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$\begingroup$ I have trouble proving $$abla\times(abla f)=$0 using index notation. I started with: $$(\hat{e_i}\partial_i)\times(\hat{e_j}\partial_j f)=\partial_i\partial_jf\e_i\)\times\hat{e_j})=\epsilon_{ijk}(\partial_i\partial_j f)partial_i partial_j partial_j partial_i\hat{e_k}$$ $\endgroup$ Gradient, divergence and curl, commonly referred to as grad, div and curl, refers to a family widely used by differential operators and related notations that we will reach shortly. We'll see later that each has a physical significance. But even if they were only short for 1, they would be worth using. The good transcript is not only shorter, but also helps understanding the forest by hiding trees. For example, one of Maxwell's equations (relating to the electric field \(\vE\) and the magnetic field \(\vB\)) written without use this notation is \begi{align*} &\Big(\frac{\partial E_3}{\partialy y} -\frac{\partially E_2}{\partialz}\Big)\hi -\Big(\frac{\partially E_3}\partial x} -\frac{\partial E_1}{\partialz}\Big)\hj +\Big(\frac{\partial E_2}{\partial x} -\frac{\partial E_1}{\partially y}\Big)\hk\ &\hskip2.5in\-\frac{1}{\Big(\frac{\partial B_1}\partial t} \hi +\frac{\partial B_2}{\partial t} \hj +\frac{\partial B_3}{\partial t} \hk\Big) \end{align*} The same equation written using this notation is \hk\Big) \end{align*} The same equation written using this notation is \h begins{equation*} \nabla\times\vE = -\frac{1}{c}\frac{\partial \v}{\partial t} \end{equation*} The shortest way to write (and the easiest way to remember) gradient , divergence and curl uses the symbol \(\nabla\) which is a differential operator as \(\frac{\partial }{\partial x}\text{.} \) It is defined by \begin{equation*} \nabla = \hi\frac{\partial }{\partial x} +\hj\frac{\partial }{\partial y} +\hk\frac{\partial }{\partial z} \end{equation*} and is called del or nabla. Here are the definitions. The gradient of a scalar function \(f(x,y,z)\) is the vector field \begin{equation*} \text{grad}\,f=\nabla f = \frac{\partialf}{\partial x}\hi +\frac{\partial f}{\partial y}\hj +\frac{\partial f}{\partial z} \hk \end{equation*} Note that the entry, \(f\text{,}\) for the gradient is a scalar function, while output,\(\nabla f\text{,}\) is a vector value function. The divergence of a vector field \(\vF(x,y,z)\) is the evaluated scalar function \begi{equation*} \text{div}\,\vF=\nabla\cdot\vF = \frac{\partially F_1}{\<\<\<\vF=\nabla\cdot\vF = \frac{\partially F_1}{\<\<\<\partially x} +\frac{\partial F_2}{\partialy y} +\frac{\partial F_3}{\partial z} \end{equation*} Note that the entry, \(\vF\text{,}\) for the divergence is a function with vector value, while output, \(\nabla\cdot\vF\text{ ,}\) is a scalar function. Loop of a vector field is the vector field \begin{equation*} \text{curl}\,\vF= \nabla\times\vF = \Big(\frac{\partial F_3}{\partial y} y} F_2}{\partial z}\Big)\hi -\Big\frac{\partial F_3}{\partial x} -\frac{\partial F_1}{\partial z}\Big)\Big\Big\frac{\partial F_2}{\partial x} -\frac{\partial F_1}{\partial y}\Big)\hk \end{equation*} Note \(\vF\text{,}\) for the loop is a vector value function, and production, \(\nabla\times\vF\text{,}\) is again a vector value function. Laplacian 2 of a scalar function \(f(x,y,z)\) is the evaluated scalar function \begin{equation*} \Delta f= \nabla^2 f =\nabla\cdot\nabla f = \frac{\partial^2 f}{\partial x^2} +\frac{\partial^2 f\partial^2 partial^2 f\partial^2 f\partial^2 f\partial^2 f\partial^2 f\partial^2 f\partial^2 f\partial^2 f2\partial^2 f\partial^2 f\partial^2 f\partial^2 frac{\partial^2 f}{\partial z^2} \end{equation*} Laplacian of a vector field \(\vF(x,y,z)\) is the vector field \begin{equation*} \Delta\vF= \nabla^2\vF =\nabla\cdot\nabla \vF = \frac{\partial^2 \vF}\partial^ 2} +\frac{\partial^2 \vF{}\partial y^2} +\frac}\partial^2 \vF{\partial y^2} +\frac{\partial^2 \vF{\partial y^2} +\frac{\partial^2 \vF{\partial y^2} +\frac{\partial^2 \vF{\partial y^2} +\frac{\partial^2 \vF{\partial y^2} +\frac{\partial^2 \vF{}\partial y^2} +\frac{\partial^2 \vF{\partial y^2} +\frac{\partial^2 \vF{}\partial y^2} +\frac{\ partial^2 \vF}{\partial z^2} \end{ecacy*} Note that the laplacian maps either a scalar function with a scalar value to a scalar value function or a vector value function to a vector value function. Pierre-Simon Laplace (1749–1827) was a French mathematician and astronomer. He is also Laplace from Laplace's equation, Laplace's transformation, and Laplace-Bayes estimator. He was Napoleon's examiner when Napoleon attended the Ecole Militaire in Paris.The gradient, divergence and laplacian all have obvious generalizations to dimensions other than three. That's not the case for the loop. It has a generalization, far from being obvious, that uses differential forms. Differential forms are well beyond our scope, but are introduced in optional §4.7. As an example of an application in which both divergence and loop appear, we have Maxwell's equations 3 4 5 , which form the foundation of classical electromagnetism. \begin{align*} \nabla\cdot\vE &= 4\pi\rho\\ \nabla\cdot\vB &= 0\\ \nabla\times\vE +\frac{1}{c}\frac{\partial\vB}{\partial t}&a 0\\ \nabla\times\vB -\frac{1}{c}\frac{\partial\v}{\partial t}&=\frac{4\pi}{c}\vJ \end{align*} Here \(\vE\) is the electric field, \(\vB\) is the magnetic field, \(\rho\) is the load density, \(\vJ\) is the current density and \(c\) is the speed of light. To be pretentious, these are Maxwell's equations in the absence of a material environment and in Gaussian units. An important consequence of Maxwell's equations is that electromagnetic radiation, it would be light, propagates at the speed of light. James Clerk Maxwell (1831–1879) was a physicist In a survey of prominent physicists, Maxwell was voted the third greatest physicist of all time. Only Newton and Einstein defeated him. Two extremely computationalally important properties of the derivative \(\diff{\ }{x}\) are the linearity and rule of the product. \begin{align*} \diff{\ }{x}\big(af(x)+bg(x)\big) &=a\diff{f}{x}(x)+b\diff{g}{x}(x)\\ \diff{\ }{x}\big(f(x)\,g(x)\big) }{x}\big(f(x)\,g(x)\big) \end{align*} Gradient, divergence and curl also have properties like these, which really stem (often easily) from them. First, here are the statements of a bunch of them. (A memory help and evidence will come later.) Actually, there are a huge number of them here. Many are included only for completeness. Only a relatively small number are used a lot. I'm in red. \(\displaystyle \color{red}{\nabla(f+g)=\nabla f+\nabla g}\) \(\color{red}{\nabla(cf)=c\,\nabla f}\text{,}\) for any constant \(c\) \(\displaystyle \{{}\nabla({{\nabla({\nabla({{\nabla({\nabla({{}\nabla({\nabla({{}\nabla()\({{\nabla()\(\displaystyle \{{}\nabla({{\nabla({{\nabla()\({{\nabla()\({{\nabla()\(\displaystyle\{{}\nabla({{}\nabla()\({{\nabla({{\nabla({}\nabla({{\f g)=(\nabla f)g+ f(\nabla g)}\) \(\nabla(f/g)=\big(g\,\nabla f-f\,\nabla g\big)/g^2\) at \(\vx\) where \(g(\vx)e 0\text{.