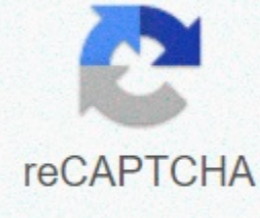I'm not robot

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Change base formula for logarithms

Change of Base Formula A formula that lets you rewrite a logarithm in terms of logs written with a different basis. It is especially useful when using a calculator to evaluate a log to any base other than 10 or e. Assume that x, a and b are all positive. Also assume that a ≠ 1, b ≠ 1. Change of base formula: Example 1: Example 2: (note that) Example 3: See also Logarithm rules, general logarithm, natural logarithm Changing base formula is a formula for expressing a logarithm in one base in terms of logarithms in other bases. For any positive real numbers so that not, we have allowed us to rewrite a logarithm in terms of logarithms in any basis . This formula can also be written Proof Late . Then. And, taking that from both sides, we get through the properties of logarithms, replacement for y, Used for calculations The change of base formula is useful for simplifying certain calculations regarding logarithms. For example, we have by changing base formula that The formula can also be useful when calculating logarithms on a calculator. Many calculators only have features for calculating base-10 and base e logarithms. But you can still calculate logs in other bases, you just need to use the change of base formula to insert into base 10. For example, if you wanted to calculate, you would first convert it into the form. Then you'll evaluate it using the base-10 log function on the calculator. Special cases and consequences Many other logarithm rules can be written in terms of changing base formula. For example, we have it . The use of the second form of changing base formula gives . One consequence of changing base formula is that for positive constants, the functions and difference by a constant factor, for everyone . This biographical article reseals a stub. Help us by expanding it. Most calculators can only evaluate common and natural logs. In order to evaluate logarithms with a base other than 10 or [latex]e[/latex], we use the change-of-base formula to rewrite the logarithm as the quotation of logarithms of any other basis; when we use a calculator, we will change it to common or natural logs. To initiation the change-of-base formula, we use the one-on-one property and power rule for logarithms. Given any positive real numbers M, b and n, where [latex]ne 1 [/latex] and [latex]be 1[/latex], we show [latex]{\mathrm{log}}_{b}M\text{=}\frac{{\mathrm{log}}_{n}M}{{\mathrm{log}}_{n}b}[/latex] Let [latex]y={\mathrm{log}}_{b}M[/latex]. By taking the log base [latex]n[/latex] from both sides of the equation, we arrive at an econsential form, namely [latex]{b}^{y}=M[/latex]. This follows after {b}^{y}\right)\hfill & ={\mathrm{log}}_{n}M\hfill & \text{Apply the one-on-one property}.\hfill \\y{\mathrm{log}}_{n}b\hfill & ={\mathrm{log}}_{n}M\hfill & \text{Apply the power line for \\}\hfill & =\frac{\mathrm{log}}_{n}M}{\mathrm{log}}_{n}b}\hfill & \text{Isolate }y.\hfill \\ {\mathrm{log}}_{b}M\hfill & =\frac{{\mathrm{log}}_{n}M}{{{\mathrm{log}}_{n}b}\hfill & \text{Substitutes for }y.\hfill \end{cases}[/latex] For example, to evaluate [latex]{\mathrm{log}}_{5}36[/latex] using a calculator, we must first rewrite the expression as a quote from common or natural logs. We will use the common log. [latex]\start{cases}{\mathrm{log}}_{5}36\hfill & =\frac{\mathrm{log}\left(36\right)}{\mathrm{log}\left(5\right)}\hfill & \text{Apply changing base formula using base 10}\text{.} \hfill \\\hfill & \about 2.2266\text{ }\hfill & \text{Use a calculator to evaluate to 4 decimal places}\text{.} \hfill\end{cases}[/latex] The change-of-base formula can be used to evaluate a logarithm with any base. For any positive actual numbers M, b and n, where [latex]ne 1 [/latex] and [latex]be 1[/latex], [latex]{\mathrm{log}}_{b}M\text{=}\frac{{\mathrm{log}}_{n} {\mathrm{log}}_{n}b}[/latex]. This follows that the change-of-base formula can be used to rewrite a logarithm with any basis as the quote of common or natural logs. [latex] {\mathrm{log}}_{b}M=\frac{\mathrm{ln}M}{\mathrm{ln}b}[/latex] and [latex]{\mathrm{log}}_{b}M=\frac{\mathrm{log}} {\mathrm{log}b}[/latex] Determine the new base n, remember that the general log, [latex]\mathrm{log}\left(x\right)[/latex], has base 10, and the natural log, [latex]\mathrm{ln}\left(x\right)[/latex], has base e. Rewriting the log as a quote using the change-of-base formula The numerator of the quote will be a logarithm with base n and argument M. The denominator of the quotation will be a logarithm with base n and argument b. Change [latex]{\mathrm{log}}_{5}3[/latex] to a quote from natural logarithms. Because we will express [latex]{\mathrm{log}}_{5}3[/latex] as a quote from natural logarithms, the new base, n = e. We rewrite the log as a quote using the change-of-base formula. The suppose of the quote will be the natural log with argument 3. The denominator of the quote will be the natural log with argument 5. [latex]\begin{cases}{\mathrm{log}}_{b}M\hfill & =\frac{\mathrm{ln}M}{\mathrm{ln}b}\hfill \\ {\mathrm{log}}_{5}3\hfill & =\frac{\mathrm{ln}3}{\mathrm{ln}5}\hfill \end{cases}[/latex] Change [latex]{\mathrm{log}}_{0.5}8[/latex] to a quote from natural logarithms. Solution Can we change common logarithms to natural logarithms? Yes. Remember that [latex]\mathrm{log}9[/latex] means [latex]\text{log}}_{\text{10}}\text{9}[/latex]. So, [latex]\mathrm{log}9=\frac{\mathrm{ln}9}{\mathrm{ln}10}[/latex]. Evaluate [latex]{\mathrm{log}}_{2}\left(10\right)[/latex] using the formula with a calculator. According to the change-of-base formula, we can rewrite the log base 2 as a logarithm of any other basis. Since our calculators can evaluate the natural log, we can choose to use natural logarithms, which is the log base e. [latex]\begin{cases}{\mathrm{log}}_{2}10=\frac{\mathrm{ln}10}{\mathrm{ln}2}\hfill & \text{Apply changing base formula using base }e.