π 2 , B = π 2 , we can find the stretching / compression factor and period. The period π π 2 =2, π π 2 =2, so the asymptotes are on x = ±1. x=±1. In the quarter period origin, we have f(0.5)=0.5tan(0.5π 2) = 0.5tan(π 4) = 0.5 f(0.5)=0.5tan(0.5π 2) =0.5tan(π 4) =0.5 That is, that the curve must pass through points (0.5,0,5), (0,5,0,5), (0,0), (0,0) and (-0,5,−0,5). (-0.5,-0.5). The only inflection point is at the beginning. Figure 2 shows a graph of one period of function. Sketch the chart f(x)=3tan(π 6 x). f(x)=3tan(π 6 x). Now that we can chart the tangent function that is stretched or compressed, we add a vertical and/or horizontal (or phase) offset. In this case, we add C C and D D to the general shape of the tangential function. f(x)=Atan(Bx−C)+D f(x)=Atan(Bx−C)+D The graph of the transformed tangential function differs from the basic tangential function of tanx tanx in several ways: The stretching factor is | And |. | A |. The period is π | B | . π | B | . Domain is x≠ C B + π | B | k, x≠ C B + π | B | k, where k k is an integer. The range is (−∞.∞). (−∞,∞). Vertical asymptotes occur at x= C B + π 2| B | k, x= C B + π 2| B | k, where k k is the odd integer. There is no amplitude. y=Atan(Bx-C)+D y=Atan(Bx-C)+D is an odd function because it is the proportion of odd and even functions (sine and cosine). For y=Atan(Bx−C)+D, y=Atan(Bx−C)+D, sketch a graph of one period. Express the function indicated in y=Atan(Bx−C)+D. y=Atan(Bx−C)+D. Identify the stretching/compression factor, | And |. | A |. Identify B B and specify period, P = π | B | . P = π | B | . Identify C C and specify phase shift C B . C B . Draw a graph y=Atan(Bx) y=Atan(Bx) shifted to the right c B C B and up D. D. Sketch the vertical asymptotes that occur at x = C B + π 2| B | k, x= C B + π 2| B | k, where k k is the odd integer. Plot any three reference points and draw a chart with those points. Graph of one part of function y=−2tan(πx+π)−1. y=−2tan(πx+π)−1. Step 1. The function is already written in the form y=Atan(Bx−C)+D. y=Atan(Bx−C)+D. Step 2. A=−2, A=−2, so stretching is | A |=2. | A |=2. Step 3. B=π, B=π, so the period is P= π | B | = π π =1. P = π | B | = π π =1. Step 4. C=−π, C=−π, so the phase shift is C B = −π π =−1. C B = −π π =−1. Step 5-7. Asymptotes are at x=− 3 2 x=− 3 2 and x=− 1 2 x=− 1 2 and the three recommended reference points are (-1,25,1), (-1,25,1), (-1,−1), (-1,−1) and − 0.75,−3). (-0.75,−3). The graph is shown in Figure 3. Note that this is a decreasing function because A<0. A<0. How would the chart in example 2 look different if we created A=2 A=2 instead of −2? −2? Identify horizontal and vertical segments relative to the graph of the dot function. Find the P P period from the gap between consecutive vertical asymptotes or x-intercepts. Write f(x)=Atan(π P x). π P x). Specify the appropriate point (x,f(x)) (x,f(x)) in the chart and use it to determine A.A. Find the formula for the function shown in Figure 4. Figure 4 Stretched tangent The graph takes the form of a tangent function. Step 1. One cycle extends from -4 to 4, so the period is P = 8. P=8. Since P = π | B | , P = π | B | , we have B = π P = π 8 . B = π P = π 8 . Step 2. The equation must be f(x)=Atan(π 8 x). f(x)=Atan(π 8 x). Step 3. To find the vertical section A, A, we can use the point (2,2). (2.2). 