} \) \(\nabla(\vF\cdot\vG)=\vF\times(\nabla\times\vG)- (\nabla\times\vF)\times\vG+(\vG\cdot\nabla)\vF+(\vF\cdot\nabla)\vG\) Here 6 \{{ begi equation*} (\vG\cdot\nabla)\vF =\vG_1\frac{\partial\vF}{\partial x} +\vG_2\frac{\partial\vF}{\partial y} +\vG_3\frac{\partial\vF}{\partial z} \end{equation*} This is actually the only definition that makes sense. For example \(\vG\cdot(\nabla\vF)\) does not make sense , because you cannot take the gradient of a function with a vector value. \(\displaystyle \color{red}{\nabla\cdot(\vF+\vG)=\nabla\cdot\vF+\nabla \cdot\vG}\) \(\color{red}{\nabla\cdot(c\vF)=c\,\nabla\cdot\vF}\text{,}\) for any constant \(c\) \(\displaystyle \color{red}{\nabla\cdot(f\vF)=(\nabla f)\cdot\vF+ f\,\nabla\cdot\vF}\) \(\displaystyle \nabla\cdot(\vF\times\vG)=(\nabla\times\vF)\cdot\vG - \vF\cdot(\nabla\times\vG)\) \(\displaystyle \color{red}{\nabla\times(\vF+\vG)=\nabla\times\vF+\nabla \times\vG} \) \(\color{red}{\nabla\times(c\vF)=c\,\nabla\times\vF} \text{,}\) for any constant \(c\) \(\displaystyle \color{red}{\nabla\times(f\vF)=(\nabla f)\times\vF+f\,\nabla\times\vF} \) \(\nabla\times(\vF\times\vG) =\vF(\nabla\cdot\vG)-(\nabla\cdot\vF)\vG + (\vG\cdot\nabla)\vF-(\vF\cdot\nabla)\vG\) Here \begin{equation*} (\vG\cdot\nabla)\vF =\vG_1\frac{\partial\vF}{\partial x} +\vG_2\frac{\partial\vF}{\partial y} +\vG_3\frac{\partial\vF}{\partial z} \end{equation*} \(\displaystyle \color{red}{ \nabla^2(f+g)=\ \nabla^2 f+\nabla^2 g }\) \(\color{red}{\nabla^2(cf)=c\,\nabla^2 f}\text{,}\) for any constant \(c\) \(\displaystyle \nabla^2(fg)=f\,\nabla^2 g+2\nabla f\cdot\nabla g+g\,\nabla^2 f\) \(\color{red}{\nabla\cdot(\nabla\times\vF)=0 \qquad}\) (curl divergence) \(\color{red}{\nabla\times(\nabla f)=0 \qquad}\) (curl of gradient) \(\displaystyle \nabla\cdot\big(f\{\nabla g\times\nabla h\}\big) =\nabla f\cdot(\nabla g\times\nabla h)\) \(\displaystyle \nabla\cdot(f\nabla g-g\nabla f)=f\,\nabla^2g-g\,\nabla^2f\) \(\nabla\times(\nabla\times\vF)=\nabla(\nabla\cdot\vF)-\nabla^2\vF \qquad\) (curl of curl) Memory Aid. Most of the vector identities (in fact, however, with the exception of the Theorem Theorem 4.1.5.d and Theorem 4.1.7) are very easy to guess. Guessed. combines conventional linearity and product rules with the facts that if the left side is a vector (scaler), then the right side must be, also, a vector (scaler) and the only valid products of the two vectors are dot and cross products and the product of a scalar with a scaler or vector cannot be either a point product or a cross product and \(\vA\times\vB = \ vB\times\vA\text{.} \) (Cross product is antisymmetric.) For example, consider Theorem 4.1.4.c, which says \(\nabla\cdot(f\vF)=(\nabla f)\cdot\vF+ f\,\nabla\cdot\vF\text{.} \) The left side, \(\nabla\cdot(f\vF)\text{,}\) is a scaler, so the right side must also be a scaler. The left side, \(\nabla\cdot(f\vF)\text{,}\) is a derivative of the product \(f\) and \(\vF\text{,}\) thus, mimitting the product rule, the right side will be a sum of two terms, one with \(\vF\) multiplying a derivative of \(f\text{,}\) and one with \(f\) multiplying a derivative of \(\vF\text{.} \) The derivative acting on \(f\) must be \(\nabla f\text{,}\) because \(\nabla\cdot f\) and \(\nabla\times f\) are not well defined. To arrive with a scaler, rather than a vector, we need to take the product point of \(\nabla f\) and \(\vF\text{.} \) So that term is \((\nabla f)\cdot\vF\text{.} \) The derivative acting on \(\vF\) must be either \(\nabla\cdot\vF\) or \(\nabla\times\vF\text{.} \) Also we need to multiply with the \(f\) scaler and end up with a scaler. So the derivative must be a scalar, i.e. \(\nabla\cdot\vF\) and that term is \(f\{\nabla\cdot\vF\}\text{.} \) Our final assumption is \(\nabla\cdot(f\vF)=(\nabla f)\cdot\vF+ f\,\nabla\cdot\vF\text{,} fortunately, it's correct. All the evidence (except for that in Theorem 4.1.7.c,d, to which we will return later) consists of writing the definition of the left side and writing the definition of the right side and observing (possibly after a small manipulation) that they are the same. For Theorem 4.1.3.a,b, Theorem 4.1.4.a,b, Theorem 4.1.5.a,b and Theorem 4.1.6.a,b, the calculation is trivial — a line on identity, if an effective notation is used. Rename the coordinates \(x,y,z\) to \(x_1,x_2,x_3\) and standard unit base vectors \(\hi\text{,}\) \(\hj\text{,}\) \(\hk\) to \(\hi_1\text{{ ,}\) \(\hi_2\text{,}\) \(\hi_3\text{.} \) Then \(\nabla = \sum_{n=1}^3\hi_n\frac{\partial\ }{\partial x_n}\) and proof, for example, Theorem 4.1.4.a is \begin{align*} \nabla\cdot(\vF+\vG) &=\sum_{n=1}^3\frac{\partial\ }{\partial x_n} \hi_n\cdot(\vF+\vG)\\ &=\sum_{n=1}^3\ frac{\partially sum_}{partial x_n} \hi_n\cdot\vF +\sum_{n=1}^3\frac{\partially x_n} \hi_n\cdot\vG =\nabla\cdot\vF+\nabla \cdot\vG \end{*} For Theorem 4.1.3. c,d, Theorem 4.1.4.c, Theorem 4.1.5.c and Theorem 4.1.6.c, calculation is easy — lines per identity. For example, the proof of Theorem 4.1.5.c is \begin{align*} \begin{align*} &=sum_{n=1}^3\frac{\partially }{\partially x_n}\hi_n\times(f \vF) =\sum_{n=1}^3\frac{\partial }{\partially x_n}\big(f\, \{\hi_n\times\vF\}\big)\\ &=sum_{n=1}^3\frac{\partial f}{\partial x_n}\hi_n\times\vF +f\sum_{n=1}^3\frac{\partial }{\partial x_n}\hi_n\times\vF\&=(\nabla f)\times\v ,\nabla\times\vF \end{align*} In the second line we used Theorem 1.1.2.b. For Theorem 4.1.4.d, calculation is also easy if you use the fact that \begin{equation*} \va\cdot(\vb\times\vc)==va\times\vb)\cdot\vc \end{equation*} which is Lemme 4.1.8.a below. This leaves the evidence of the theorem 4.1.3.e, Theorem 4.1.5.d, Theorem 4.1.7.a,b,c,d,e, which we write explicitly. Theorem 4.1.3.e: First write the left side as \begin{gather*} \nabla(\vF\cdot\vG) =\sum_{n=1}^3\hi_n\frac{\partial }{\partial x_n}(\vF\cdot\vG) =\sum_{n=1}^3\hi_n\Big\frac{\partial\vF}{\partial x_n}\cdot\vG\Big) +\sum_{n=1}^3\hi_n\Big(\vF\cdot\frac{\partial\vG}{\partial x_n hi_n <3> <4>}\Large) \end{gather*} Then rewrite \(\va\times(\vb\times\vc)=(\vc\cdot\ va)\vb-(\vb\cdot\va)\vc\ text{,}\) which is Lemme 4.1.8.b below, as \begin{equation*} (\vc\cdot\va)\vb=\va\times(\vb\times\vc)+(\vb\cdot\va)\vc \end{equation*} Apply it with \(\vb=\hi_n\text{,}\) \} \}(\vc=\frac{\partial\vF}{\partial x_n}\text{,}\) \(\va=\vG\) and once with \(\vb=\hi_n\text{,}\) \(\vc=\frac{\partial\vG}{\partial x_n}\text{ ,}\) \(\va=\vF\) gives \begii{align*} \nabla(\vF\cdot\vG) &=\sum_{n=1}^3\bigg[\vG\times\Big\hi_n\times\frac{\partial\ \vF}{\x_n partial}\Mare) +(\vG\cdot\hi_n)\frac{\partial\vF}{\partial x_n}\bigg]\\ &\hskip1in+\sum_{n=1}^3\bigg[\vF\times\Big\hi_n\times\frac{\partial\vG}{\partial x_n}\Big) + (\vF\cdot\hi_n)\frac{\partial\vG}{\partial x_n}\bigg]\\ &==\vG\times(\nabla\times\vF) +(\vG\cdot\nabla)\vF +\ \vF\times(\nabla\times\vG) +(\vF\cdot\nabla)\vG \end{align*} Theorem 4.1.5.d : We use the same trick. Write the left side as \begin{align*} \nabla\times(\vF\times\vG) &==sum_{n=1}^3\hi_n\times\frac{\partial }{\partial x_n}(\vF\times\vG)\\ &==sum_{n=1}^3\hi_n\times\Big\\partial\vF}{\x_n}\times\vG\Big) +\sum_{n=1}^3\hi_n\times\Big\\times\frac{\partial\vG}{\partial x_n\Mare) \end{align*} Application \(\va\times\\times\vc)=(\vc\cdot\va)\vb\\cdot\va ,\) which is Lemme 4.1.8.b below, \begin{align*} \nabla\times(\vF\times\vG)&= \sum_{n=1}^3\Big[G_n\frac{\partial \vF}{\partial x_n} -\frac{\partial F_n}{\partial x_n}\vG\Big] +\sum_{n=1}^3\Big\frac{\partial G_n}{\partially x_n}\vF F_n\frac{\partial \vG}{\partial x_n}\Large]\cr }\cr &=(\vG\cdot\nabla)\vF -(\nabla\cdot\vF)\vG+(\nabla\cdot\vG)\vF -(\vF\cdot\nabla)\vG \end{align Theorem 4.1.7.a : Substitution in \begin{align*} \nabla\times\vF &=Big\\frac{\partial F_3}{\partial y} -\frac{\partial F_2}{\partial z}\Big)\hi --\Big\frac{\partial F_3}{\partial x} F_1}{\partialz}\Big)\hj +\Big\\frac{\partial F_2}{\partial x} x} F_1}{\partial y}\Big)\hk\\ \end{align*} gives \begin{align*} \nabla\cdot\big(\nabla\times F\big) &=\frac{\partial }{{{partial }{{\partial x}\Big\\frac{\partially F_3}{\partialy y} -\frac{\partial F_2}{\partially z}\\frac{\partial }{\partial y}\Big\frac{\partial F_3}{\partial x} -\frac{\partial F_1}{\partial z}\Big) +\frac{\partial }{\partial z}\Big\frac{\partial F_2}{\partial x} -\frac{\partial F_1 ly}{\partialy y}\Big)\cr &=color{fr 2 F_3}{\ partial x\partial y}} -\color{blue}{\frac{\partial^2 F_2}{\partial x\partial z}} -\color{red}{\frac{\partial^2 F_3}{\partial y\partial x}} +\frac{\partial^2 F_1}{{\partial y\partial z} +\color{blue}{\frac{\partial^2 F_2}{\partial z\partial x}} -\frac{\partial^2 F_1}{\partial