\hfill \\\\\about 3.321 Use a calculator to evaluate to 4 decimal places}.\hfill\\\hfill\\321 Use a calculator to evaluate to 4 decimal places}.\hfill\\\hfill\\\end{cases}[/latex] Evaluate [latex]{\mathrm{log}}_{5}\left(100\right)[/latex] using the change-from-base formula. Solution I discussed most of the log rules in a separate lesson. However, I deliberately left out one to discuss it in detail here. The log rule is called the Change-of-Base Formula. If you are interested in why the Change-of-Formula works, click the following link to view the proof: Evidence from Logaritm Properties. The logarithm that uses base-10 is known as the common logarithm, while the one that uses base-e is known as the natural logarithm. Note: The number of e is a mathematical constant that has a numeric value of e \approx. 2.71828. This is an irrational number because it cannot be expressed as a ratio of two integers or as a fraction. More so, the number of e is the basis of the natural logarithm. Therefore, the general logarithms and natural logarithms use the standard bases 10 and e, respectively. common logarithm is written as \large{\color{red}\log x = {\log _{10}}x} natural logarithm written as \large{\color{blue}\ln x = {\log _e}x} Before proceeding, I would like to point out some nuances or subtleties regarding the mathematical expressions of common and natural logarithms. The LOG and LN Buttons of a Graph Calculator Most chart calculators have features or keys that directly calculate the logarithms of numbers in base-10 and base-e. So, you'll only see two buttons: LOG for general logarithm and LN for natural logarithm. Obviously, a problem arises when we want to calculate the logarithm of a number using non-standard bases such as 2, 3, 7, 0.5, and 0.25. The logarithms above use NON-STANDARD bases because they do not have \large{\color{green}10} or the number \large{\color{green}e}. How do we proceed to punch in the numbers on a chart calculator? As I mentioned before, most calculators are limited to only computer logarithms with base 10 and base e. This is where the Change-of-Base Formula comes to save. It can convert a non-standard-base logarithm as a ratio of two logarithmic operations that use the standard base either 10 or the constant e. What is the Change-of-Base Formula? The Change-of-Base Formula is an instruction on how to rewrite or transform a given logarithmic expression as a relationship or fraction of two logarithm operations using any valid basis. That means, if we use a logarithm using a Base, then we can turn it into an equivalent relationship or fraction of two logarithmic operations so we choose any base we want. We can literally choose any basis as long as it is positive, but equals \color{red}1. But if we want to calculate or know the value of a logarithm, we need to choose base-10 or base e, since most calculators have these feature keys. The log key [log] calculates the common (using base-10) logarithm while the ln key [ln] calculates the natural (using base-e) logarithm. Let's analyze how the formula transformed the original logarithm into an equivalent expression as a

proportion of two log operations. The argument of the original logarithm becomes the argument of the logarithm of the numerator. The basis of the original logarithm becomes the argument of the logarithm of the denominator. The logarithms in the numerator and denominator have the same base. The value of the base $\large\color{green}c$ is any basis we selected. Examples of the Change-of-Base Formula The first two examples (Example #1 and #2) are perfect textbook problems because the argument and the basis of a logarithm can be expressed as powers of a common number (a positive number that does not equal 1) that serve as the new base when the change-of-base rule applyes. Example 1: Evaluate $\large{\log {}\_48}$. The first thing I acknowledged is that both the argument and its base can be expressed as a force of 2. Note that it therefore only makes sense to transform $\log {}\_48$ as a ratio of two logarithmic operations with base-2 using the Change-from-Base Formula. Then simplify further using the following rules logarithms which are: $\large{\color{blue}{\log \_b}\left({{x^k}} \right) = k\cdot {\log \_b}\left(x\right)}$ $\large{\color{red}{\log \_b}b = 1}$ Here's the complete solution. Example 2: Evaluate $\large{\log {}\_{\large{{1\over {27}}}}\left(9\right)}$. You might think there's a problem