I'm not robot

reCAPTCHA

Continue

Nth term formula for triangular numbers

 This is the Order of The Triangle Numbers: 1, 3, 6, 10, 15, 21, 28, 36, 45, ... This is simply the number of points in each triangular pattern: By adding another line of points and calculating all the points we can find the next sequence number. The first triangle has only one dot. The second triangle has another row with 2 additional points, makes 1 + 2 = 3 The third triangle has another line with 3 additional points, makes 1 + 2 + 3 = 6 Fourth has 1 + 2 + 3 + 4 = 10 etc! What about the dots in the 60th triangle? Rules We can create Rules so that we can calculate any triangular number. First, rearrange the dots like this: Then multiply the number of points, and the shape into a rectangle: It is now easy to know how many points: simply multiply n by n +1 The point in the rectangle = n(n+1) But remember we are doubling the number of points, so Point in the triangle = n(n+1)/2 We can use xn to mean the points in the n triangle, so we get the rule: Rule: xn = n(n+1)/2 Example: 5th Triangle Number is x5 = 5(5+1)/2 = 15 Example: the 60th is x60 = 60(60+1)/2 = 1830 Wouldn't it be much easier to use a formula than to add all those points? Example: You are nesting logs. There is enough ground for you to put 22 logs side by side. How many logs can you fit in a stack? x22 = 22(22+1)/2 = 253 Stacks may be very high, but you can enter 253 logs in them! Activity: A Walk in the Desert © 2018 MathsIsFun.com as a result of the European Union's General Data Protection Regulation (GDPR). We do not allow internet traffic to Byju websites from countries in the EU at this time. Tracking or performance measurement cookies are not served with this page. +100Berg with Yahoo Answers and earn 100 points today. Terms • Privacy • RSS • HelpAbout Answers • Community Guidelines • Leaderboards • Knowledge Partners • Points & When I think of triangle numbers, I imagine bowling pins. Let's draw a diagram. O O O OO O OO OOThat's 4th triangle number, equal to 10. Let's add a few more bowling pins to the chart. O O O OO O O NO O N NO N N NAha! Now we have a square, and it's easy to figure out how many bowling pins are in the square. Since there are 4 lines, 4^2 = 16.Note that we add an N amount that is almost the same as we have O to complete the square, except for the number of pins on the diagonal. That's always true - here's the same diagram for the 5th triangle number. O O O N N N What happens if we reduce Half of the square? We got rid of all the N's, but we also got rid of some O. In this case, because we have 5^2 = 25 pins, and 25 / 2 = 12.5. So, we reduced 12.5 pins, which left us with 12.5 pins. Note that we end up with remaining pins, which I will represent with ☻. Deleted Pins are fully represented by lowercase n and o(dots for spacing only). O O O O .nO O ☻ n .nO.o .n .n.n.o .n .n .n .nNow, we need to re-add the pins we subtract from the original triangle number. We will always end up subtracting half of the diagonal, so we need to add back half of the diagonal. How many pins are there on the diagonal? This equals the number of lines! So, here, the number of the 5th triangle is5^2 - 5^2 / 2 + 5 / 2(5^2 + 5) / 2This should be quite clear how to generalize it to the nth triangle number. I hope that helps! +100Berg with Yahoo Answers and earn 100 points today. Terms • Privacy • AdChoices • RSS • HelpAbout Answers • Community Guidelines • Leaderboards • Knowledge Partners • Points & Feedback Level) Formulas are usually written in the form n(n+1)/2. Google the triangle number to find this. Order squares in the form an^2 + bn + c. You first calculate the difference between the first few terms. It's called the first difference. First, the difference between n = 1 and n = 2, is 3a + b. The next one is 5a+ b.Then you calculate the difference between the differences. It's called the second difference. It's all the same, same as 2a. So a = (2nd difference)/2.Plug into the first difference and solve it for b.The first term is a*1^2 + b*1 + c = a + b + c. So plug in a and b, set the same as the first term, and solve for c.Example: Triangular number.1 difference: 2, 3, 4, 5, ... 2nd difference: 1, 1, 1, 1,... So 1 = 2a, or a = 1/2.First difference = 2 = 3a + b = 3/2 + b. So b = 2 - 3/2 = 1/2.First term = 1 = a + b + c = 1/2 + 1/2 + c. So c = 0.General n-th term = (1/2)n^2 + (1/2)n which is equivalent to (n^2 + n)/2 = n(n+1)/2 1 3 6 10 15 21 wt is the nth term and you can tell how you got it Think about how the order is defined. What do you add to each term? 1 + 2 + 3 + ... + (n-1) + n Find the number of this series to give the nth triangle number. You know how to get the order by adding numbers from 1 to n. so 1 is 1 2 is 1 + 2 = 3 3 is 1 + 2 + 3 = 6 nth is 1 + 2 + 3 + ... + n = S and so on. So you are looking for the number of numbers up to n and for now we will call this S. You can see that if n is five then one way of doing this is to say: 1 + 2 + 3 + 4 + 5 = S 5 + 4 + 3 + 2 + 1 = S 6 + 6 + 6 + 6 + 6 = 2S so S = 0.5 (5x6) can you see the pattern there? Can you apply this to n numbers? if n is 6 we will have S = 0.5(6x7) 0.5n ^2 +/- something ??? picture of several triangular numbers using dots forming right-angled triangles. what happens if you replicate the triangle, rotate it 180 and then put it together What is the nth term for the number of points on each side of the rectangle? so the number of dots in it? so that the number of points in the Triangle? The above principles are illustrated quite well here. Triangle 1,3,6,10,15,21, n(n+1)/2 I can remember that