z\partial y}}&=0 \end{align*} because the two red terms were canceled , the two blue terms were cancelled, and the two black terms were canceled. Theorem 4.1.7.b: Replace in \begin{align*} \nabla f &=\frac{\partial f}{\partial x}\hi +\frac{\partial f}{\partial y}\hj +\frac{\partial f}{\partial z}\hk\cr\\ \end{align*} gives \begin{align*} \nabla\times\big(\nabla f\big) &=Big\frac{\partial }{\partial y}\frac{\partial f}{\partial z} -\frac{\partial }{\partial z}\frac{\partial f}{\partial y}\Big) \hi -\\partial z}\frac{\partial f}{\partial y}\Big) \hi -\\partial z}\frac{\partial f}{\partial y}\Big) \hi -\\ partial z}\frac{\partial f}{\partial y}\Big) \hi -\Big\\frac{\partial }{\partial x}\frac{\partial f}{\partial z} -\frac{\partial }{\partial z}\frac{\partial f}{\partial x}\Big) \ hj\ &\hski p1in+\Big\frac{\partial }{\partial x}\frac{\partial f}{\partial y} -\frac{\partial }{\partial y}\frac{\partial f}{\partial x}\Big) \hk\\ &=0 \end{align*} Theorem 4.1.7.c : By Theorem 4.1.4.c, followed by Theorem 4.1.4.d, \begin{align*} \nabla\cdot\big[f(\nabla g\times\nabla h)\big] &=\nabla f\cdot(\nabla g\times\nabla h) +f\nabla\cdot(\nabla g\times\nabla h)\\ &=\nabla f\cdot(\nabla g\times\nabla h) +f\big[(\nabla\times\nabla g)\ cdot\nabla h-\nabla g\cdot(\nabla\times\nabla h)\big] \end{align*} By Theorem 4.1.7.b , \(\nabla\times\nabla g=\nabla\times\nabla h=0\text{,}\) so \start{equation*} \nabla\cdot\big[f(\nabla g\times\nabla f)\big] =\nabla f\cdot(\nabla g\times\nabla h) \end{equation*} Theorem 4.1.7.d: By Theorem 4.1.4.c, \begin{align*} \nabla\cdot(f\nabla g\g\nabla f) &= (\nabla f)\cdot(\nabla g) +f\ ,\nabla\cdot(\nabla g) -(\nabla g)\cdot(\nabla f) +g\ ,\nabla\cdot(\nabla f)\\ &=f\,\nabla^2g-g\,\nabla^2f \end{align*} Theorem 4.1.7.e: \begin{align*} \nabla\times(\nabla\times\vF) &=\sum_{\ell=1}^3\hi_\ell\frac{\partial } {\partial x_\ell} \times\bigg(\sum_{m=1}^3 \hi_m\frac{\partial } {\partial x_m} \times\sum_{n=1}^3\hi_n F_n\bigg)\\ &=sum_{\ell ,m ,n=1}^3\hi_\ell\times \frac{\partial^2 F_n } {\partial x_\ell\partial x_m} \end{align*} Using \(\va\times(\vb\times\vc)=(\vc\cdot\va)\vb-(\vb\cdot\va)\vc\text{,}\) avem \begin{ecuație*} \hi_\ell\times \big(\hi_m\times\hi_n\big) = (\hi_\ell\cdot\hi_n)\hi_m =(\hi_\ell\cdot\hi_n)\hi_m =\delta_{\ell,n}\hi_m -\delta_{\ell,m}\hi_n \end{equation*} where 7 \begin{equation*} \delta_{m,n} =\begin{houses} 1& \text{if } m=n \\ 0& \text{if } me n \end{houses} \end{equeay*} Therefore \\start{align*} \nabla\times(\nabla\times\vF) &=\sum_{\ell,m,n=1}^3\delta_{\ell,n}\hi_m\ \frac{\partial^2 F_n } {\partial x_\ell\partial x_m} -\sum_{\ell ,m,n=1}^3\delta_{\ell,m}\hi_n\ \frac{\partial^2 F_n } {\partial x_\ell\partially x_m}}&=\sum_{m,n=1}^3\hi_m\ \frac{\partial {\partial x_m} \frac{\partial F_n x_m\partial }{\partial partial x_n} -\sum_{m,n=1}^3\hi_n\ \frac{\partial^2 F_n}{\partial x_m^2}\cr &= \nabla(\nabla\cdot\vF)-\nabla^2\vF \end{align*} \(\delta_{m ,n}\) is called the Kronecker delta function. It is named after the German theorist and logician Leopold Kronecker (1823–1891). He is famous for being toldGod made integers. All others are the works of man. \(\displaystyle \va\cdot(\vb\times\vc)=(\va\times\vb)\cdot\vc\) \(\displaystyle \va\times(\vb\times\vc)=(\vc\cdot\va)\vb-(\vb\cdot\va)\vc\) (a) Here are two pieces of evidence. For the first time, just write both parts \begin{align*} \va\cdot(\vb\times\vc) &=(a_1,a_2,a_3)\cdot(b_2c_3-b_3c_2\,\,b_3c_1-b_1c_3\,\,b_1c_2-b_2c_1)\\&=a_1b_2c_3-a_1b_3c_2+a_2b_3c_1-a_2b_1c_3+a_3b_1c_2-a_3b_2 a_2b_3 a_3b_2c_1 ,\,a_3b_1-a_1b_3\,,\,a_1b_2-a_2b_1)\cdot(c_1,c_2,c_3)\\ &=a_2b_3c_1-a_3b_2c_1+a_3b_1c_2-a_1b_3c_2+a_1b_2c_3-a_2b_1c_3 \end{align*} and notice that they are the same. For the second proof, we will write both sides again, but this time we express them in terms of determinants. \begii{align*} \va\cdot\vb\times\vc &=(a_1,a_2,a_3)\cdot\det\left\\begiin{matrix}\hi&\hj&\hk \\ b_1&b_2&b_3 \\ c_1&c_2&&c_3\end{matrix}\right]\]&=a_1\det\left[\beginner{matrix} b_2&b_3 \\ c_2&c_3\end{}\right] -a_2\det\left[\begs{matrix} b_1&b_3 \\ c_1&c_3\end{matrix}\ right] +a_3\det\left[\start{matrix} b_1&b_2 \\ c_1&c_2\end{matrix}\right]]&=\det\left[\start{matrix}a_1&a_2&a_3 \\ b_1&b_2&b_3 \\ c_1&c_1&b_3;c_2&c_3\end{matrix}\right]]\ \va\times\vb\cdot\vc &=\det\left\\begin{matrix}\hi&\hj&\h \\ a_1&a_2&a_3 \\ b_1&b_2 &b_3\end{matrix}\right]\cdot(c_1 ,c_2,c_3)\\ &=c_1\det\left[\begin{matrix} a_2&a_3 \\ b_2&b_3\end{matrix}\right] -c_2\det\left[\{matrix} a_1&;a_3 \\ b_1&b_3\end{matrix}\right] +c_3 a_3 <8>\det\left[\begin{matrix} a_1&a_2 \\ b_1&b_2\ {matrix}\right]]\\&=\det\left[\begin{matrix} c_1&c_2&c_3\cr a_1&a_2&a_3 \\ b_1&b_2&b_3 \end{matrix}\ right] \end{align*} The exchange of two rows in a determinant changes the determinant sign. Moving the top row of a \(3\times 3\) to the bottom row requires two row shifts. So the two factors \(3\times 3\) are equal. (b) Proof is not extremely difficult - just write both sides and grind. Substitution in \begin{equation*} \vb\times\vc \ =\ (b_2c_3-b_3c_2)\hi-(b_1c_3-b_3c_1)\hj + (b_1c_2-b_2c_1)\hk \end{equation*} gives, for left side, \begin{align*} \va\times(\vb\times\vc) =\!\!\!\det\left[\start{matrix}\hi&\hj &\hk \\ a_1&a_2&a_3 \\ b_2c_3 &b_3c_2&b_1c_3+b_3c_1&b_1c_2-b_2c_1 \end{matrix}\right]\\ =&\hi\big[a_2(b_1c_2-b_2c_1)-a_3(-b_1c_3+b_3c_1)\big]\\&\hj\big[b_ b_1c_2 a_1 2c_1)-a_3(b_2c_3-b_3c_2)\large]\\ +&\hk\big[a_1(-b_1c_3+b_3c_1)-a_2(b_2c_3-b_3c_2)\large] \end{align*} On the other hand, right side \begin{align*} (\va\cdot\vc)\vb-(\va\cdot\vb)\vc \ =\ &(a_1c_1+a_2c_2+a_3c_3)(b_1\hi+b_2\hj+b_3\hk)\\ &\hskip0.5in-(a_1b_1+a_2b_2+a_3b_3)(c_ 1\hi+c_2\hj+c_3\hk)\\ =\ \hi\ \big[\ color{blue}{a_1b_1c_1} +a_2b_1c_2+a_3b_1c_3- \color{blue}{a_1b_1c_1} -a_2b_2c_1-a_3b_3c_1\big]\\ {+} &\hj\ \big[a_1b_2c_1 +\color{blue}{a_2b_2c_2 } +a_3b_2c_3-a_1b_1c_2 -\color{blue}{a_2b_2c_2} -a_3b_3c_2\big]]\\ {+}&hk\ \big[a_1b_3c_1+a_2b_ 3c_2 +\color{blue}{a_3b_3c_3} -a_1b_1c_3-a_2b_2c_3 -\color{blue}{a_3b_3c_3}\big]{ {=}\ & \hi\ [a_2b_1c_2+a_3b_1c_3-a_2b_2c_1-a_3b_3c_1]\ {+}&\hj\ [a_1b_2c_1+a_2b_2c_1-a_3b_3c_1]\ {+}&\hj\ [a_1b_2c_1+a_3b_2c_3 a_2b_2c_1-a_3b_3c_1]\ {+}&a_ a_1b_2c_1\h 2b_2c_1-a_3b_3c_1]\\ {+}&hj\ [a_1b_2c_1+a_2b_2c_1-a_3b_3c_1]\\ {+}&hj\ [a_1b_2c_1+a_2b_2c_1-a_3b_3c_1]\\ {+}&hj\ [a_1b_2c_1 a_2b_3c_2+a_2b_2c_1 a_2b_2c_3 a_1b_1c_3-a_3b_3c_1]\ {+}&h <2>-a_1b_1c_2-a_3b_3c_2]\\ {+}&\hk a_1b_3c_1\ end{align*} The last formula I had for the left side is the same as the last formula I had for the right side. I've seen vector identity theorem 4.1.7.b before. It is said that if a vector field \(\vF\) is from the \(\vF = \nabla\varphi\) form for some \(\varphi\) functions (i.e., if \(\vF\) is conservative), then \begin{equation*} \nabla\times\vF = \nabla\times(\nabla\varphi) = 0 \end{equation*} Inverse, we also saw in Theorem 2.4.8, that if \(\vF\) is defined and has partial derivatives of continuous first order on all \(\bbbr^3\text{ ,}\) and if \(\nabla\times\vF=0\text{,}\) then \(\vF\) is conservative. Vector identity theorem 4.1.7.b is our screening test for conservatory. Because the right side is zero, vector identity Theorem 4.1.7.a is suggestive. It is said that if a vector field \(\vF\) is from the \(\vF = \nabla\times\vA\) form for a vector field \(\vA\text{,}\) then \begin{equation*} \nabla\cdot\vF = \nabla\cdot\cdot\\cdot\\ \nabla\times\vA) = 0 \end{equation*} When \(\vF = \nabla\times\vA\text{,}\) \(\vA\) is called vector potential for \(\vF\text{.} \) We will see in Theorem 4.1.12, below, that, conversely, if \(\vF(\vx)\) is defined and has continuous first-order partial derivatives on all \(\bbbr^3\text{ ,}\) and if \(\nabla\cdot\vF=0\text{,}\) then \(\vF\) has a vector potential 8 . Identity theorem 4.1.7.a is indeed another screening test. Does that remind you of Theorem 2.4.8? You should. As an example, consider the Maxwell \begin{align*} \nabla\cdot\vB &= 0\\ \nabla\times\vE +\frac{1}{c}\frac{\partial\v}{\partial t}&=0 \end{align*} that we saw in Example 4.1.2. The first equation implies that (assuming that \(\vB\) is sufficiently smooth and decomposing) there is a vector field \(\vA\text{,}\) \(\vA\text{,}\) magnetic potential, with \(\vB=\nabla\times\vA\text{.} \) Replacing this in the second equation gives \begin{equation*} 0= \nabla\times\vE +\frac {1}{c}\frac{\partial\ }{\partial t}\nabla\times\vA =\nabla\times\Big(\vE+\frac{1}{c}\frac{\partial\vA }{\partial t}\Big) \end{equation*} So \