2=Atan(π 8)=Atan(π 4) 2=Atan(π 8)=Atan(π 4) Because tan(π 4)=1, tan(π 4)=1, A=2. A=2. This function should formula f(x)=2tan(π 8 x). f(x)=2tan(π 8 x). In Figure 5, you will find a formula for the function. The lace was defined by the reciprocal identity secx= 1 cosx . secx= 1 kosx . Note that the function is not defined if the cosine is 0, resulting in vertical asymptotes in π 2 , π 2 , 3π 2 , 3π 2, etc. Since the cosine is never more than 1 in absolute value, the secant, which is reciprocal, will never be less than 1 in absolute value. We can chart y = secx y = secx by observing the cosine function graph, because the two functions are mutual. See Figure 6. The cosine chart is displayed as an intermitteen orange wave, so we can see the relationship. Where the cosine function graph decreases, the secan function graph increases. Where the cosine function graph increases, the secan function graph decreases. If the cosine function is zero, the secan is not defined. The Sekan chart has vertical asymptotes at each x x value, where the cosine crosses the x-axis; we display them in the chart below with intermitteen vertical lines, but we do not display all asymptotes explicitly on all later charts involving secan and kosecant. Note that because the cosine is an even function, the secant is also an even function. That is, sec(−x)=secx. sec(−x)=secx. Figure 6 Secan function graph, f(x)=secx= 1 cosx f(x)=secx= 1 cosx As we did for the tangential function, we will refer again to the constant | A | | A | like stretching, not amplitude. Stretching is | And |. | A |. The time is 2π | B | . 2π | B | . Domain is x≠ π 2| B | k, x≠ π 2| B | k, where k k is the odd integer. The range is (−∞,−| A |] ∪[| And |,∞). (−∞,−| A |] ∪[| And |,∞). Vertical asymptotes occur at x= π 2| B | k, x= π 2| B | k, where k k is the odd integer. There is no amplitude. y=Asec(Bx) y=Asec(Bx) is an even function because cosine is an even function. Like the secual, the cosecant is defined by the reciprocal identity cscx= 1 sinx . cscx= 1 sinx . Note that the function is not defined if the sinus is 0, resulting in a vertical asymptote in the graph in 0, 0, π, π, and so on. Since the sinus is more than 1 in absolute value, cosecant, is reciprocal, will never be less than 1 in absolute value. We can chart y = cscx y = cscx by observing the sinus function graph, because the two functions are mutual. See Figure 7. The sinus chart is displayed as an intermitterupted orange wave, so we can see the relationship. If the sinus function graph decreases, the cossant function graph increases. If the sinus function graph increases, the cossant function graph decreases. The Cosecant chart has vertical asymptotes in each x x value, where the sine chart intersesses the x-axis; we display them in the chart below with intermitteer vertical lines. Note that because the sinus is an odd function, the kosecant function is also an odd function. This means that csc(−x)=−cscx. csc(−x)=−cscx. The graph of the orca, shown in Figure 7, is similar to the seance chart. Figure 7 Graph of the kosecant function, f(x)=cscx= 1 sinx f(x)=cscx= 1 sinx Stretching Factor is | And |. | A |. The time is 2π | B | . 2π | B | . Domain is x≠ π | B | k, x≠ π | B | k, where k k is an integer. The range is (−∞,−| A |]∪[| A |,∞). (−∞,−| A |]∪[| A |,∞). Asymptotes occur at x = π | B | k, x = π | B | k, where k k is an integer. y=Acsc(Bx) y=Acsc(Bx) is an odd function because the sinus is an odd function. For shifted, compressed and/or stretched versions of sequential and coseant functions, we can proceed using similar methods as for tangential and corrective. This means that we locate vertical asymptotes and also evaluate functions for several points (specifically local extremes). If we want to chart only one period, we can select an interval for the period in more than one way. The procedure for secan is very similar because the cofunction identity means that the secan chart is the same as the cosecant chart moved half a period to the left. Vertical and phase shifts can be applied to the cossant function in the same way as for secan and other functions. The equations become as follows. y=Asec(Bx−C)+D y=Asec(Bx−C)+D y=Acsc(Bx−C)+D y=Acsc(Bx−C)+D Stretching is | And |. | A |. The time is 2π | B | . 