Negative exponential distribution graph

Learning Results Recognize exponential probability distribution and apply it appropriately Exponential distribution often takes care of the amount of time until a specific event occurs. For example, the amount of time (which begins now) until an earthquake occurs has an exponential distribution. Other examples include the duration, in minutes, of long-distance business phone calls, and the amount of time, in months, a hard car battery. It can also be shown that the value of the change you have in your pocket or purse approximately follows an exponential distribution. The values of an exponential random variable occur as follows. There are fewer large values and more small values. For example, the amount of money customers spend on a trip to the supermarket follows an exponential distribution. There are more people who spend small amounts of money and fewer people who spend large amounts of money. Exponential distribution is widely used in the field of reliability. Reliability takes care of the amount of time a product lasts. Let X - the amount of time (in minutes) a postal employee spends with your client. Time is known to have an exponential distribution with the average amount of time equal to four minutes. X is a continuous random variable because time is measured. It's μ 4 minutes. To perform any calculation, you must know m, the decay parameter. [latex] m, frac{1}, mu [/latex]. Therefore, [latex]-m-frac{1}{4}-0.25-[/latex] The standard deviation, σ, is the same as the mean. μ - σ Distribution notation is X - Exp(m). Therefore, X to Exp(0.25). The probability density function is f(x) - me–mx. The number e 2.71828182846... It is a number that is often used in mathematics. Scientific calculators have the ex key. If you enter one for x, the calculator displays the value e. The curve is: f(x) at 0.25e–0.25x, where x is at least zero and m to 0.25. For example, f(5) to 0.25e-(0.25)(5) to 0.072. The postal worker spends five minutes with customers. The graph is as follows: Notice that the chart is a decreasing curve. When x is 0, f(x) to 0.25e(-0.25)(0) a (0.25)(1) to 0.25 x m. The maximum value on the Y axis is m. The amount of time spouses buy anniversary cards can be modeled by an exponential distribution with the average amount of time equal to eight minutes. Type the distribution, specify the probability density function, and plot the distribution. Solution: X - Exp (0.125); f(x) to 0.125e–0.125x; Using the information in Example 1, find the probability that an employee will spend four to five minutes with a randomly selected customer. The curve is: X - Exp (0.125); f(x) to 0.125e–0.125x a) Find P(4 x < 5). Solution: The Cumulative Distribution Function (CDF) gives the area to the left. P(x < 5) to 1 – e(–0.25)(5) to 0.7135 and P(x < 4) to 1 – e(–0.25)(4) to 0.6321 You can these calculations easily in a calculator. The probability of a postal employee going from four to five minutes with a randomly selected customer is from P(4 < x < 5) to P(x < 5) – P(x < 4) to 0.7135 to 0.6321 to 0.0814. On the home screen, enter (1 – e-(–0.25*5))–(1–e-(–0.25*4)) or enter e-(–0.25*4) – e-(–0.25*5). b) Half of all customers are finished within how long? (Find the 50th percentile) Solution: Find the 50th percentile. P(x < k) at 0.50, k to 2.8 minutes (calculator or computer) Half of all clients are finished in 2.8 minutes. You can also do the calculation as follows: P(x < k) at 0.50 and P(x < k) to 1 –e–0.25k Therefore, 0.50 to 1st e-0.25k and e-0.25k to 1 to 0.50 to 0.5 Take natural records: ln(e–0.25k) to ln(0.50). Therefore, –0.25k to ln(0.50) Resolve for k: [latex],k-frac-ln0.50-0,25,0,25,25,2,8[/latex] minutes c) What is larger, mean or median? Solution: From part b, the median or the 50th percentile is 2.8 minutes. The theoretical mean is four minutes. The average is bigger. The number of days ahead that travelers buy their air tickets can be modeled by exponential distribution with the average amount of time equal to 15 days. Find the likelihood of a traveler buying a ticket less than ten days in advance. How many days do half of all travelers wait? Solution: P(x < 10) to 0.4866 50th percentile at 10.40 On average, a certain part of the computer lasts ten years. The duration of the part of the equipment is distributed exponentially. a) What is the probability that a piece of computer will last more than 7 years? Solution:Leave x : Lasts the amount of time (in years) that a part of the computer lasts. [latex],mu, {10}[/latex] so that m a [latex]{1},mu, frac{1}{10}, 0.10, P(x > 7). Draw the chart. Since P(X < x) to 1 –e–mx and then P(X > x) to 1 –(1 –e–mx) to e-mx P(x > 7) á e(–0.1)(7) to 0.4966. The probability of a piece of computer lasting more than seven years is 0.4966. On the home screen, type e-(-.1*7). b) On average, how long would five parts of the computer last if used one after the other? Solution: On average, a part of the computer lasts ten years. Therefore, five parts of the computer, if used just after the other would last, on average, (5)(10) to 50 years. c) Eighty percent of computer parts last at most how long? Solution: Find the 80th percentile. Draw the chart. Leave k the 80th percentile. Resolve to k: [latex]-k-frac-ln(1-0.80)-0.1-16.1-[/latex] Eighty percent of computer parts last a maximum of 16.1 years. Solution: Find P(9 < x < 11). Draw the chart. d) What is the probability that a piece of computer will last between nine and Years? Solution: P(9 < x < 11) to P(x < 11) – P(x < 9) á (1 – e(–0.1)(11)) – (1 – e(—0.1)(9)) to 0.6671 – 0.5934 to 0.0737. The probability of a piece of computer lasting between nine and 11 years is 0.0737. Suppose the length of a call, in minutes, is an exponential random variable with decay parameter 112. If someone else reaches a pay phone just before you, find a chance you'll have to wait more than five minutes. Leave X - the duration of a phone call, in minutes. What is m, μ and σ? The probability that you should wait more than five minutes is _______ . Solution: m a [latex]-frac{1}{12}[/latex] [latex]-mu [/latex] to 12 [latex]-sigma [/latex] at 12 P(x > 5) to 0.6592 The waiting time between events is often modeled using exponential distribution. For example, suppose an average of 30 customers per hour arrive at a store and the time between arrivals is distributed exponentially. On average, how many minutes pass between two successive arrivals? When the store opens for the first time, how long does it take on average three customers to arrive? After a customer arrives, find the probability that the next customer will take less than a minute to arrive. After a customer arrives, find the probability that the next customer will take more than five minutes to arrive. Seventy percent of customers arrive within how many minutes of the previous customer? Is an exponential distribution reasonable for this situation? Solutions: Since we expect 30 customers per hour (60 minutes) to arrive, we expect a customer to arrive on average every two minutes on average. Because a customer arrives every two minutes on average, it will take six minutes on average for three customers to arrive. Let X time between arrivals, in minutes. On the a side, μ 2, so m to 12 to 0.5. Therefore, X ∼ Exp(0.5). The cumulative distribution function is P(X < x) to 1 – e(–0.5x)e.Therefore P(X < 1) a 1 – e(–0.5)(1) ≈ 0.3935 P(X > 5) to 1 – P(X < 5) to 1 – (1 – e(–5)(0.5)) to e–2.5 ≈ 0.0821. We want to resolve 0.70 to P(X < x) for x. Substitution in the cumulative distribution function gives 0.70 x 1 – e–0.5x, so e–0.5x to 0.30. When converting this to a logarithmic form, it is given –0.5x to ln(0.30) or x-ln(0.30)–0.5≈2.41 minutes. Thus, seventy percent of customers arrive within 2.41 minutes of the previous customer. This model assumes that a single customer arrives at once, which may not be reasonable as people can shop in groups, leading multiple customers to arrive at the same time. It also assumes that the customer flow does not change throughout the day, which is not true if some hours of the day are busier than others. Asedornation of exponential distribution In Example 1, remember that the amount of time between clients is distributed exponentially with an average of two minutes (X to Exp (0.5)). Let's say it's been five minutes since the last customer arrived. Since an unusual amount has elapsed time, it would seem more likely that a customer will arrive in the next minute. With exponential exponential This is not the case: the additional time spent waiting for the next customer does not depend on how much time has elapsed since the last client. This is known as the memoryless property. Specifically, the memoryless property says that P (X > r + t ? X > r) to P (X > t) for all r ≥ 0 and t ≥ 0 For example, if five minutes have elapsed since the last customer arrived, then the probability that more than one minute will elaps before the next customer arrives is calculated using r s 5 and t to 1 in the previous equation. P(X > 5 + 1 ? X > 5) to P(X > 1) to e(–0.5)(1) ≈ 0.6065. This is the same probability as waiting more than a minute for a customer to arrive after the previous arrival. Exponential distribution is often used to model the longevity of an electrical or mechanical device. In example 1, the lifespan of a certain part of the computer has the exponential distribution with an average of ten years (X - Exp(0.1)). The memoryless property says that knowledge of what has happened in the past has no effect on future probabilities. In this case, it means that an old part is no more likely to decompose at a particular time than a new part. In other words, the part stays as good as new until it suddenly breaks. For example, if the part has already lasted ten years, then the probability of it lasting another seven years is P(X > 17o X > 10) -P(X > 7) at 0.4966. See Example 1, where the time a postal employee spends with their client has an exponential distribution with an average of four minutes. Suppose a customer has spent four minutes with a postal employee. What is the probability that I will spend at least three additional minutes with the postal clerk? The X disintegration parameter is m to 14 x 0.25, so X ∼ Exp(0.25). The cumulative distribution function is P(X < x) at 1 – e–0.25x. Do we want to find P(X > 7? X > 4). The memoryless property says P(X > 7? X > 4) - P (X > 3), so we just need to find the probability that a customer will spend more than three minutes with a postal employee. These are P(X > 3) to 1 – P (X < 3) to 1 – (1 – e–0.25·3) to e–0.75 ≈ 0.4724. Relationship between Poisson and Exponential Distribution There is an interesting relationship between exponential distribution and Poisson distribution. Assume that the time between two successive events follows the exponential distribution with an average of μ units of time. Let's also assume that these times are independent, which means that the time between events is not affected by the times between previous events. If these assumptions are maintained, the number of events per unit time follows a Poisson distribution with the average of Remember that if X has the Poisson distribution with the mean, then [latex]P(X-k)-frac-lambda-k-e-lambda-k!