((c)\frac{\partial\vA }{\partial t}\Big) \end{equation*} So \((c)\frac{\partial\vA }{\partial t}\Big) \end{equeay*} So \((c)\frac{ \partial\vA\vE+\frac{1}{c}\frac{\partial\vA }{\partial t}\) passes the Theorem 4.1.7.b and there is a function \(\partial t}\) passes the screening test of Theorem 4.1.7.b and there is a function \(\partial t}\) passes the screening test of Theorem 4.1.7.b and there is a function \(\partial t}\) passes \varphi\ (called electrical potential) with \begin{equation*} \vE+\frac{1}{c}\frac{\partial\vA }{\partial t} = -abla \varphi \ end {equation*} I put the minus sign in just to provide compatibility with the usual physical terminology. To \(\vr(x,y,z) = x\,\hi+y\,\hj+z\,\hk\) and to \(\psi(x,y,z)\) be an arbitrary function. Verify that \begin{equation*} \nabla\cdot\big(\vr\times\nabla\psi\big) = 0 \end{equation*} Solution After Vector Identity Theorem 4.1.4.d, \begin{gather*} \nabla\cdot\big(\vr\times\nabla\psi\big) =(\nabla\times\vr)\cdot\nabla \psi -\vr\cdot\big(\nabla\times(\nabla\psi\big) \end gather*} By vector identity Theorem 4.1.7.b, the second term is zero. Now from \begin{equation*} \nabla\times\vr = \Big(\frac{\partial}{\partial z}\Big)\hi -\Big(\frac{\partial}{\partial z}\Big)\hj +\Big(\frac{\partial}{\partial x} -\frac{\partial}{\partial y}\Big)\hk =\vZero \end{equation*} the first term is also zero. Indeed \(\nabla\cdot\big(\vr\times\nabla\psi\big) = 0\) holds for any free curl \(\vr(x,y,z)\text{.} \) We will now continue to explore the vector potential that has been entered in example 4.1.9. First, here is the formal definition. Vector field \(\vA\) is declared to be a vector potential for vector field \(\vB\) if \begin{equation*} \vB=\nabla\times\vA \end{equation*} That's how we saw in Example 4.1.9, if a vector field \(\vB\) has a vector potential, then vector identity theorem 4.1.7.a implies that \(\nabla\cdot\vB=0\text{.} \) This fact deserves to be called a theorem. If there is a vector potential for the vector field \(\vB\text{,}\) then \begin{equation*} \nabla\cdot\vB=0 \end{equation*} Of course, we will consider the reverse soon. Also note that vector potential, when it exists, is far from unique. Two vector fields \(\vA\) and \(\tilde\vA\) both vector potentials for the same vector field if and only if \begin{equation*} \nabla\times\vA=\nabla\times\times\tilde\vA \iff \nabla\times(\vA-\tilde\vA)=\vZero \finished\equation*} I mean, if and only if the difference \(\vA-\tilde\vA\) passes the conservative field screening test of theorems 2.3.9 and 2.4.8. In particular, if \(\vA\) is a vector potential for \(\vB\) (i.e. if \(\vB=\nabla\times\vA\)) and if \(\psi\) is any function, then \begin{equation*} \nabla\times(\vA+\nabla\psi) = \nabla\times\vA + \nabla\times\nabla\psi =\vB \end{equation*} by identity theorem 4.1.7.b. I mean, \(\vA+\nabla\psi\) is another vector potential for \(\vB\text{.}\) To simplify calculations, we can always choose \(\psi\) so that, for example, the third component of \(\vA+\nabla\psi\text{,}\) i.e. \(\big(\vA+\nabla\psi\big)\cdot\hk= \vA_3+\frac{\partial\psi}{\partial z}\text{,}\) is zero — choose only \(\psi = -\int \vA_3\, \de{z}\text{.}\) We just proved whether vector field \(\vB\) has vector potential , then, in particular, there is a vector potential \(\vA\) for \(\vB\) with 9 \(\vA_3=0\text{.}\) There is nothing special about the \(3\) index here. By exactly the same argument, we could come up with another vector potential whose second component is zero, and a third vector potential whose first component is zero. Here's an example that exploits this choice to simplify the calculations used to find a vector potential. Let \begi{equation*} \vB = yz\,\hi + zx\,\hj + xy\,\hk \end{equation*} This vector field has been carefully configured to obey \begin{equation*} \nabla\cdot\ vB = \frac{\partial }{\partial x}(yz) +\frac{\partial }{\partial y}(zx) +\frac{\partial }{\partial z}(xy) =0 \end{equation*} and thus passes the screening test of Theorem 4.1.12. Let's try to find a vector potential for \vB\text{.} \) I mean, let's try to find a vector field \(\vA= A_1\,\hi + A_2\,\hj + A_3\,\hk\) that is subject to \(\nabla\times\vA = \vB\text{,}\) or equivalent, \begin{align*} \frac{\partial A_3}{\partial y} -\frac{\partial A_2}{\partial z} &=B_1 = yz\\ \frac{\partial A_1}{\partial z} -\frac{\partial A_3}{\partial x} &=B_2=zx\\ \frac{\partial A_2}{\partial x} -\frac{\partial A_1}{\partial y}&=B_3=xy \end{align*} This system is ugly to resolve, because each equation contains several of the three unknowns, \(A_1\text{,}\) \(A_2\text{,}\) \(A_3\text{.}\) Let's take advantage of our observation above that if there is any vector potential, then, in particular, there is a vector potential \(\vA\) that also submits to \(A_3=0\text{.}\) , let's also ask as \(A_3=0\text{.}\) Then the above equations simplify \begin{align*} -\frac{\partially A_2}{\partial z} &=yz\\ \frac{\partially A_1}{\partial z} &=zx\\ \frac{\partial A_2}{\partial x} -\frac{\partial A_1}{\partial y} &=xy \end{align*} This system is much easier, because now that I have chosen \(A_3=0\text{,}\) the first equation contains only one unknown, \(A_2\) and we can find all \(A_2\) that are subject to the first equation simply by integrating with \(z\text{:}\) \begin{equation*} A_2 = -\frac{yz^2}{2} + N(x,y) \end{equation*} Note that, because \(\frac{\partial}{\partial z}\) treats \(x\) and \(y\) as constants, the integration constant \(N\) is allowed to depend \(x\) and \(y\text{.}\) Similarly, the second equation contains only one unknown, \(A_1\text{,}\) and is easy to resolve by integrating in The second equation is satisfied if and only if \begin{equation*} A_1 = \frac{xz^2}{2} + M(x,y) \end{equation*} for some functions \(M\text{.}\) Finally, the third equation is also satisfied if and only if \(M(x,y)\) and \(N(x,y)\) are subject to \begin{equation*} \frac{\partial }{\partial x}\Big(-\frac{yz^2}{2} + N(x ,y)\Big) -\frac{\partial }{\partial y}\Big(\frac{xz^2}{2} + M(x,y)\Big)=xy \end{equation*} that simplifies \begin{equation*} \frac{\partial N}{\partial x}(x ,y) -\frac{\partial M}{\partial y}(x,y) =xy \end{ecuacy*} This is a linear equation in two unknowns, \(M\) and \(N\text{.}\) Usually, we can easily solve a linear equation in an unknown. So, we are free to remove one of the unknowns by setting, for example, \(M=0\text{,}\) and then choose any \(N\) that obeys \begi{equation*} \frac{\partial N}{\partial x}(x,y) = xy \end{equation*} Integration with \(x\) gives, as a possible choice, \(N(x,y) = \frac{x^2 y}{2}\text{.}\) So We found a vector potential. Namely \begin{equation*} \vA = \frac{xz^2}{2} \hi +\Big(-\frac{yz^2}{2} + \frac{x^2y}{2}\Big)\hj \end{equation*} It can and really should, quickly verify that \(\nabla\times\vA=\vB\text\)(\ Let \begi{equation*} \vB = (2x)\,\hi+(2z-2x)\,\hj + (2x-2z)\,\hk \end{equation*} This vector field is subject to \begi{equation*} \nabla\cdot\vB = \frac{\partial }{\partial x}(2x) +\frac{\partial }{\partial y}(2z-2x) +\frac{\partial }{\partial z}(2x-2z) =0 \end{equation*} and thus passes the screening test of Theorem 4.1.12. We will now find a vector potential \(\vA= A_1\,\hi + A_2\,\hj + A_3\,\hk\) for \(\vB\text{.