2π | B | . The domain is x≠ C B + π 2| B | k, x≠ C B + π 2| B | k, where k k is the odd integer. The range is (−∞,−| |+D]∪[| A |+D,∞). (-∞,−| |+D]∪[| A |+D,∞). Vertical asymptotes occur at x= C B + π 2| B | k, x= C B + π 2| B | k, where k k is the odd integer. There is no amplitude. y=Asec(Bx-C)+D y=Asec(Bx-C)+D is an even function because cosine is an even function. Stretching is | And |. | A |. The time is 2π | B | . 2π | B | . Domain is x≠ C B + π | B | k, x≠ C B + π | B | k, where k k is an integer. The range is (−∞,−| |+D]∪[| A |+D,∞). (−∞,−| A A |+D,∞). Vertical asymptotes occur at x = C B + π | B| k, x = C B + π | B| k, where k k is an integer. There is no amplitude. y=Acsc(Bx-C)+D y=Acsc(Bx-C)+D is an odd function because the sinus is an odd function. Since the function forms y = Asec(Bx), y = Asec(Bx), chart one period. Express the function given in the form y=Asec(Bx). y=Asec(Bx). Identify the stretching/compression factor| And |. | A |. Identify B B and specify period, P= 2π | B | . P= 2π | B | . Sketch graph y=Acos(Bx). y=Acos(Bx). Use the relationship between y=cosx y=cosx and y=secx y=secx to draw a graph y=Asec(Bx). y=Asec(Bx). Sketch asymptotes. Plot any two reference points and draw a chart with those points. Chart one period f(x)=2.5sec(0.4x). f(x)=2.5 s(0.4x). Step 1. The given function is already written in general form, y=Asec(Bx). y=Asec(Bx). Step 2. A=2.5 A=2.5, so stretching is 2.5. 2.5. Step 3. B=0.4 B=0.4, so P= 2π 0.4 =5π. P = 2π 0.4 =5π. The time is 5π 5π units. Step 4. Sketch the graph of the function g(x)=2.5cos(0.4x). g(x)=2.5cos(0.4x). Step 5. Using the relationship between cosine and sekanus, draw the function of kosecant. Steps 6-7. Sketch two asymptotes at x=1.25π x=1.25π and x=3.75π. x=3.75π. We can use two reference points, the local minimum (0,2,5) (0,2,5) and the local maximum at (2.5π,−2.5). (2.5π,−2.5). Figure 8 shows the graph. Chart one period f(x)=−2.5sec(0.4x). f(x)=−2.5 s(0.4x). Do vertical displacement and stretching/compression affect the secan range? Yes. Range f(x)=Asec(Bx−C)+D f(x)=Asec(Bx−C)+D is (−∞,−| |+D]∪[| A |+D,∞). (−∞,−| |+D]∪[| A |+D,∞). Due to the function of the shape f(x)=Asec(Bx−C)+D, f(x)=Asec(Bx−C)+D, graph one dot. Express the function indicated in y=Asec(Bx−C)+D. y=Asec(Bx−C)+D. Identify the stretching/compression factor, | And |. | A |. Identify B B and specify a period of 2π | B | . 2π | B | . Identify C C and specify phase shift C B . C B . Draw a graph y=Asec(Bx) y=Asec(Bx) , but slide it to the right c B C B and up d. D. Sketch the vertical asymptotes that occur at x = C B + π 2| B | k, x= C B + π 2| B | k, where k k is the odd integer. Graph one period y=4sec(π 3 x- π 2)+1. y=4sec(π 3 x- π 2)+1. Step 1. Express the function indicated in the form y=4sec(π 3 x- π 2)+1. y=4sec(π 3 x- π 2)+1. Step 2. The stretch/compression factor is | A |=4. | A |=4. Step 3. The time is 2π | B| = 2π π 3 = 2π 1 · 3 π =6 2π | B| = 2π π 3 = 2π 1 π 3 π =6 Step 4. The phase shift is C B = π 2 π 3 = π 2 π 3 π =1.5 C B = π 2 π 3 = π 2 3 π × 1.5 step 5. Draw graph y=Asec(Bx), y=Asec(Bx), but shift on the right c B =1.5 C B =1.5 and up by D=6. D=6. Step 6. Sketch the vertical asymptotes that occur when x=0.x=3, x=0,x=3, and x=6. x=6. The local minimum is (1,5,5) (1,5,5) and the local maximum (4.5,−3). (4.5,−3). Figure 9 shows the graph. Graph one period