- [/latex]. On the contrary, if the number of events per unit of time a Poisson distribution, then the amount of time between events follows the exponential distribution. (k! á k*(k-1*)(k–2)*(k-3)... 3*2*1) At a police station in a large city, calls arrive at an average rate of four calls per minute. Assume that the time that elapses from one call to the next has the exponential distribution. Note that we are only concerned about the speed at which calls arrive, and we are ignoring the time spent on the phone. We must also assume that the times spent between calls are independent. This means that a particularly long delay between two calls does not mean that there will be a shorter waiting period for the next call. We can then infer that the total number of calls received over a period of time has the Poisson distribution. Finds the average time between two successive calls. Find the probability that after a call is received, the next call occurs in less than ten seconds. Find the probability that exactly five calls will occur in one minute. Find the probability of fewer than five calls occurring in one minute. Find the probability of more than 40 calls occurring over an eight-minute period. Solutions: On average there are four calls per minute, so 15 seconds occur, or [latex]{15}{60} [/latex] 0.25 minutes between successive calls on average. Let T - time elapsed between calls. From part a, [latex],mu, from 0.25o [/latex], so m a [latex]{1} frac{1}-0.25o [/latex] to 4. Therefore, T - Exp(4). The cumulative distribution function is P(T < t) at 1 – e–4t. The probability of the next call occurring in less than ten seconds (ten seconds at 1/6 minutes) is P(T < [latex]-frac{1}{6}[/latex]) - 1 – [latex]-e-4-4-frac{1}{6}-approx-0.4866o [/latex] Let X - the number of calls per minute. As noted above, the number of calls per minute has a Poisson distribution, with an average of four calls per minute. Therefore, X ∼ Poisson(4), and therefore P(X to 5) - [latex]-frac-{4}-{5}-e-4-5!- Approx.[/latex] 0.1563. (5! (5)(4)(3)(2)(1)) Note that X must be an integer, so P(X < 5) - P(X ≤ 4). To calculate this, we could take P(X to 0) + P(X to 1) + P(X to 2) + P(X to 3) + P(X to 4). Using technology, we see that P(X ≤ 4) to 0.6288. Leave Y: The number of calls that occur over an eight-minute period. Since there are an average of four calls per minute, there is an average of (8)(4) - 32 calls during each eight-minute period. Therefore, Y ∼ Poisson(32). Therefore, P(Y > 40) to 1 – P (Y ≤ 40) to 1 – 0.9294 to 0.0707. Review of the concept If X has an exponential distribution with the mean then the decay parameter is [latex]m{1}-mu-[/latex], and we write X ∼ Exp(m) where x ≥ 0 and m > 0 . The probability density function of X is f(x) - me-mx (or equivalently [latex]f(x)-frac{1}-mu-e-frac-x-mu-[/latex]. The cumulative distribution function of X is P(X≤ x) at 1 – e–mx. The exponential exponential has the property without memory, which says that future probabilities do not depend on any past information. Mathematically, it says that P(X > x + k X > x) to P(X > k). If T represents the timeout between events, and if T ∼ Exp(-), the number of X events per unit time follows the Poisson distribution with the mean . The probability density function of [latex]P-left(X-k-right)-frac-lambda-k-e-lambda-k! [/latex]. This can be calculated using a TI-83, 83+, 84, 84+ calculator with the poissonpdf(o, k) command. The cumulative distribution function P(X ≤ k) can be calculated using the TI-83, 83+.84+ 84+ calculator with the poissoncdf(-, k) command. Review of the exponential formula: X - Exp(m) where m - the decay parameter pdf: f(x) to m[latex]-e-mx-[/latex] where x ≥ 0 and m > 0 cdf: P(X ≤ x) - 1 –[latex]-e-mx-[/latex] means [latex]-mu - .frac{1}-m-[/latex] standard deviation σ - μ percentile, k: k á [latex]-frac-ln(-text-AreaToTheLeftOfK-)-m-[/latex] Additionally P(X > x) á e(–mx) P(a < X < b) á e(–ma) – e(–mb) Memoryless property: P(X > x + k- X > x) - P (X > k) Poisson probability: P(X-k)-[latex]-frac-lambda-k-e-lambda-k-k!- [/latex] with mean [latex]-lambda[/latex] k! k*(k-1)*(k-2)*(k-3)... 3*2*1 Data from the United States Census Bureau. World Earthquakes data, 2013. Available online at (accessed June 11, 2013). No hitter. Baseball-Reference.com, 2013. Available online at (accessed June 11, 2013). Zhou, Rick. Exponential distribution conference slides. Available online at www.public.iastate.edu/~riczw/stat330s11/lecture/lec13.pdf (accessed June 11, 2013). 2013).