}\) As in the last example, we will simplify calculations by continuing to request 10 that \(A_3=0\text{.}\) Of course, we might as well choose \(A_1=0\) or \(A_2=0\text{.}\) Requirements that \(\nabla\times\vA = \vB\) and \(A_3=0\) descend to \begin{align*} -\frac{\partial A_2}{\partial z} &=2x\\ \frac{\partial A_1}{\partially z} &=2z-2x\\ \frac{\partial A_2}{\partial x} -\frac{\partial A_1}{\partialy y} &=2x-2z \end{align*} Because \(\frac{\partialz}\) treats \(x\) and \(y\) as constants, the first equation is satisfied if and only if \(N(x ,y)\) \begin{equation*} A_2 = -2xz + N(x,y) \end{equation*} and the second equation is satisfied if and only if there is a \(M(x) function \(y)\) \begin{equation*} A_1 = z^2-2xz + M(x,y) \end equation*{ Finally, the third equation, is also satisfied if and only if M(x,y)\) and N(x,y)\) are subject to \begin{align*} \frac{\partial }{\partial x}\Big(-2xz + N(x,y)\Mare) -\frac{\partial }{\partial y}\Big(z^2-2xz + M(x,y)\Big)=2x-2z\\ &\iff -2z+\frac{\partial N}{\partial x}(x ,y) -\frac{\partial M}{\partial y}(x,,y) =2x -2z\\ &\iff \frac{\partial N}{\partial x}(x,y) M}{\partial y}(x,y) =2x \end{align*} All \(z\)'s of this equation have canceled 11 , , we can choose, for example, \(M(x,y)=0\) and \(N(x,y) = x^2\text{.}\) So we found a vector potential. Namely \begin{equation*} \vA = (z^2-2xz)\,\hi + (x^2-2xz)\hj \end{equation*} Again it is a good idea to verify that \(\nabla\times\vA=\vB\text{.} \) If \(z\)'s have not cancelled, not \(N(x,y)\) and \(M(x,y)\) which, after all, are independent of \(z\text{,}\) could satisfy the equation. That would have been a sure sign of a user error. We can use easily the strategy of the latest examples to prove that \(\vB\) is a vector field that is defined and has all the partial derivatives of continuous first order on all \(\bbbr^3\text{.}\) Then there is a vector potential for \(\vB\) if and only if it passes the screening test \(\v nabla\cdot\vB=0\text{.}\)We already know that the existence of a vector potential implies that \(\nabla\cdot\vB=0\text{.}\) So we just have to assume that \(\nabla\cdot\vB=0\) and prove that this implies the existence of a vector field \(\vA\) that is subject to \(\nabla\times\vA = \vB\text{.}\) must resolve \begin{align*} \frac{\partial A_3}{\partial y} -\frac{\partial A_2}{\partial z} &=B_1(x,y,z)\\ -\frac{\partial A_3}{\partial x} +\frac{\partial A_1}{\partial z}&=B_2(x,y,z)\\ \frac{\partial A_2}{\partial x} -\frac{\partial A_1}{\partial y}&=B_3(x,y,z) \end{align*} We will explicitly find such a \(\vA\) using exactly the 4.1.14 example strategy. In particular, we will look for a \(\vA\) that has, also \(A_3=0\text{.}\) Then the equation simplified to \begin{align*} -\frac{\partial A_2}{\partial z} &=B_1(x,y,z)\\ \frac{\partial A_1}{\partial z} &=B_2(x,y,z)\\ \frac{\partial A_2}{\partial x} -\frac{\partial A_1}{\partially A_1 B_3} ,z) \end{align*} The first equation is met if and only if \begi{equation*} A_2(x ,y,z) = -\int_0^z B_1(x,y,\tilde z) \de{\tilde z} +N(x,y) \end{equation*} for some functions \(N(x,y)\text{.} \) And the second equation is satisfied if and only if \begin{equation*} A_1(x,y,z) = \int_0^z B_2(x ,y,\tilde z) \de{\tilde z} +M(x,y) \end{equation*} So all three equations are met if and only if we can find \(M(x,y)\) and \(N(x,y)\) that subject to \begin{align*} &\frac{\partial }{\partial x}\Big(\overbrace{-\int_0^z B_1(x ,y,\tilde z)\ \de{\tilde z} +N(x,y)}^{A_2(x ,y,z)}\Big)\\ &\hskip1in -\frac{\partial }{\partial y}\Big(\overbrace{\int_0^z B_2(x,y,\tilde z)\ \de{\tilde z} +M(x,y)}^{ A_1(x,y,z)}\Big) =B_3(x,y,z) \end{align*} which is the case if and only if \begi{gather*} \frac{\partial N}{\partial x}(x ,y) -\frac{\partial M}{\partial y}(x ,y) = B_3(x,y,z) + \int_0^z\Big(\frac{\partial B_1}{\partial x}} (x,y,\tilde z) + \frac{\partial B_2}{\partial y}(x,y,\tilde z)\Big) \de{\tilde z} \end{gather*} At first glance, we seem to have a very big problem here. Regardless of what \(N\) and \(M\) we choose the left side will depend only on \(x\) and \(y\) — not on But it looks like the right side depends on \(z\) too. Fortunately, the screening test (which we have not used at this point in the proof) rides the rescue and ensures that the right hand actually does not depend on \(z\text{.}\) By the screening test, \begin{equation*} \nabla\cdot\vB =\frac{\partial B_1}{\partial x} +\frac{\partial B_2}{\partial y} +\frac{\partial B_3}{\partial z}=0 \end{equeay*} and we have \becui {equation*} \frac{\partial B_1}{\partial x} +\frac{\partial B_2}{\partial y} =-\frac{\partial B_ 3}{\partial z} \end{equation*} so that the right side is \begin{align*} B_3(x ,y,z) + \int_0^z\Big(-\frac{\partial B_3}{\partial z}(x,y,\tilde z) \Big) \de{\tilde z } &= B_3(x,y,z) + \Big[-B_3(x,y,\tilde z)\Big]^{\tilde z=z}_{\tilde z=0}\\ &=B_3(x,y,0) \end{align*} of the fundamental theorem of calculus. So we just have to choose \(M\) and \(N\) so as to obey \begi{gather*} \frac{\partial N}{\partial x}(x,y) -\frac{\partial M}{\partial y}(x,y) = B_3(x,y,0) \end{gather*} For example, \(M=0\text{,}\) \(N(x,y) = \int_0^x B_3(\tilde x,y,0) \de{\tilde x}\) working. So not only have we proven that there is a vector potential, but we have found a formula for it. Note that in Theorem 4.1.12 we assume that \(\vB\) passes the screening test on all \(\bbbr^3\text{.}\) If this is not the case, for example because the vector field is not defined on all \(\bbbr^3\text{,}\) then \(\vB\) may not have a vector potential. An example (point source) is provided in example 4.4.8. In this section we will develop an interpretation of the gradient \(\nabla f(\vr_0)\text{.}\) This should only be a review of the material you have seen before. Let's say you're moving through space and your position is in the space. at \(t\) is \(\vr(t)=\big(x(t),y(t),z(t)\big)\text{.}\) As you move, measure, for example, temperature. If the temperature in position \((x,y,z)\) is \(f(x,y,z)\text{,}\) then the temperature you measure on time \(t\) is \(\fbig(x(t),y(t),z(t)\big)\text{.}\) So the change rate temperature you feel is \begin{align*} &\diff{ }{t}f\big(x(t),y(t),z(t)\big)\\ &\hskip0.3in =\frac{\partial f}{\partial x}\big(x(t) ,y(t),z(t)\large) \diff{x}{t}(t) +\frac{\partial f}{\partial y}\big(x(t),y(t),z(t)\big) \diff{y}{t}(t)\\ &\hskip1in +\frac{\partial f}{\partial z}\big(x(t) ,y(t)\t)\big) \diff{z}{t}(t) \qquad \text{(by the chain rule)}\\\hskip0.3in=\fbig(\vr(t)\big)\cdot\vr\big(t)\big) &\hskip0.3in=\big|\nabla f\big(\vr(t)\big)\big| \ ,\big|\vr'(t)\big|\,\cos\theta \end{align*} where \(\theta\) is the angle between the gradient vector \(\nabla f\big(\vr(t)\big)\) and the speed vector \(\vr'(t)\text{.}\) This is the change rate per unit time. We can get the change rate per unit of distance travelled by moving at speed one, \(\big|\vr'(t)\big|=1\) and then \begin{gather*} \diff{ \diff{ =\big|\nabla f\big(\vr(t)\big)\big|\,\cos\theta \end{gather*} If, at some point \(t=t_0\text{,}\) you are at \(\vr(t_0)=\vr_0\text{,}\) and then \begin{gather*} \diff{ }{t}f\big(\vr(t)\big)\Big|_{t=t_0 <> <> =\big|\nabla f(\vr_0)\big|\,\cos\theta \end{gather*} Recall that \(\theta\) is the angle between our direction of motion and the gradient vector \(\nabla f(\vr_0)\text{.}\) So , in order to maximize the rate of temperature change we feel, so we would go through \ (\vr_0\text{,}\) we would have to choose our direction of motion to be in the direction of the gradient vector \(\nabla f(\vr_0)\text{.}\) In conclusion \begin{ignat*}{2} \nabla f(\vr_0) &= \text{ has direction } &\left\{ \begin{array}{l} \text{direction of maximum rate of change}}\text{of f at $\vr_0$ \end{array}\right.