Average rate of change practice worksheet answers

Nelson took a summer job for five weeks, where he got a weekly wage plus tips. His take-home salary is recorded in the table on the right. What was the average change rate in his weekly take-home pay during the five weeks of his job? Select: To continue to enjoy our website, we ask that you confirm your identity as a human being. Thank you very much for your cooperation. Error : Click Not a Robot, and then try downloading again. Determine the average change rate for $$\displaystyle f(x) = \frac{x+1}{x+2}$$ from $$x = 0$$ to $$x = 4$$. Step 1 Calculate the average change rate. $$ \begin{align*} \frac{\Delta f}{\Delta x} & = \frac{\blue{f(4)} - \red{f(0)}}{4 - 0}\\6pt] & = \frac{\blue{\frac {4+1}{4+2}} - \red{\frac{0+1}{0+}} 4\\[6pt] & = \frac{\blue{\frac 5 6} - \red{\frac 1 2}} 4\\[6pt] & = \frac{1/3} 4\\[6pt] & = \frac 1 {12} \end{align*} $$ Reply $$\displaystyle \frac{\Delta f}{\Delta x} = \frac 1 {12}$$ Determine the average change price of $$f(x) = \sin x$$ from $$x = \pi$$ to $$x = 2\pi$$ (where $$x$$ is measured in radians). Step 1 Calculate the average change rate. $$ \begin{align*} \frac{\Delta f}{\Delta x} & = \frac{\blue{f(2\pi)}-\red{f(\pi)}}{2\pi}\\6pt] & = \frac{\blue{\its 2\pi}-\red\their\pi}}{\pi}\\6pt] & = \frac{\blue{0}-\red{0}}{\pi}\\[6pt] & = 0 \end{align*} $$ Reply $$\displaystyle \frac{\Delta f}{\Delta x} = 0$$ Determine the average change rate for the function below from $$t = -2$$ to $$t = 8$$$ $$ f(x) = 60e^{0.5t} $$ Step 1 Calculate the average change rate. $$ \begin{align*} \frac{\Delta f}{\Delta t} & = \frac{\blue{f(8)} - \red{f(-2)}}{8 - (-2)}\\[6pt] & = \frac{\blue{60e^{0.5(8)}} - \red{60e^{0.5(-2)}}{8 - (-2)}\\[6pt] & = \frac{60\left(e^4 - e^{-1}\\right)}{10}\\[6pt] & = 6\left(e^4 - e^{-1}\right)\\[6pt] & \ca. 325.3816 \end{align*} $$ Reply $$ \displaystyle \frac{\Delta f}}{\Delta t} = 6\left(e^4 - e^{-1 1}\right)\ca. $325.3816 $ Determine the average change rate for the function below from $$x = -6$$ to $$x = -3$$. $$ f(x) = 2 - 8x - 5x^3 $$ Step 1 Calculate the average change rate. $$ \begin{align*} \frac{\Delta f}{\Delta x} & = \frac{\blue{f(-3)} - \red{f(-6)}}{-3 - (-6)}\\\6pt] & = \frac {\blue{(2 - 8-3) - 5(-3)^3)} - \red{(2 - 8-6) - 5(-6)^3)}{-3 +6}\\[6pt] & = \frac{\blue{161} - \red{1130}} 3\\[6pt] & = -\frac{969} 3\\[6pt] & = - 323 \end{align*} $$ Answer $$\frac{\\Delta f}{\Delta x} = -323$$ Antag, that the average size of a particular population of sweet, fluffy rabbits can be described by the $$ P(t) function = \frac{250}{1+4e^{-0.75t}, $$ where $$t$$ is measured this year, and $$P(t)$$ is measured in number of rabbits. As time increases from $$t = $5 to $$t = 10, what is the average change in the bunny population? Step 1 Calculate the average change rate. $$ \frac{\Delta P}{\Delta t} & = \frac{\blue{P(10)} - \red{P(5)}}{10 \red{P(5)}}{10 & = \frac{\blue{\frac{250}{1+4e^{-0.75(10)}}} - \red{\frac{250}{1+4e^{-0.75(5)}}}}{5}\\[6pt] & = \frac{250\left(\frac 1 {1+4e^{-7.5}} - \frac 1 {1+4e^{-3...75}}\right)}{5}\\6pt] & = 50\left(\frac 1 {1+4e^{-7.5}} - \frac 1 {1+4e^{-3.0 75}\right)\\[6pt] & \ca. 4.2 \end{align*} $$ Response From year 5 to year 10 increases the population of cute, fluffy rabbits at an average rate of about 4.2 rabbits per year. In a particular company, the cost of producing $$x$$ pallets of goods can be described by the function $$ C(x) = 25x + 4500, $$ where $$C(x)$$ is measured in dollars. Determine the average rate of change in costs as production decreases from 150 pallets to 120 pallets. Step 1 Calculate the average change rate. $$ \begin{align*} \frac{\Delta C}{\Delta x} & = \frac{\blue{C(120)} - \red{C(150)}}{120 - 15 }\\[6pt] & = \frac{\blue{25(120)+4500} - \red{25(150)+4500}}{-030}\\[6pt] & = \frac{\blue{7500} - \red{8250}}{-30}\\[6pt] & = \frac{-750}{-30 }\\ & = 25 \end{align*} $$ Response As the amount of manufactured goods decreases from 150 pallets