with the question, because the basis of the logarithm is a fraction. Remember that the basis of a logarithm should be positive, but it cannot equal 1. So, the base $\large{1 \over {27}}$ is completely okay. This is obviously a positive number and it does not equal 1. The fraction $\large{1 \over {27}}$ can again be written a force of 3. This is where the Negative Rule of Econent comes in handy. The Negative Rule of Econctory allows us to reverse the fraction (recite), but we need to switch the sign of the econting. For $\Large{1 \over the {27}}$, we can express it as a force of 3 with a negative econteacher. For 9, we can also rewrite it as a force of 3 that is $9 = {3^2}$. Since the base and argument of $\large{\log {}\_{{{1\over {27}}}}\left(9\right)}$ can be rewritten as powers of 3, we will calculate this logarithm using the Change-of-Base Formula with base-3. On top of the Negative Rule of Econtensent we will also need the rules of logarithms like Indicated. $\large{\color{blue}{\log \_b}\left({{x^k}} \right) = k\cdot {\log \_b}\left(x\right)}$ $\large{\color{red}{\log \_b}b = 1}$ Now, below is the complete solution. Example 3: Calculate the value of $\large{\log {}\_{{\{5}}\left) {} \_{{\{5}}\left) \right)}$. Round out your answer to the nearest thousandth. This is no longer a pretty problem, because the argument and the basis of the logarithm cannot be expressed as forces of a common number. In other words, there's no scenario where we can express 5 and 12 as epsonic numbers so they have the same base. To resolve this, we can use the change-of-base rule to rewrite the original logarithm as a ratio of two logarithms of the basis of our choice. We have two options: use base-10 or base-e. It doesn't matter which one we choose, because the answer will be the same. For this problem, let's use base-10. Don't forget to round up your answer to three decimal places because we're asked to round it off to the nearest thousandth. Our calculator must confirm that our answer is correct. Example 4: Calculate the value of $\large{\log {}\_{\large{7}}\left(right{9)}}$. Round your answer to the nearest hundredth. In the previous example, we used base-10 to evaluate the logarithm. This time, we'll use the natural number $\color{red}e$ as the basis of choice when applying the Change-of-Base formula. Note that we don't need to write the natural logarithm as $\large{{\log }\_e}\left(x\right)}$. We can leave that step and write it immediately as $\large{\ln \left(x\right)}$. I added this as one of the steps below for clarity and emphasis purposes. Let's continue and apply the change-of-base rule to call $\large{\log {}\_7\left(9\right)}$ as a relationship or fraction of two natural log operations. Also, be sure to round up your answer in two decimal places, as the problem requires expressing our final answer in the nearest hundredth. Your calculator must perform similar result as the one below. Example 5: Change $\large{\log {}\_{\large{6}}\left({0.1} \right)}$ as a quote from two natural logarithms. Calculate its value and then round to the nearest tide. This problem requires us to change the given logarithm as a quote from natural logarithms. This means we have no choice but to use the natural number $\large\color {save}e$ as the basis when we apply the change-of-base formula. Don't forget so well that we're told to round up our answer to the nearest tenth (one decimal place). Here's our solution: Our calculator agrees with our answer. Example 6: Change $\color{blue}\large{\log\left(7\right)}$ as a quote in terms of natural logarithms. Then calculate its value. Round out your answer to the nearest ten thousandth. I acknowledge that although we can resolve directly to the value of $\color{red}\log\left(7\right)$ with a calculator as it has a LOG key, want problem we have to take the long route. Not because it's a futile exercise of our time, but more importantly, it's an opportunity for us to apply our solid understanding or understanding of the change-of-base formula. Remember that when you see a log operation without a base, it is assumed to have a base of 10. Have. our very first step is to rewrite $\log\left(7\right)$ as $\{\log \_{10}}\left (7\right)$ so that it's much easier to see what are the numbers we're doing during the change-of-base step. It's a great feeling when a calculator verifies the value outputs that our answer. Example 7: Change $\color{blue}\large\ln\left({13}\right)$ as a relationship in terms of common logarithms. Then calculate its value. Round out your answer to the nearest ten thousandth. Just as example #6, you don't need to apply the change-of-base formula because we can calculate it directly with a calculator. However, the intention of this problem is to showcase our in-depth understanding of general and natural logarithms, and how to properly handle the formula. So, let's transform $\large\ln\left({13}\right)$ into LOG form where the base is $\large\color{blue}e$, therefore, $\large{\log \_{\large{e}}}\left({13}\right)$. Now we use the change-of-base formula to express it as a proportion of two common logarithms. Remember, the common logarithm uses base-10. Yes! The calculator agrees with our answer. Answer.