\\ & \text{ has magnitude } & \left\{ \begin{array}{l} \text{magnitude of maximum change rate}} \\ \text{ (per unit distance) of f to $\vr_0$ \end{array} \right. \end{ignat*} In this section we will develop an interpretation of the divergence \(\nabla\cdot\vv(\vr_0)\) of the vector field \(\vv(\vr)\) in point \(\vr_0\text{.}\) We will do this in two stages. First we will express \(\nabla\cdot\vv(\vr_0)\) in terms of full flow. We will then use the interpretation of data flow integrals in Lemme 3.4.1 to obtain an interpretation of \(\nabla\cdot\vv(\vr_0)\text{.}\) Think of \(\vv(x,y,z)\) as the speed of a fluid at \((x,y,z)\) and pin any point \(\vr_0=(x_0,y_0,z_0)\text{.}\) Be the sphere centered at \(\vr_0\) for any \(\paint \gt 0\text{ ,}\) \(S_\paint\) to be the sphere centered at \(\vr_0\) of radius \(\paint\text{.} \) Denotes by \(\hn(x,y,z)\) the outer normal to \(S_\paints\) at \((x,y,z)\text{.} \) We will prove, in Lemme 4.1.20, below, that we can write \(\nabla\cdot\vv(\vr_0 >0>)\) as the limit \begin{gather*} \nabla\cdot\vv(x_0,y_0,z_0) =\lim_{\vops\rightarrow 0} \frac{1}{\frac{4}{3}\pi\vops^3} \dblInt_{S_ ,y,z)\cdot\hn(x,y,z)\,\dee{S} \end{gather*} Once we have that lamme we can use that \(\frac{4}{3}\pi\paints^3\) is the volume of the inner sphere \(S_\paint\) and Lemme 3.4.1, \(\dblInt_{S_\paints}\vv(x,y,z)\cdot\hn(x,y,z)\,\dee{S}\) is the rate 12 at which the fluid exits \(S_\paints\) to conclude that Lemma 3.4.1 applies with the density \(\rho\) set equal to one , therefore, the rate is the number of units of volume of liquid coming out \(S_\paint\) per unit of time \begin{align*} \nabla\cdot\vv(\vr_0) &=\left\{ \begin{array}{l} \ text{the rate at which the fluid exits from a }\\ \text{infinitesimal sphere centered }\\ text{at $\vr_0 per unit of time, per unit of volume} \end{array} \right.\\ &=\text{strength of the source at $\vr_0$} \end{align*} Here is the critical calculation. \begin{gather*} \nabla\cdot\vv(x_0 ,y_0,z_0) 0} \frac{1}{\frac{4}{3}\pi\paints^3} \dblInt_{S_\vv}\vv(x,y,z)\cdot\hn(x,y,z)\,\dee{S} \end{gather*} Here is one 4.1.20. There is another, more understandable evidence of this result given in paragraph 4.4.1. We cannot give this proof here because it uses the divergence theorem, which we will reach later in the chapter. By translating our coordinate system, it is sufficient to consider \(\vr_0=(x_0,y_0,z_0) = (0.0.0)\text{.} \) Then \begin{gather*} S_\paints = \Set{(x,y,z)}{|(x,y,z)|=\vops}\\ qquad \hn(x,y,z) = \frac{1}{\vops}(x,y,z) \end{gather*} Expand \(\vv (x,y,z)\) in a Taylor extension to the powers \(x\text{,}\) \(y\text{ ,}\) and \(z\text{,}\) on the first order, with a second-order error term. \begin{gather*} \vv(x,y,z) = \vA + \vB\,x +\vC\,y +\vD\, z +\vR(x,y,z) \end{gather*} where \begin{gather*} \vA=\vv(0.0.0) \qquad \vB=\frac{\partial \vv}{\partial x}(0.0.0) \qquad \vC=\frac{partial \vv}{\\ partial y}((0.0.0) \qquad \vC=\frac{partial \vv}{\partial y}((3 0.0.0) \qquad \vD=\frac{\partial \vv}{\partial z}(0.0.0) \end{gather*} and error term \(\vR(x ,y,z)\) is delimited by a constant times \(x^2+y^2+z^2\text{.} is delimited by a constant times \(x^2+y^2+z^2\text{.} \) In particular, there is a constant \(K\) so that on \(S_\paint\text{,}\) \begin{equation*} |\vR(x,y,z)|\le K\vops^2 \end{equation*} So \begin*align*} &\dblInt_{S_\vv(x,y ,z)\cdot\hn(x,y,z)\,\dee{S}\\ &\hskip1in= \frac{1}{\vps} \dblInt_{S_\vps}\big(\vA + \vB\,x +\vC\,y +\vD\, z +\vR(x,y,z) \big)\cdot (x ,y,z)\,\dee{S} \end{align*} Multiply the point product so that integrating becomes \start{aligned*} {3} &\ \vA\cdot\hi\,x &&\ \vA\cdot\hj\,y &&++vA\cdot\hk\ ,z\\ &&+vB\cdot\hi\,x^2 &&++\ \vB\cdot\hj\,xy &&++\vB\cdot\hk\,xz\ &&+\vC\cdot\hi\,xy &&++\ \vC\cdot\hj\,y^2 &&+c\c ,yz\\ &&+\vD\cdot\hi\ ,xz\ &&+\ \vD\cdot\hj\,yz &&+\vD\cdot\hk\,z^2\ &&+\vR(x,y,z)\cdot (x,y,z) \end{alignat But most of them integrate at zero, simply because the integral of a strange function over a hair field is zero. Because \(S_\paint\) is invariant under \(x\rightarrow -x\) and under \(y\rightarrow -y\) and under \(z\rightarrow -z\) we have \begin{gather*} \dblInt_{S_\paint}\!\! x\,\dee{S} =\dblInt_{S_\paint}\!\! y\,\dee{S} =\dblInt_{S_\paint}\!\! z\,\dee{S} =\dblInt_{S_\paint}\!\! xy\,\dee{S} =\dblInt_{S_\paint}\!\! xz\,\dee{S} =\dblInt_{S_\paint}\!\! yz\,\dee{S} =0 \end{gather*} which is a relief. Now we're stuck with \start{align*} \dblInt_{S_\vops}\vv(x,y,z)\cdot\hn(x,y,z)\,\dee{S} &= \frac{1} {\veps} \dblInt_{S_\paints}\big(\vB\cdot\hi\,x^2 +\vC\cdot\hj\,y^2 +\vD\cdot\hk\, z^2 \big)\,\dee{S}\hskip1in +\frac{1}{\paints} \dblInt_{S_\vps}\vR(x,y,z)\cdot (x,y,z)\ ,\dee{S} \end{align*} Also \(S_\vps\) is invariant 15 under the interchange \(x\) and \(y\) and also under the interchange \(x\) and \(z\text{.}\) Accordingly \dblInt_{S_\vops} x^2\,\dee{S} &=\dblInt_{S_\vops} y^2\,\dee{S} =\dblInt_{S_\vops} z^2\,\dee{S} =\frac {1}{3}\dblInt_{S_\vops}\big[x^2+y^2+ z^2\big]\,\dee{S}\\ &=\frac{1}{3}\dblInt_{S_\veps}\vops^2\,\dee{S\qquad\text{since &=\frac{1}{3}\dblInt_{S_\veps}\vops^2\ ,\dee{S\qquad\text{since on the } S_\paint\\ &=\frac{4}{3}\pi\paint^4 \end{align*} because the sphere surface \(S_\paint\) is \(4\pi\vps^2\text{.} \) So far, we have \begin{align*} &\dblInt_{S_\vv}\vv(x,y,z)\cdot\hn(x,y,z)\,\dee{S} = \frac{4}{3}\pi\vops^3 \big(\vB\cdot\hi +\vC\cdot\hj +\vD\cdot\hk\big)\\ &\hskip2in+\frac{1}{\veps} \dblInt_{S_\vps}\vR(x ,y,z)\cdot (x,y,z)\,\dee{S}\\ &\hskip0.75in= \fra c{4}{3}\pi\vops^3 \nabla\cdot\vv(\vZero) +\frac{1}{\vops} \dblInt_{S_\vps}\vR(x ,y,z)\cdot (x,y,z)\,\dee{S}\\ &\hskip2in\text{(review definitions } \vB, \vC, \vD) \end{align*} involving \begin{align*} &\lim_{\paint\rightarrow 0} \frac{1}{\frac{4}{3}\pi\vops^3} \dblInt_{S_\vops}\vv(x ,y,z)\cdot\hn(x,y,z)\,\dee{S}}\hskip1in= \nabla\cdot\vv(\vZero) +\lim_{\paint\rightarrow 0} \frac{3}{4\pi\paint^4} \dblInt_{S_\vps}\vR(x ,y,z)\cdot (x,y,z)\,\dee{S} \end{align*} Finally, it is sufficient to remember that \(|\vR(x,y,z)|\le K\vops^2\) and, on \(S_\paint\text{ ,}\) \(| (x,y,z)|==paint\text{,}\) so that \begin{align*} \frac{3}{4\pi\paint^4}\bigg| \dblInt_{S_\vps}\vR(x,y,z)\cdot (x,y,z)\,\dee{S} \bigg| &\le \frac{3}{4\pi\paint^4} \dblInt_{S_\vps}|\vR(x,y,z)|\,| (x,y,z)|\,\dee{S}}\le \frac{3}{4\pi\vops^4} \dblInt_{S_\paint}K\paints^3\,\dee{S} = \frac{3}{4\pi\vops^4} \ K\paints^3\,\big(4\pi \paint^2\)\\ &= 3K\paints \end{align*} converges to zero as \(\paints\rightarrow 0\text{.} \) So we left with the desired result. Terms such as \(xy\text{,}\) \(xz\) and \(yz\) are not required because, for example, \(|xy|\le \frac{1}{2}(x^2+y^2)\text{.} \) This inequality is equivalent to \(\big(|x|-|y|\big)^2\ge 0\text{.} \) Here is a outline of the vector field \(\vv(x,y,z) = x\,\hi+y\,\hj+z\,\hk\) and an origin-centered sphere, would be \(S_\paint\text{.} \) This speed field has fluid being created and pushed through the sphere. We have \begin{gather*} \nabla\cdot\vv(\vZero) = 3 \end{gather*} in accordance with our 4.1.19 interpretation. Here is a outline of the vector field \(\vv(x,y,z) = -y\,\hi+x\,\hj\) and an origin-centered sphere, would be \(S_\paint\text{.} \) This speed field has only fluid going around in circles. No fluid crosses the sphere. Divergence \begin{gather*} \nabla\cdot\vv(\vZero) = 0 \end{gather*} consistent with our 4.1.19 interpretation. Here is a outline of the vector field \(\vv(x,y,z) = \hi\) and an origin-centered sphere, would be \(S_\vps\text{.} \) This speed field has only fluid that moves evenly to the right. The fluid enters the sphere from the left and goes right at exactly the same rate, so the net rate of fluid crosses the sphere is zero. Divergence \begin{gather*} \nabla\cdot\vv(\vZero) = 0 \end{gather*} again consistent with our 4.1.19 interpretation. We will now develop the interpretation of the loop or, more precisely, the for any drive vector \(\hn\text{.