formula for a reason, I'll think about how I got the answer and get back to you if you want, if you just quote me and ask so I remember. (Original post by Elongar) The above principles are illustrated quite well here. I have absolutely no idea where it's going, but it seems to take a while to get there! I taught him a slightly different way of skipping all the pointless algebra they seem to be doing... It's not the best resource in the world, and I feel a little long. The main reason I refer to it is simply because of the geometric derivative of the triangular number formula, i.e. consider the rectangle x (which I think animation helps in communicating). Nth triangle number given by: 1 + 2 + 3 + ... + (n-2) + (n-1) + n Now to add it all: Let S be the number of integers n first S = 1 + 2 + 3 + ... + (n-2) + (n-1) + n Now rewrite this again to the back: S = n + (n-1) + n-2) + ... + 3 + 2 + 1 Add these two equations together -equal: 2S = (n+1) + (n+1) + ... + (n+1) + (n+1) i.e. 2S = (n+1) added to itself n times 2S = n(n+1) S = n(n+1)/2 We provide three proofs here that the number of the 1+2+3+...+n is n(n+1)/2. The first is a visual that only involves formulas for rectangular areas. This was followed by two pieces of evidence using algebra. The first to use ... notation and the second introduce you to Sigma notation which makes the evidence more precise. We can visualize the sum of 1 + 2 + 3 + ... + n as a dot triangle. Numbers that have such a period pattern are called Triangles (or triangles) of numbers, written T(n), the number of integers from 1 to n : n123456 T(n) as the sum of11+21+2+31+2+3+41..51..6 T(n) as a triangle ... T(n)=136101521 For proof, we will calculate the number of points in T(n) but, instead of summarizing the numbers 1, 2, 3, etc. up to n we will find a total using only one multiplication and one division! To do this, we'll load two triangular copies of the dot together, one red and a green inverted copy. For example T(4)=1+2+3+4 + = Note that we get a rectangle that has the same number of rows (4) but has one additional column (5) so the rectangle is 4 by 5 therefore contains 4x5 = 20 balls but we take two copies of T (4) to get this so we have to have 20/2 = 10 balls in T(4), which we can check easily. This visual evidence applies to the size of any triangular number. Here it is again on T(5): + = So T(5) is half of the rectangle point 5 height and width of 6, ie half of 30 points, T(5)=15. Try the formula for yourself with this Quiz (click the button) that opens in a new window. After quizzes, close the window and try this button again for more Quiz questions. For T(n)=1+2+3+...+n we take two copies and get a rectangle i.e. n by (n+1). So there you have to have - Our visual proof that T(n) = 1 + 2 + 3 + ... + n = n(n + 1)/2 The same evidence uses algebra! Here's how a mathematician can write down the above evidence using algebra: T(n)+T(n) = 1 + 2 + 3 + ... + (n-1) +n +n + (n-1) + (n-2) +... + 2 + 1 Two copies, one red and the other, reversed, in green = (1 + n) + (2 + n-1) +(3 + n-2) + ... +(n-1 + 2) +(n + 1) pair the term, red with green = (n +1) + (n+1) + (n+1) 1) + (n+1)1) + (n+1) + ... +(n+1) + (n+1)All n pairs are equal to (n+1) 2 T(n) = n (n+1) T(n) = n (n+1) / 2 Using Sigma notation Some people assume ... too vague and want a more appropriate alternative. For this reason, in summing the series, sigma notation is used. Sigma is the Greek letter name for the English s, written as (like an M on its side) as a capital letter and (like a small falling b) in lowercase. In this case, s stands for sum. (The high curly shape S provides a mathematical symbol for integration - another type of quantity). Mathematicians use sigma modal for the number of series as follows: the formula describes the ith term of the summed series. It is written after sigma; the starting value for i is written below sigma; The final value for i is written above sigma i=the final value(formula for the term ith) i=the starting value In fact, the formula after sigma can be written in terms of any variable not only i, e.g. k, but then we must indicate which one varies in quantity below sigma. Often variables are omitted above sigma but never omitted below sigma. Here are some examples: Sum of 102 + 112 + 122, where the number added is the square number i2: i = 12i2 i = 10 The same amount can also be written in many other ways, for example, as the number of square numbers (i +9)2 where this time I went from 1 to 3 i = 3 (i +9)2 i = 1 or as the number of square numbers (i +11)2 where this time I went from -1 to 1 (ie. i = -1, 0 and 1) i=1(i+11)2 i=-1 Sum of 1+2+3+.. +9 is T(9) or i=9i T(9) = i=1 Here is T(n) which is 1+2+3+...+n, this time eliminating the second use of i above sigma: ni = T(n) i=1 and this time, we have T(n) but it is written backwards: n+ (n-1) + ... 3 + 2 + 1 where the term ith is now n +1–i for i from 1 to n: i=n(n+1–i) = T(n) i=1 Finally, note that if all terms are independent of variables, for example if there is no i in the formula but the variable below sigma is i, then all terms are constant. The number of terms will be given by the start and end values. Here, all terms are fixed (constants) at 3: i= 73 = 3+3+3+3 = 12 i=4 Here is evidence of algebra from above but now written using sigma notation: T(n)+T(n) = i=ni i=1 + i=n(n+1–i) i=1 Two copies, one red and the other, reversed, in green 2 T(n) = i=n(i i=1 pair of terms, red with green 2 T(n) = i=n(n+1) i=1 n copies i do not appear in the formula so all terms are the same 2 T(n)=n (n+1) T(n)= n (n+1) / 2 Back to the Results page runsums