to 120 pallets, production costs fall on average of $25 per pallet. Note 1: We could have saved ourselves the effort to calculate $$\Delta C/\Delta x$$ by simply noting $$C (x) $$ is a linear feature. The average rate of change for any linear function is just its tilt. Note 2: When the average change rate is positive, the function and variable change in the same direction. In this case, since the amount of goods produced decreases, so do the cost. Suppose you invest $2000 in an account that earns 8% interest each year, but the interest is composed every month. The amount you have in the account is then described by the $$ A(t) function = 2000\left(1 + \frac{0.08}{12}\right)^{12t}. $$ If you do not make deposits or withdrawals, what is the average change rate in the amount of money in the account... during the first 5 years? over the other 5 years? Part (a) - Step 1 Calculate the average change rate. $$ \begin{align*} \frac{\Delta A}{\Delta t} & = \frac{\blue{A(5)} - \red{A(0)}}{5-0}\\6pt] & = \frac{\blue{2000\left(1 + \frac{0.08}{12}\right)^{12(5)}} - \red{2000\left\(1 + \frac{0.08}{12}\right)^{12(0)}}{5}\\[6pt] & \\\blue{400\left(1 + \frac{0.08}{12}\right)^{60}} - \red{400\left(1 + \frac{0.08}{12}\right)^{0}}\\6pt] & = \blue {400\left(1 + \frac{0.08}{12}\right)^{60}} - \red{400}\\6pt] & \approximately 195.94 \end{align*} $$ Part (b) - Step 1 Calculate the average change rate. $$ \begin{align*} \frac{\Delta A}{\Delta t} & = \frac{\blue{A(10)} - \red{A(5)}}{10-5}\\[6pt] & = \frac{\\\blue{2000\left(1 + \frac{0.08}{12}\right)^{12(10)}} - \red{2000\left}} - \red{2000\left(1 + \frac{0.08}} {12}\right)^{12(5)}}} 5\\[6pt] & = \blue{400\left(1 + + - \red {400\left(1 + \frac{0.08}{12}\right)^{60}}\\[6pt] & \ca. 291.92 \end{align*} $$ Response During the first five years the account grows with averaged $195.94 a year. Over the next five years, the account grows by an average of $291.92 a year. Suppose a particular electrical circuit is designed to hold the current, $$I $$$ on a constant 0.02 amps. But both the excitement, $$V $$ and the resistance, $$R $$$, can vary. So according to Ohm's Law, $$R = \frac {0.02} V,$$ where $$R $ $ is measured in Ohms and $$V $ $ measured in volts. What is the average speed of resistance change on the track as the voltage increases from 1.5 volts to 9 volts? Step 1 Calculate the average change rate. $$ \begin\{align*} \frac{\Delta R}{\Delta V} & = \frac{\blue{R(9)}-\red{R(1.5)}}{9-1.5}\\\6pt] & = \frac{\blue{\frac{0.02} 9}-\red{\frac{0.02}{1.5}}{7.5}\\[6pt] & = \left(\blue{\frac{ 0.02} 9}-\red{\frac{0.02}{1.5}}\right)\cdot \frac 1 {7.5}\\[6pt] & = -\frac 1 {675}\\[6pt] & \ca. -0.00148 \end{align*} $$ Response As voltage increases from 1.5 volts to 9 volts , resistance will decrease at an average rate of $$\frac 1 {675}$ohm per volt, or approximately 0.00148 ohm per volt. Suppose $$P (t) $$ represents the skills gained on a particular task after receiving $$t $$ hours training. Suppose the following equation applies when $$t$$ is increased from 3 to 12. Interpret the equation in an entire sentence. $$ \frac{\Delta P}{\Delta t} = 12\% $$ Step 1 Rewriting the average change rate as a fraction with a denominator of 1. $$ \frac{\Delta P}{\Delta t} = 12\% = \frac{12\%} 1 $$ Response As $$t$$ increases from 3 hours to 12 hours of training, the skill increases at an average speed of 12% per hour. Suppose $$R (x) $$ represents revenue (in thousands of dollars) earned by a particular company from the sale of $$x $$ tons of goods. Suppose the following equation applies when sales increase from 0.8 tons to 1.4 tons. Interpret the equation in an entire sentence. $$ \frac{\Delta R}{\Delta x} = -0.2 $$ Step 1 Rewrite the average change rate so that it has a 1 in the denominator. $$ \frac{\Delta R}{\Delta x} = -0.2 = -\frac{0.2} 1 Response When sales increase from 0.8 to 1.4 tons, the company's revenue decreases at an average rate of $200 per tonne of