} \) \) we have made in developing the interpretation of divergence, we will first express \(\nabla\times\vv(\vr_0)\cdot\hn\) as a limit of integrals, and then we will interpret integrals. To specify the integrals involved, be \(C_\paint\) the circle centered at \(\vr_0\) has the radius \(\paint\) is in the plane by \(\vr_0\) perpendicular to \(\hn\) is oriented as standard in relation to \(\hn\text{.} \) Imagine sitting on circle with feet on the plane through \(\vr_0\) perpendicular to \(\hn\text{,}\) with the vector from the feet to the head in the same direction as \(\hn\) and with the left arm pointing to \(\vr_0\text{.} \) Then you are facing in the positive direction for \(C_\paint\text{.} \) We will show in Lemme 4.1.25, below, that \begin{equation*} \nabla\times v(\vr_0)\cdot \hn =\lim_{\paint\rightarrow 0}\frac{1}{\pi\vops^2} \oint_ {C_\paint} \vv(\vr)\cdot \dee{\vr} \end{equation*} Now let's work on interpreting the right side, especially interpreting the integral \(\oint_{C_\veps} \vv(\vr)\cdot \de{\vr}\ text{ ,}\) which is called the \(\vv\) movement around \(C_\paint\text{.} \) Place a small paddle wheel in fluid with its operating bone along \(\hn\) and its paddles along \(C_\paint\text{,}\) as in the figure below, except that paddlewheel is very expensive and has much more than just four paddles. Pretend 16 that you're one of the paddles. If the roods rotate to \(\Om\) radians per unit of time, then move at speed \(\Om \paint\text{.} \) If you are at \(\vr\text{,}\) the fluid speed component in the direction of motion, i.e. tangential to \(C_\paint\text{,}\) is \(\vv(\vr)\cdot\diff{\vr}{s}\text{,}\) because \(\diff{\vr}{s}\text{,}\) with \(s\) denotes the length of the arc along the circle, is a tangential unit vector at \(C_\veps\text{.}\) All paddles must move at the same speed. So the paddle speed, \(\Om\paint\text{,}\) should be the average value of \(\vv(\vr)\cdot\diff{\vr}{s}\) around the circle. Thus, the rate of rotation, \(\Om\text{,}\) of the flywheel must be determined by \begin{align*} \Om \paint &= \frac{\oint_{C_\vv}\vr)\cdot\diff{\vr}{s}\dee{s }} {\oint_{C_\paint}\dee{s}} = \frac{\oint_{C_\vv(\vr)\cdot\dee{\vr}} {2\pi\vops} \end{align*} Accordingly, \(\vnab la\times v(\vr_0)\cdot \hn\) is the limit that \(\paint\) (dadlewheel radius) tends to zero of \begin{gather*} \frac{1}{\pi\paint^2} \oint_{C_\ paint} \vv(\vr)\cdot \de{\vr} =2\ Man \end{gather*} That's our interpretation. The acting method could help you here. If a fluid has the speed field \(\vv\) and you place an infinitesimal paddle wheel at \(\vr_0\) with the bone in the direction \(\vn\text{,}\) then it rotates to \(\frac{1}{2}\nabla\times \vv(\vr_0)\cdot \hn\) radians per unit of time. In particular, to maximize the rotation rate, orient the wheel with the oars, so that \vv(\vr_0)\text{.} \There will be some examples at the end of this section. First, we show \begin{equation*} \nabla\times \vv(\vr_0)\cdot \hn =\lim_{\paint\rightarrow 0}\frac{1}{\pi\paint^2} \oint_{C_\paint} \vv(\vr)\cdot \de{\vr} \end{equation*} Here is proof 17 of Lemma 4.25. There is another, more understandable evidence of this result given in paragraph 4.4.1. We can't give this proof here because it uses Stokes' theorem, which we'll get to later in the chapter. So we did in the evidence of Lemme 4.1.20, we can always translate our coordinate system, so that \(\vr_0=(x_0,y_0,z_0) = (0.0.0)\text{.} \) Also, we can rotate our coordinate system so that \(\hn=\hk\text{.}\) Because \(\vr_0=(0.0.0)\) and \(\hn=\hk\text{,}\) so that \(C_\paint\) to be in the \(xy\)-plane, we can parametric \(C_\paint\) by \begin{equation*} \vr(t) =\paint\cos t\ ,\hi +\paint\sin t\,\hj \end{equation*} Again so I did in the lemme proof 4.1.20, expands \(\vv(x,y,z)\) in a Taylor expansion in the powers \(x\{ text ,}\) \(y\text{,}\) and \(z\text{,}\) to the first order, with second order error term. \begin{gather*} \vv(x,y,z) = \vA + \vB\,x +\vC\,y +\vD\, z +\vR(x,y,z) \end{gather*} where \begi{gather*} \vA=\vv(0.0.0) \qquad \vB=\frac{\partial \vv}{\partial x}(0.0.0) \qquad \vC=\frac{\partial \vv}{\partial y}(0.0 ,0) \qquad \vD=\frac{\partial \vv}{\partial z}(0.0.0) \end{gather*} and the error term \(\vR(x,y,z)\) is delimited by a constant times \(x^2+y^2+z^2\text{.} is bounded by a constant times \ (x^2+ y^2+z^2\text{.} is delimited by a constant times \(x^2+y^2+z^2\text{.} \) In particular, there is a constant \(K\) so that, \(C_\paint\text{,}\) \begin{equation*} |\vR(x,y,z)|\le K\paints^2 \end{equation*} So \begin{align*} &\le K\paints^2 \end{equation*} So \begin{align*} &\ amp;\oint_{C_\veps} \vv(\vr)\cdot \dee{\vr}}\hskip0.25in=\int_0^{2\pi} \big(\vA + \vB\ ,\paints\,\cos t +\vC\,\vps\,\sin t +\vR(\vr(t)\big) \cdot \big(-\paint\sin t\,\hi +\vps\cos t\ ,\hj\big)\ \dee{t} \end{align*} Again, multiply the point product so that integrating becomes \begin{aignat*}{2} & -\paint\vA\cdot\hi\,\sin t &&+\ \vps\vA\cdot\hj\,\cos t\ &&-\vops^2\vB\cdot\hi\,\sin t\cos t &&+\vops^2\vB\cdot\hj\ ,\cos^2t\ &&-\vps^2\vC\cdot\hi\,\sin^2 t &&+\ \vps^2\vC\cdot\hj\ ,\sin t\cos t\ amp;+\vR(\vr(t))\cdot \big(-\paint\sin t\,\hi +\paint\cos t\,\hj\big) \end{aignat*} Again most of these terms integrate to zero, because \begin{assigned*}{2} \int_0^{2\pi}\sin t\ \dee{t} &=\hskip10pt\int_0^{2\pi}\cos t\ \dee{t} &&=0\\ \int_0^{2\pi}\sin t\cos t\ \dee{t} &= \frac{1}{2}\int_0^{2\pi}\sin(2t)\ \dee{t} &&=0 \end{occurred*} and \(\sin^2t\) and \(\cos^ Terms 2 t\) are easy to integrate using (see Example 2.4.4) \begin{equation*} \int_0^{2\pi}\sin^2 t\ \de{t}=int_0{2\pi}\cos^2 t\ \dee{t} \big]\ \dee{t}=\pi \end{ecuație*} Deci rămânem cu \begi{align*} \oint_{C_\vv(\vr)\cdot \dee{\vr} &= \pi\vps^2\vB\cdot\hj - \pi\vps^2\vC\cdot\hi +\int_0^{2\pi} \ \vR(\vr(t)) \cdot \big(-\vops\sin t\,\hi +\vops\cos t\,\hj\big)\ \dee{t} \end{align*} implying that \begin{align*} \lim_{\paint\rightarrow 0}\frac{1}{\pi\paint^2} \oint_{C_\paint} \vv(\vr)\cdot \dee{\vr} &= \frac {\partial \vv_2}{\partial x}(0.0.0) - \frac{\partial \vv_1}{\partial y}(0.0.0)\\ &\hskip0.5in +\lim_{\paint\rightarrow 0}\! \frac{1}{\pi\paint^2} \int_0^{2\pi}\!\!\! \vR(\vr(t)) \cdot \big(-\vps\sin t\,\hi +\vops\cos t\,\hj\big)\ \dee{t}\\ &= \big(\nabla\times\vv(0.0.0)\big)\cdot\hk\\ &\hski p0.5in +\lim_0} \frac{1}{\pi\vops^2} \int_0^{2\pi} \vR(\vr(t)) \cdot \big(-\dup\sin t\,\hi +\vops\cos t,\hj\big)\ \de{t} \end{align*} Finally, it is enough to remember that \(|\vR(x ,y,z)|\le K\paints^2\text{,}\) so that \begin{align*} \frac{1}{\pi\vops^2}\bigg| \int_0^{2\pi} \vR(\vr(t)) \cdot \big(-\paint\sin t\,\hi +\paint\cos t\,\hj\big)\ \dee{t} \bigg| &\le \frac{1}{\pi\vops} \int_0^{2\pi}|\vR(\vr(t))|\,\dee{t}}\le \frac{1}{\pi\vops} \int_0^{2\pi}K\veps^2\,\de {t}\\ &= \frac{1}{\pi\paints} \ K\vops^2\,(2\pi)\\ &= 2K\paints \end{align*} converges to zero as \(\veps\rightarrow 0\text{.}\) Here are some examples. We will use the same vector fields as in examples 4.1.21, 4.1.22, and 4.1.23. In all examples, we will orient the wheel with paddles so that \(\hn=\hk\) and sketch the view from above, so that the paddle wheel looks like Here is a outline of the vector field \(\vv(x,y,z) = x\,\hi+y\,\hj+z\,\hk\) and an origin-centered circle, would be \(C_\vps\text{.} \) This speed field has fluid moving parallel to the paddles so paddlewheel shouldn't rotate at all. Calculation \begin{align*} \nabla\times\vv(\vZero) = \det\left[\begin{matrix} \hi & \hj &\hk \\ \frac{\partial }{\partial x} & \frac{\partial }{\partial y} & \frac{\partial }{\partial z} \\ x & y & z \end{matrix} \right] =\vZero \involved \nabla\times\vv(\vZero)\cdot\hk = 0 \end{align*} is compatible with our 4.1.24 interpretation. Here is a outline of the vector field \(\vv(x,y,z) = -y\,\hi+x\,\hj\) and an origin-centered circle, would be \(C_\paint\text{.