In this section, you will: apply the rules of cosine to solve the oblique triangle problem. Solve problems using cosine rules Use Heron's formula to find the area of the triangle. Let's say the ship leaves the port, travels 10 miles, turns 20 degrees and travels another 8 miles as shown (pictured). How far is the boat from the port? Unfortunately, while the rules of Sines allow us to deal with many non-right triangular cases, it does not help us with a triangle at a known angle between two known triangles. SAS (side angle), or when all three sides are known, but no angle is known, the SSS triangle (side). In this section, we will examine other tools for solving this oblique triangle solution described by the last two cases. The tool we need to solve the ship's distance from the dock is the law of Cosines, which determines the relationship between measuring the angle and the lateral length in the oblique triangle. Three recipes make up the rules of cosine. At first glance, the formula can seem complicated because there are so many variables. However, once the model is understood, the laws of cosine are easier to work with than most formulas at this mathematical level. Understanding how cosine's rules come in, how to use formulas The acquisition begins with a common Pythagorean theorem, an extension of the pythagorean theorem to a non-scene triangle. Here's how it works: The non-right triangle arbitrarily [latex]\ ABC\, [/latex] is placed in a coordinate plane with the top of [latex]\,A\,[/latex] at the beginning. The side [latex], c\,[/latex], drawn along the x-axis and the vertex [latex]\,C\,[/latex] is located at a certain point. In general, triangles are available anywhere in the plane, but for this description we will place the triangle as indicated. Figure 2 We can place the perpendicular from [latex]\, C\,[/latex] to the axis. x (นี่คือระดับความสูงหรือความสูง) Recalling the basic trigonometric identities, we know that [latex]\mathrm{cos}\,\theta =\frac{x\text{(adjacent)}}{b\text{(hypotenuse)}}\text{ and }\mathrm{sin}\,\theta =\frac{y\text{(opposite)}}{b\text{(hypotenuse)}}[/latex] In terms of[latex]\,\theta ,\text{ }x=b\mathrm{cos}\,\theta \,[/latex]and[latex]y=b\mathrm{sin}\,\theta .\text{ }[/latex]The[latex]\,\left(x,y\right)\,[/latex]point located at[latex]\,C\,[/latex]has coordinates[latex]\,\left(b\mathrm{cos}\,\theta ,\,\,b\mathrm{sin}\,\theta \right).\,[/latex]Using the side[latex]\,\left(x-c\right)\,[/latex]as one leg of a right triangle and[latex]\,y\,[/latex]as the second leg, we can find the length of hypotenuse[latex]\,a\,[/latex]using the Pythagorean Theorem. So [latex]\begin{array}{llllll} {a}^{2}={\left(x-c\right)}^{2}+{y}{2}\hfill & \hfill & \hfill & \hfill & \hfill \hfill \hfill \\text{{{left(b\math@math@{s},theta \right)}^{2}\hfill & \hfill & \hfill & \hfill & \hfill & \text{Substitute }\left(b\mathrm{cos}\,\theta \right)\text{ for}\,x\,\,\text{and }\left(b\mathrm{sin}\,\theta \right)\,\text{for }y.\hfill \\ \text{ }=\left({b}^{2}{\mathrm{cos}}^{2}\theta -2bc\mathrm{cos}\,\theta +{c}^{2}\right)+{b}^{2}{\mathrm{sin}}^{2}\theta \hfill & \hfill & \hfill & \hfill & \hfill & \text{Expand the perfect square}.\hfill \\ \text{ }={b}^{2}{\mathrm{cos}}^{2}\theta +{b}^{2}{\mathrm{sin}}^{2}\theta +{c}^{2}-2bc\mathrm{cos}\,\theta \hfill & \hfill & \hfill & \hfill & \hfill & \text{Group terms noting that }{\mathrm{cos}}^{2}\theta +{\mathrm{sin}}^{2}\theta =1.\hfill \\ \text{ }={b}^{2}\left({\mathrm{cos}}^{2}\theta +{\mathrm{sin}}^{2}\theta \right)+{c}^{2}-2bc\mathrm{cos}\,\theta \hfill & \hfill & \hfill & \hfill & \hfill & \text{Factor out }{b}^{2}.