goods sold. Note 1: Because the average change rate is negative, the two volumes change in opposite directions. As the volume of goods sold is increasing, revenues should fall. Note 2: Although the average change in revenue is negative, it does not mean that the company loses money. This only means that they earn less per tonne than in the past. This can happen if the company lowers the price of their goods. They sell more items but earn less per item. Suppose the power in an electrical circuit increases at an average speed of 0.03 amps per second. Write an equation that expresses this idea. Step 1 Let $ $I = $ $ the amount of electrical current flowing through the circuit, measured in amplifiers. Represent $$t$$ measured in seconds. Answer $$\displaystyle \frac{\Delta I}{\Delta t} = 0.03$$ Suppose someone is driving at an average speed of 85 kilometers per hour. Write an equation that expresses this idea. Step 1 Let $$d$$ represent the individuals distance from their starting point, in kilometers. Let $$t$$ represent time, in hours. Answer $$\displaystyle \frac{\Delta d}{\Delta t} = $85$$ Suppose someone has been driving for 45 minutes at a constant 50 kilometers per hour. Then they increased their speed and drove for the additional 1.5 hours. When they arrived at their destination, their average speed for the entire trip was 80 kilometers per hour. How fast have they been running over the last 1.5 hours? Step 1 Find the total distance if the person had been driving at 80 km/h for a full 2.25 hours. $$ \frac{80\mbox{ km}}{1\mbox{ hour}} \cdot \frac{2.25\mbox{ hours}} 1 = (80)(2.25) \mbox{ kilometers} = 180 km $$ Step 2 Determine the remaining distance to run over the last 1.5 hours. The driver spent $$3/4 of an hour driving at 50 km/h, and so had raised $$50\cdot 0.75 = 37.5 $ kilometers. This left $180-37.5 = $142.5 miles to travel. Step 3 Determine the speed needed to cover the remaining distance in the remaining time. The person had to travel 142.5 kilometers in 1.5 hours. So the speed should be $$ \frac{142.5\mbox{ kilometers}{1.5\mbox{ hours}} = 95\mbox{ kph.} $$ Reply The person was driving at a speed of 95 kilometers per hour for the last 1.5 hours. In electrical circuits, energy is measured in joules (pronounced jools) and power is measured in watts. The ratio of the two is $$ 1\mbox{ watt} = \frac {1\mbox{ joule}}{\mbox{second}} $$ So wattage is the speed of energy change relative to time (like speed is the speed of changing distance relative to time). Suppose a variable wattage bulb (like a light bulb on a dimmer switch) has been pulling 30 watts for the last 15 minutes. Wattage is then increased so that after another 5 minutes the average change rate for the entire 20 minutes is 50 watts. What was the higher power bulb was set to achieve this? Step 1 Determine the total amount of energy spent during the 20 minutes. $$ \frac{50\mbox{ joules}}{\mbox{second}} \cdot \frac{20\mbox{ minutes}} 1 = \frac{50\mbox { joules}}{\mbox{second}} \cdot \frac{1200\mbox{ seconds}} 1 = 60{,}000\mbox{ joules}. $$ Step 2 Determine the amount of energy needed in the last 5 minutes. By the time the bulb had burned at 30 watts for 15 minutes, it had already used $$ \frac{30\mbox{ joules}}{\mbox{second}} \cdot \frac{15\mbox{ minutes}} 1 = \frac{30\mbox{ joules}}{\mbox{second}} \cdot \frac{900 \mbox { seconds}} 1 = 27{,}000\mbox{ joules}. $$ The remaining energy to be used would be 60,000-27,000 = 33,000 joules. Step 3 Determine the speed (in joule/sec) that would be to use the remaining energy during the last 5 minutes. The remaining energy should be used for 5 minutes, which is the same as 300 seconds. So we have $$ \frac{33{,}000\mbox{ joules}}{300\mbox{ seconds}} = \frac{110\mbox{ joules}}{1\mbox{ second}} = 110\mbox{ watts}. $$ Answer The bulb would have burned at 110 watts over the last 5 minutes. Return to lesson Error: Click Not a Robot, try downloading again. Again.