f(x)=−6sec(4x+2)−8. f(x)=−6sec(4x+2)−8. The domain cscx cscx was given to be all x x so that x≠kπ x≠kπ for all integer k. k. Would domain y = Acsc(Bx−C)+Dbex≠ C+kπ B ? y=Acsc(Bx−C)+Dbex≠ C+kπ B ? Yes. Excluded domain points follow vertical asymptotes. Their locations show the horizontal offset and compression or extension implied by the transformation to the input of the original function. Since the function forms y =Acsc(Bx), y = Acsc(Bx), graph one period. Express the function indicated in the form y=Acsc(Bx). y=Acsc(Bx). | A |. | A |. Identify B B and specify period, P= 2π | B | . P= 2π | B | . Draw a graph y = Asin(Bx). y=Asin(Bx). Use the relationship between y=sinx y=sinx and y=cscx y=cscx to draw a graph y=Acsc(Bx). y=Acsc(Bx). Sketch asymptotes. Plot any two reference points and draw a chart with those points. Chart one period f(x)=−3csc(4x). f(x)=−3csc(4x). Step 1. The given function is already written in general form, y=Acsc(Bx). y=Acsc(Bx). Step 2. | A |=| −3 |=3, | A |=| −3 |=3, so stretching is 3. Step 3. B=4, B=4, so P= 2π 4 = π 2 . P = 2π 4 = π 2 . The time is π 2 π 2 units. Step 4. Sketch the graph of the function g(x)=−3sin(4x). g(x)=−3sin(4x). Step 5. Use the sinus and cosecant functions to draw the kosecant function. Steps 6-7. Sketch three asymptotes at x=0.x= π 4 , x=0,x= π 4 and x= π 2 . x = π 2 . We can use two reference points, local maximum to (π 8 ,−3) (π 8 ,−3) and local minimum (3π 8,3). (3π 8 ,3). Figure 10 shows the graph. Chart one period f(x)=0.5csc(2x). f(x)=0.5c (2x). Due to the function of the shape f(x)=Acsc(Bx−C)+D, f(x)=Acsc(Bx−C)+D, graph one dot. Express the function indicated in y=Acsc(Bx−C)+D. y=Acsc(Bx−C)+D. Identify the stretching/compression factor, | And |. | A |. Identify B B and specify a period of 2π | B | . 2π | B | . Identify C C and specify phase shift C B . C B . Draw a chart y=Acsc(Bx) y=Acsc(Bx), but slide it to the right c B C B and up d. D. Sketch the vertical asymptotes that occur at x = C B + π | B | k, x = C B + π | B | k, where k k is an integer. Sketch graph y=2csc(π 2 x)+1. y=2csc(π 2 x)+1. What is the domain and scope of this feature? Step 1. Express the function indicated in the form y=2csc(π 2 x)+1. y=2csc(π 2 x)+1. Step 2. Identify the stretching/compression factor| A |=2. | A |=2. Step 3. The time is 2π | B | = 2π π 2 = 2π 1 · 2 π 2π | B | = 2π π 2 = 2π 1 · 2 π =4. Step 4. The phase offset is 0 π 2 =0. 0 π 2 =0. Step 5. Draw a chart y=Acsc(Bx) y=Acsc(Bx), but move it up D=1. D=1. Step 6. Sketch the vertical asymptotes that occur when x=0,x=2,x=4. x=0,x=2,x=4. The graph for this function is shown in Figure 11. Figure 11 Transformed cosecant functions Vertical asymptotes shown on the chart indicate one period of function, and local extremes at this interval are displayed by periods. Notice how the graph of the transformed cosecant is related to the graph f(x)=2sin(π 2 x)+1, f(x)=2sin(π 2 x)+1, displayed as an orange dashed wave. For the graph f(x)=2cos(π 2 x)+1 f(x)=2cos(π 2 x)+1 shown in Figure 12, sketch the graph g(x)=2sec(π 2 x)+1 g(x)=2sec(π 2 x)+1 on the same axis. The last trigonometric function we need to examine is conclusive. The cortangen is defined by the reciprocal identity cotx= 1 tanx . cotx= 1 tanx . Note that a function is not defined if the tangential function is 0, resulting in a vertical asymptote in the graph at 0.π, 0.