\hfill \\ {a}^{2}={b}^{2}+{c}^{2}-2bc\mathrm{cos}\ ,\theta \hfill & \hfill & \hfill & \hfill & \hfill & \hfill \end{array}[/latex] สูตรที่ได้รับมาจากเป็นหนึ่งในสามสมการของกฎของโคไซน์ Other equations are found in similar fashion. Please note that triangular drafting is always useful when solving corners or sides. In real situations, try drawing a situation diagram. When more information appears, the diagram may need to be changed. Make those changes to the diagram, and ultimately the problem will be easier to fix. Cosines's rule states that the square of either side of the triangle is equal to the sum of the squares of the other two negative sides, twice the product of the other two sides, and the cosine of the included corners. For triangles labeled (pictured) with corners [latex]\,alpha,\beta,[/latex]\,gamma,[/latex] and the opposite side[latex]\, a,b,[/latex] and [latex]\,c,\,[/latex] respectively [latex]\begin{{a}^{2}={b}{2}+c}^c}^{2}-2bc\,{mathrm{s}\alpha {b}{2}={a^}{2}+{c}^{2}-2ac\mathrm{cos}_beta\{2}=a}^{2}+b}^{2}-2ab\,\\mathrm{s}\gamma\{end{[/latex] Figure 3 To solve the problem for measuring the missing side, it is necessary to measure the corresponding opposite angle. When solving problems for angles, corresponding opposite side measurements are required. We can use another version of cosine law to solve the angle problem.\begin{array}{l}\hfill_begin{array}{l}array}{l}hfill\hfill\mathrm{cos}\text{ }\alpha =\frac{{b}^{2}+{c}^{2}-{a}^{2}}{2bc}\hfill \end{array}\hfill \\ \mathrm{cos}\text{ }\beta =\frac{{a}^{2} +{c}^{2}-{b}^{2}}{2ac}\hfill \\ \mathrm{cos}\text{ }\gamma =\frac{{a}^{2}+{b}^{2}-{c}^{2}}{2ab}\hfill \end{array}array\hfill \end{array[/latex] Make two sides and angles between each other (SAS), find the measurement of the sides and the remaining corners of the triangle. Triangular draft Specifies the unit of measure of the known side and angle. Use variables to represent The triangular pool measures 40 feet on one side and 65 feet on the other side, creating an angle that measures 50° on the third side for how long (up to the nearest ten). The pilot flew on a straight route for 1 hour and 30 minutes, then she corrected the course, headed 10° to the right of her traditional course and flew 2 hours in a new direction. If she maintains a constant speed of 680 mph, how far is her starting point? Chicago is 714 km from New York and New York is 2,451 km from Los Angeles. Washington, D.C.C Washington, D.C is 25 miles from Boston, and Boston is 315 km from Philadelphia. Two planes left the same airport at the same time. One flies at 20° east of the north at 500 mph. The second fly is at 30° to the south at 600 mph. How far apart are the planes after 2 hours? Two planes depart in different directions. One travels 300 mph due to the west and the other travels 25° north of the west at 420 mph. After 90 minutes, how far are they apart, assuming they are flying at the same altitude? One square is 15.4 units long and 9.8 units, its area is 72.9 square meters. Find longer diagonal measurements Four sides, respectively, of the quadrilateral, are 4.5 cm, 7.9 cm, 9.4 cm, and 12.9 cm, the angle between the two smallest sides is 117° The area of this quadrilateral is? Four sides, respectively, of the quadrilateral, are 5.7 cm, 7.2 cm, 9.4 cm, and 12.8 cm, the angle between the two smallest sides is 106° what is this area of quadrilateral? Find areas of triangular land that measure 30 feet on one side and 42 feet on the other; the total angle measures 132° of rounding into the nearest square footage. Find areas of triangular land that measure 110 feet on one side and 250 feet on the other. Total angle measured 85°.