} \) This speed field has fluid that spins in circles, counterclockwise. So paddlewheel should rotate counterclockwise too. I mean, it should have a positive angular speed. Our interpretation 4.1.24 predicts an angular speed of half \begin{align*} \nabla\times\vv(\vZero)\cdot\hk = \det\left[\start{matrix} \hi & \hj &\hk \\ \frac{\partial }{\partial x} \ frac{\partial }{\partial y} & \frac{\partial }{\partial z} \\ -y & x & 0 \end{matrix} \right] \cdot\hk =2\hk\cdot\hk =2 \{align*} that is positive 18 . Even for the small values of \(2\text{.} \) Here is a outline of the vector field \(\vv(x,y,z) = \hi\) and an origin-centered circle, would be \(C_\paint\text{.} \) Fluid pushing on from the top tries to make paddlewheel wheels clockwise. The fluid pushing on the bottom palette tries to make paddle wheel rotate counterclockwise at the same rate. So paddlewheel paddlewheel they don't rotate at all. Our interpretation 4.1.24 predicts an angular speed of \begin{align*} \frac{1}{2}\cdot\nabla\times\vv(\vZero)\cdot\hk =\frac{1}{2} \det\left[\start{matrix} \hi & \hj &\hk \\ \frac{\partial }{\partial x} & \frac{\partial }{\partial y} & \frac{\partial }{\partial z} \\ 1 & 0 & 0 \end{matrix} \right] \cdot\hk =\vZero\cdot\hk = 0 \end{align*} was expected. Let \(\vF = P\,\hi + Q\,\hj\) be the two-dimensional vector field shown below. Assuming that the vector field in the image is a force field, the work performed by the vector field on a particle moving from point \(A\) to \(B\) along the given path is: Zero negative positive Not enough information for determination. Which statement is most true about line integral \\(\int_{C} \vF\cdot\de{\vr}\text{.}\) \(\displaystyle \int_{C_2} \vF\cdot\de{\vr} \gt 0\) \(\displaystyle \int_{C_2} \vF\cdot\dee{\vr}=0\) \(\displaystyle \int_{C_2} \vF\cdot\dee{\vr} \lt 0\) There is not enough information to determine. \(\nabla\cdot\vF\) from point \(N\) (in the image) is: Positive negative zero Not enough information for determination. \(Q_x - P_y\) in point \(Q\) is: Positive negative zero Not enough information to determine. Assuming that \(\vF = P\,\hi + Q\,\hj\text{,}\) which of the following instructions is correct about \(\frac{\partial P}{\partial x}\) at point \(D\text{?} \) \) \(\frac{\partial P}{\partial x}=0\) to \(D\text{,} \) \) \(D\text{.} \) \(\frac{\partial P}{\partial x}=0\) to \(D\text{.} \) \(\frac{\partial p}{\partial x}=0\) to \(D\text{.} \) \(\frac{\partial p}{\partial x}=0\) to \(D\text{.} \) \(\frac{\partial P}{\partial x}=0\) to \(D\text{.} \) \(\frac{\partial p}{\partial x}=0\) to \(D\text{.} \) \(\frac{\partial P}{\partial x} \gt 0\) to \(D\text{.} \) \(\frac{\partial P}{\partial x} \lt 0\) to \(D\text{.} \) The sign \(\frac{\partial P}{\partial x}\) at \(D\) cannot be determined by the information given. \(\nabla\times\vF\) must be perpendicular to \(\vF\text{?} \) Check vector identities \(\displaystyle \nabla\cdot(f\vF)=f\nab la\cdot\vF+\vF\cdot\nabla f\) \(\displaystyle \nabla\cdot(\vF\times\vG) =\vG\cdot(\nabla\times\vF)- \vF\cdot(\nabla\times\vG)\) \(\displaystyle \nabla^2(fg)=f\,\nabla^2 g+2\nabla f\c dot\nabla g+g\,\nabla^2 f\) Rating \(\nabla\cdot\vF\) and \(\nabla\times\vF\) for each of the following vector fields. \(\displaystyle \vF=x\,\hi+y\,\hj+z\,\hk\) \(\displaystyle \vF=xy^2\hi-yz^2\hj+zx^2\hk\) \(\vF=\frac{x\hi+y\hj}{\sqrt{x^2+y^2}}\) (polar base vector \(\hat{\bf r}\) in 2d) \(\vF=\frac{-y\hi+x\hj}{\sqrt{x^2+y^2}}\)) (polar base vector \(\hat{\pmb{\theta}}\) in 2d) Calculate and simplify \(\nabla\cdot\big(r(\vr)\big)\) \(\vr=(x ,y,z)\) and \(r=|(x,y,z)|\text{.}\) Express your response in terms of \(r\text{.}\) Calculate \(\nabla\big(yz\,\hi + 2xz\,\hj + e^{xy}\,\hk\big)\text{.}\) we use notation \(\vr = x\,\hi + y\,\hj + z\,\hk\) \(r = |\vr|\text{,}\) and \(k\) is a number \(k = 0, 1, -1, -2, -2, \dots \text{.} \) Find the \(k\) value for which \begin{equation*} (r^k) = -3\frac{\vr}{r^5} \end{equation*} Find the \(k\) value for which \begin{equation*} \nabla^2 (r^k) = \frac{2}{r^4} \end{equation*} Be \(\vr\) vector field \(\vr = x\,\hi + y\,\hj + z\,\hk\) and be \(r\) the \(r = |\vr|\text{.}\) Let \(\va\) be the constant vector \(\va = a_1\,\hi + a_2\,\hj + a_3\,\hk\text{.}\) Calculate and simplify the following quantities. Responses must be expressed in terms of \(\va\text{,}\) \(\vr\text{,}\) and \(r\text{.}\) There must be no \(x\)'s, \(y\)'s, or \(z\)'s in replies. \(\displaystyle \nabla\cdot\vr\) \(\displaystyle \nabla(r^2)\) \(\displaystyle \nabla\times(\vr\times\va)\) \(\displaystyle \nab la\cdot\big(\nabla(r^3)\big) = ar\text{.}\) Let \begin{equation*} \vr = x\,\hi + y\,\hj + z\,\hk, \qquad r = |\vr| \end{equation*} Calculate \(\a\) where \(\nabla\cdot\big(\frac{1}{r}\big) = -r^a\,\vr\text{.}\) Calculate \(\a\) where \(\nabla\cdot\big(r\,\vr\big) = ar\text{.}\) Calculate \(\a\) where \(\nabla\cdot\big(\nabla(r^3)\big) = ar\text{.}\) Find, if possible, a vector field \(\vA\) that has the \(\hk\) component \(A_3=0\) and is a vector potential for \(\displaystyle \vF=(1+yz)\hi+(2y+xz)\hj+(3z^2+xy)\hk\) \(\displaystyle \vF= yz\hi zx\hj+xy\hk++ \) Let's start{to gather*} \vF = \frac{-z}{x^2+z^2}\hi +y\,\hj + \frac{x}{x^2+z^2}\,\hk \end{gather*} Determine the scope \(\vF\text{.}\) Determination of the loop \(\vF\text{.}\) Simplify if possible. Determine the divergence then \(\vF\text{.}\) Simplify if possible. \(\vF\) is conservative? Give a reason for your answer. A physicist studies a vector field \(\vF\) in its laboratory. It knows from theoretical considerations that \(\vF\) must be of the form \(\vF=abla\times\vG\text{,}\) for a smooth vector field \(\vG\text{.}\) Experiments show, also that \(\vF\) must be from \begin{equation*} \vF(x,y,z)=(xz+xy)\hi+\alpha(yz-xy)\hj+\beta(yz+xz)\hk \end{equation*} where \(\alpha\) and \(\beta\) are constant. Determine \(\alpha\) and \(\beta\text{.}\) Other experiments show that \(\vG=xyz\hi-xyz\hj+g(x,y,z)\hk\text{.}\) Find the unknown function \(g(x,y,z)\text{.}\) A rigid body rotates at an angular speed of \(\Om\) rad/sec about an axis that passes through the origin and has the direction \(\Ha\text{.}\) When you are standing in the head \(\Ha\) looking towards the origin, the rotation is counterclockwise. Set \(\vOm=\Om\Ha\text{.}\) Shows that the point speed \(\vr=(x,y,z)\) in the body is \(\vOm\times\vr\text{.}\) Evaluate \(\nabla\times(\vOm\times\vr)\) and \(\nabla\cdot(\vOm\times\vr)\) treat \(\vOm\) as constant. Find the speed of students in a classroom at latitude \(49^\circ\) N due to the rotation of the Earth. Ignore the Earth's motion about the Sun, the Sun in the Galaxy and so on. Earth's radius is 6378 km. Let's say that vector \(\vF\) is subject to \(\nabla\cdot\vF=0\) in all \(\bbbr^3\text{.}\) \begin{equation*} \vr(t)=tx\,\hi+ty\,\hj+tz\,\hk,\qquad 0\le t\le 1 \end{equation*} to be a setup of the line segment from origin to \((x,y,z)\text{.}\) Definition \begin{equation*} \vG(x,y,z) \end{equation*} Definition \begin{equation*} \vG(x,y,,) \end{equation*} Definition \begin{equation*} \vG(x,y,z) \end{equation*} Definition \begin{equation*} \vG(x,y,,) \end{equation*} Definition \begin{equation*} \vG(x,y,,) Definition \begin{equation*} \vG(x,y,,) \end{equation*} Definition \begin{equation*} \vG(x,y,,) \end{equation*} Definition \begin{equation*} \vG(x,y,,) \end{equation*} Definition \begin{equation*} \vG(x,y,,) \end{equation*} Definition \begin{equation*} \vG(x,y,,) \end{equation*} Definition \begin{equation*} \vG(x,y,z)=\int_0^1 t\,\vF\big(\vr(t)\big)\times\frac{d\vr}{dt}(t)\,dt \end{equation*} Shows that \(\nabla\times \vG=\vF\) throughout \(\bbbr^3\text{.}\) \\bbbr^3\text{.

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