π, and so on. Since all real numbers are the output of the tangential function, all real numbers are also the output of the cortangent function. We can chart y = cotx y = cotx by observing the graph of tangential function, because these two functions are mutual. See Figure 13. Where the tangential function graph decreases, the cortangent function graph increases. Where the tangential function graph increases, the cortangent function graph decreases. The cortangen graph has vertical asymptotes at each x x value, where tanx=0; tanx=0; we will display them in the chart below with dashed lines. Since the cortangent is a reciprocal tangent, cotx cotx has vertical asymptotes at all x x values, where tanx = 0, tanx = 0 and cotx = 0 cotx = 0 at all x x values, where tanx tanx has its vertical asymptotes. Figure 13 Of The Stretching Cortagent Function | And |. | A |. Period is P = π | B | . P = π | B | . Domain is x≠ π | B | k, x≠ π | B | k, where k k is an integer. The range is (−∞.∞). (−∞,∞). Asymptotes occur at x = π | B | k, x = π | B | k, where k k is an integer. y=Acot(Bx) y=Acot(Bx) is an odd function. We can transform the cortangent graph in much the same way we did for tangent. The equation becomes as follows. y=Acot(Bx−C)+D y=Acot(Bx−C)+D Stretching is | And |. | A |. The period is π | B | . π | B | . Domain is x≠ C B + π | B | k, x≠ C B + π | B | k, where k k is an integer. The range is (−∞.∞). (−∞,∞). Vertical asymptotes occur at x = C B + π | B | k, x = C B + π | B | k, where k k is an integer. There is no amplitude. y=Acot(Bx) y=Acot(Bx) is a special function because it is the proportion of even and odd functions (cosine and sinus) due to the modified cotangent f(x)=Acot(Bx), f(x)=Acot(Bx), graph one dot. Express the function in the form of f(x)=Acot(Bx). f(x)=Acot(Bx). Identify stretching factor| And |. | A |. Specify period, P = π | B | . P = π | B | . Draw a chart y=Atan(Bx). y=Atan(Bx). Plot any two reference points. Using the relationship between the dot and the cortangent, draw a graph y=Acot(Bx). y=Acot(Bx). Sketch asymptotes. Specify the stretching factor, period, and phase shift y=3cot(4x), y=3cot(4x), and then sketch the chart. Step 1. The expression of the function in the form of f(x)=Acot(Bx) f(x)=Acot(Bx) gives f(x)=3cot(4x). f(x)=3cot(4x). Step 2. Stretching is | A |=3. | A |=3. Step 3. Period is P = π 4 . P = π 4 . Step 4. Sketch graph y=3tan (4x). y=3tan (4x). Step 5. Plot two reference points. Two such points are (π 16.3) (π 16.3) and (3π 16 ,−3). (3π 16 ,−3). Step 6. Draw y=3cot(4x) with each other. y=3cot(4x). Step 7. Sketch asymptotes, x=0,x= π 4 . x=0,x= π 4 . The orange chart in Figure 14 shows y=3tan(4x) y=3tan(4x) and the blue chart shows y=3cot(4x). y=3cot(4x). Due to the modified function of the cortangent in the form of f(x)=Acot(Bx−C)+D, f(x)=Acot(Bx−C)+D, graph one period. Express the function in the form f(x)=Acot(Bx−C)+D. f(x)=Acot(Bx−C)+D. Identify stretching, | And |. | A |. Specify period, P = π | B | . P = π | B | . Identify phase shift C B . C B . Draw a graph y=Atan(Bx) y=Atan(Bx) shifted to the right c B C B and up D. D. Sketch asymptotes x= C B + π | B | k, x = C B + π | B | k, where k k is an integer. Plot any three reference points and draw a chart with those points. Sketch a graph of one period of function f(x)=4cot(π 8 x− π 2)−2. f(x)=4cot(π 8 x− π 2)−2. Step 1. The function is already written in general form f(x)=Acot(Bx−C)+D. f(x)=Acot(Bx−C)+D. Step 2. A=4, A=4, so stretching is 4. Step 3. B = π 8 , B = π 8 , so the period is P = π | B | = π π 8 =8. P = π | B | = π π 8 =8. Step 4. C = π 2 , C = π 2 , so the phase shift is C B = π 2 π 8 =4. C B = π 2 π 8 =4. Step 5. We draw f(x)=4tan(π 8 x− π 2)−2. f(x)=4tan(π 8 x− π 2)−2. Step 6-7. The three points we can use to guide the chart are (6.2),(8,−2), (6.2),(8,−2) and (10,−6). (10,-6). To do this f(x)=4cot(π 8 x− π 2)−2, we use the tangent-kangtan relationship. f(x)=4cot(π 8 x− π 2)−2. Step 8. Vertical asymptotes are x=4 x=4 and x=12. x=12. The graph is shown in Figure 15. Figure 15 One period of modified cortangent function Many real-world scenarios represent periodic functions and can be modeled by trigonometric functions. As an example, let's go back to the scenario from the partition opener. Have you already watched the beam formed by the rotating light on the police car and thought about moving the light beam itself over the wall? The periodic behavior of the distance that light shines as a function of time is obvious, but how do we determine the distance? We can use the tangential function. Suppose the function y=5tan(π 4 t) y=5tan(π 4 t) indicates the distance in the movement of the light beam from the top of the police car through the wall, where t t is the time in seconds and y y is the distance in tracks from the point on the wall directly opposite the police car. Find and interpret stretching factor and period. Graph in interval [0,5]. [0,5]. Exchang f(1) f(1) and discuss the value of the function on this input. From the general form y=Atan(Bt) y=Atan(Bt) we know that | A | | A | is a stretching factor and π B π B is the period. We see that the stretching factor is 5. This means that the beam of light shifts by 5 feet after the middle of the period. The time π π 4 = π 1啦 4 π =4. π π 4 = π 1 ∎ 4 π =4. That is, every 4 seconds, a beam of light sweeps the wall. The distance from the location opposite the police car increases as the police car approaches. For the function graph, we draw asymptote to t = 2 t = 2 and use the stretching factor and period. See Figure 17 period: f(1)=5tan(π 4 (1))=5(1)=5; f(1)=5tan(π 4 (1))=5(1)=5; After 1 second, the beam shifted 5 feet from the spot opposite the police car. Access these online resources for additional instruction and practice with graphs of other trigonometric functions. Charting Tangent Charting Cosecant and Secant Charting Cotangent 6.2 Section Exercise 1. Explain how you can use the sinusoidal function chart to chart y=cscx. y=cscx. 2. How do I use graph y=cosx y=cosx to create a y=secx chart? y=secx? 3. Explain why tanx tanx equals π. π. 4. Why aren't there any wiretaps in the y=cscx chart? y=cscx? 5. How is the y=cscx y=cscx period compared to the y=sinx period? y=sinx? For the following exercises, they compare each trigonometric function with one of the following graphs. 6. 7. 8. 9. For the following exercises, find the period and horizontal offset of each of the functions. 10. f(x)=2tan(4x-32) f(x)=2tan(4x-32) 11. h(x)=2sec(π 4 (x+1) h(x)=2sec(π 4 (x+1) 12. m(x)=6csc(π 3 x+π) m(x)=6csc(π 3 x+π) 13. If tanx=−1.5, tanx=−1.5, find a tan(−x). tan(−x). 14. If secx=2, secx=2, find sec(−x). sec(−x). 15. If cscx=−5, cscx=−5, find csc(−x). csc(−x). 16. If xsinx=2, xsinx=2, find (−x)sin(−x). (-x)sin(−x). For the following exercise, rewrite each expression so that the x x argument is positive. 17. cos(-x)cos(-x)+sin(-x) cos(-x)cos(-x)+sin(-x) 18. cos(-x)+tan(-x)sin(-x) cos( )+tan(−x)sin(−x) For the next exercise, outline two chart periods for each of the following functions. Identify the stretching factor, dot, and asymptotes. 19. f(x)=2tan(4x-32) f(x)=2tan(4x-32) 20. h(x)=2sec(π 4 (x+1) h(x)=2sec(π 4 (x+1) 21. m(x)=6csc(π 3 x+π) m(x)=6csc(π 3 x+π) 22. j(x)=tan(π 2 x) j(x)=tan(π 2 x) 23. p(x)=tan(x− π 2) p(x)=tan(x− π 2) 24. f(x)=4tan(x) f(x)=4tan(x) 25. f(x)=tan(x+ π 4) f(x)=tan(x+ π 4) 26. f(x)=πtan(πx−π)−π f(x)=πtan(πx−π)−π 27. f(x)=2csc(x) f(x)=2csc(x) 28. f(x)=− 1 4 csc(x) f(x)=− 1 4 csc(x) 29. f(x)=4sec(3x) f(x)=4sec(3x) 30. f(x)=−3cot(2x) f(x)=−3cot(2x) 31. f(x)=7sec(5x) f(x)=7sec(5x) 32. f(x)= 9 10 csc(πx) f(x)= 9 10 csc(πx) 33. f(x)=2csc(x+ π 4)−1 f(x)=2csc(x+ π 4)−1 34. f(x)=−s(x− π 3)−2 f(x)=−s(x− π 3)−2 35. f(x)= 7 5 csc(x− π 4) f(x)= 7 5 csc(x− π 4) 36. f(x)=5(cot(x+ π 2)−3) f(x)=5(cot(x+ π 2)−3) For subsequent exercises, locate and delineate two periodicals of periodical functions with a given stretching factor, | A |, | |, period and phase shift. 37. Dom curve, A=1, A=1, period π 3 ; π 3 ; and phase shift (h,k)=(π 4.2) (h,k)=(π 4.2) 38. The dosing curve A=−2, A=−2, period π 4 , π 4 and phase shift (h,k)=(− π 4 ,−2) (h,k)=(− π 4 ,−2) For the following exercises, find the equation for the graph of each function. 39. 40. 41. 42. 43. 44. 45. For the following exercises, use the chart calculator to chart the two periods of the function. Note: most graphics calculators do not have a cosecant button; therefore you will need to enter cscx cscx as 1 sinx . 1 sinx . 46. f(x)=| csc(x) | f(x)=| csc(x) | 47. f(x)=| cot(x) | f(x)=| cot(x) | 48. f(x)= 2 csc(x) f(x)= 2 csc(x) 49. f(x)= csc(x) sec(x) f(x)= csc(x) sec(x) 50. Graph f(x)=1+ with 2 (x)− tan 2 (x). f(x)=1+ with 2 (x)− tan 2 (x). What is the function displayed in the chart? 51. f(x)=sec(0.001x) f(x)=sec(0.001x) 52. f(x)=cot(100πx) f(x)=cot(100πx) 53. f(x)= sin 2 x+ cos 2 x f(x)= sin 2 x+ cos 2 x 54. Function f(x)=20tan(π 10 x) f(x)=20tan(π 10 x) indicates the distance in motion of the light beam from the police car through the wall for time x, x, in seconds and distance f(x), f(x), in the legs. Graph in interval [0,5]. [0,5]. Find and interpret stretching factor, period, and asymptote. Evaluate f(1) f(1) and f(2.5) f(2.5) and discuss the values of the function on these inputs. 55. Standing on the shore of the lake, the fisherman monuments the boat far in the distance to his left. Let x, x, measured in radians, be the angle of vision to the ship and the line directly north of its position. Suppose that directly north is 0 and x x is measured left and positively to the right. (See Figure 19.) The ship sails directly from the west directly to the east, and regardless of the curvature of the Earth, the distance d(x), d(x), in kilometers, from the fisherman to the boat is determined by the function d(x)= 1.5sec(x). d(x)=1.5 s(x). What is a reasonable domain for d(x)? d(x)? Graph d(x) d(x) on this domain. Find and discuss the meaning of all vertical asymptotes on chart d(x). d(x). Calculate and interpret d(− π 3). d(− π 3). Round to the second decimal place. Calculate and interpret d(π 6). d(π 6). Round to the second decimal place. What is the minimum distance between a fisherman and a boat? When will this happen? 56. The laser range meter is locked on a comet approaching Earth. The distance g(x), g(x), in kilometers, comets after x x days, for x x in an interval of 0 to 30 days, is given g(x)=250,000csc(π 30 x). g(x)=250,000 c(π 30 x). Graph g(x) g(x) in the interval [0.30]. [0.30]. Exchang g(5) g(5) and interpret the information. What is the minimum distance between a comet and Earth? When will this happen? To which constant in the equation does it correspond? Find and discuss the importance of all vertical asymptotes. 57. The camcorder is aimed at the rocket on the launch pad 2 miles from the camera. The cant angle from ground to rocket after x x π is π 120 x. π 120 x. Write a function expressing the altitude h(x), h(x), in miles of rocket above the ground after x x seconds. Ignore the curvature of the Earth. Graph h(x) h(x) in interval (0.60). (0.60). Evaluate and interpret the values h(0) h(0) and h(30). h(30). What happens to h(x) h(x) when x x is approaching 60 seconds? Interpret the meaning of this in terms of the problem. Problem.