Indexed family of sets

In mathematics, a family or indexed family is an unofficial collection of objects, each of which is related to an index set index. For example, a family of real numbers indexed by a set of integers is a collection of real numbers where a function selects a real number for each integer (probably the same). More specifically, the indexed family is a mathematical function, along with the I {\displaystyle I} range and the X {\displaystyle X} image. Items in the X set are often referred to as the family component. In this view, indexed families are interpreted as a collection of functions rather than functions. The I {\displaystyle I} set is called the family index (set), and X {\displaystyle X} is the indexed set. Mathematical statement definition. Set (\displaystyle I} and X {\displaystyle X}, and x {\displaystyle x} is a surjective function, so that x : I → X i ↦ x i = x (i) , {\displaystyle {\begin{aligned }x\colon I&\to X\\i&i&ll mapsto x_{i}=x(i),\end{aligned}}} and then this creates an element family X {\displaystyle X} indexed I {\displaystyle I} (x i) i ∈ i {\displaystyle (x_{i}_{i\in I}} or simply (x i) {\displaystyle (x_{i})} when the index set is considered known. Sometimes angular brackets or braces are used instead of brackets, and the latter is used at risk of mixing families with sets. The indexed family can be converted to the X = { x i : i ∈ I } {\displaystyle {\mathcal {X}}=\x_{i}:i\in I\}} (I\}}) set, i.e. under I image x. Because x mapping does not need to be an injective, there may be i , j ∈ I {\displaystyle i,j\in i} in addition to i ≠ j (\displaystyle ieq j}), that x i = x j {\displaystyle x_{i}=x_{j}} . Thus , | X | ≤ | 1000000 , {\display style | {\mathcal {X}}|| \leq | I|,} where | A| indicates the number of sets A. Index inventory is not limited to counting, and of course a subset of powerset can be indexed, resulting in an indexed pool family. See below for important differences between stocks and families. Examples Index Mark: When you use an index mark, indexed objects form a family. For example, consider the following sentence: Vectors v1, ..., vn are linearly independent. Here (vi)i ∈ {1, ..., n} represent a vector family. The i-th vector vi only makes sense with respect to this family, the sets are unsettled and there is no i-th vector in a set. Furthermore, linear independence can only be defined as a property of the collection; therefore, it is important whether these vectors are linearly independent as sets or families. When considering n = 2 and v1 = v2 = (1, 0), the set of them consists of only one element and is linearly independent, but the family contains the same element twice and depends linearly. Matrices Let's say that a text says: only if rows A are linearly independent. Compared to the previous example, it is important that rows A are linearly independent as families, not as inventory. For example, consider the matrix A = [1 1 1 1 1]. {\displaystyle A={\begin{bmatrix}1&1\\1&1\end{bmatrix}}.} The set of rows consists of only one element (1.1) and is linearly independent, but the matrix cannot be inverted. The family of rows contains two elements and is linearly dependent. Therefore, the statement is correct if it refers to the family of rows, but it is incorrect to refer to a set of rows. (The statement is correct even if the rows are interpreted as referring to a multiset in which the elements are also separated but which lack part of the structure of the indexed family.) Features, kits and families with Surjective features and families are officially equivalent to any function f domain I induve a family (f(i))i∈I. In practice, however, the family is considered a collection, not a function: being an element of a family is equivalent to being in the range of the corresponding function. The family contains each item exactly once, if and only if the correct function is injectable. As a set, the family is a container, and each x set gives a family (xx)x∈X. Thus, each set of course will be a family. All families (Ai)i∈I there is the set of all the items {Ai | i∈I}, but it does not carry any information about more containment or structure provided by I. Therefore, using a set instead of a family, some information may be lost. Examples: Make n the finiable set {1, 2, ..., n}, where n is a positive integer. The ordered pair is a family indexed by 2 = {1, 2} two items. n-tuple is the n. indexed family. Infinite sequence is a family indexed by natural numbers. The list is the n-tuple of an unspecified n or infinite sequence. The n×m matrix is a family indexed by the product n×m descartes. A mesh is a family indexed by a controlled set. Operations families index stocks are often used for amounts and other similar operations. For example, if (ai)i∈I have a family of numbers, the sum of all numbers ∑ i ∈ i a i . {\display style \sum _{i\in I}a_{i}.} When (Ai)i∈I have a family of kits, the union of all these kits denotes ⋃ i ∈ I A i. {\displaystyle \bigcup _{i\in I}A_{i}.} Similarly, for intersections and products in Descartes. Subfamily Family (Bi)i∈J is a family subfamily (Ai)i∈I, if and only if J is the I and all i subfamily J Bi = Ai Usage in category theory Main article: Diagram (category theory) A similar concept of category theory is called a diagram. A chart is a functor that results in a family of objects in one category C, another category J, related to two index-dependent morphine. See also: Serial co-product disjoint union tagged union index Array data type Net (mathematics) Diagram (category theory) Parametric Family References Mathematical Society of Japan, Encyclopedic Dictionary of Mathematics, 2nd edition, 2 vols., Kiyosi Itô (ed.), MIT Press, Cambridge, MA, 1993. EDM (volume). The pre-activity \(\PageIndex{1}\): The union and intersection of a set of sets in section 5.3 discussed the different properties of the actions you set. We are now focusing on the associative properties of the set union and set intersection. Notice that the definition of set union tells us how to create a union of two groups. This is the associative law that allows us to discuss the union of three sets. By using peer rights, if \(A\), \(B\) and \(C\) are subsets of a universal set, \(A \cup B \c\) \(A \cup B) \(A \cup B) or \(A \cup (B \c)\) can be. That is, \(A \cup B \cup C = (A \cup B) \cup C = A \cup (B \cup C).\) For this activity, the universal set is N, and the following four sets will be used: \(A=\) {1, 2, 3, 4, 5} \(B=\) {2, 3, 4, 5, 6} \(C=\) {3, 4, 5, 6, 7} \(D=\) {4, 5, 6, 7, 8} Use the roster method to specify sets \(A \cup B \cup C\), \(B \cup C \cup D\), \(A \cap B \cap C\), and \(B \cap \D\), \(\Cap B \Cap\), \((B \cap C\), \(B \cap C\), \((B \cap C\), \((B \cap C\), \(Cap \Cap\). Use the roster method to specify each of the following sets. In all cases, be sure to follow the order specified in parentheses. (a) \((A \cup B \cup C) \cup D\) (b) \(A \cup (B \cup C \cup D)\) (c) \(A \cup (B \cup C) \cup D\) (d) \(A \cup B) \cup (C \cup D)\e) \(A\cap B \cap C) \cap D ((A\cup f) \(A \cap (B \cap C \cap D)\) (g) \(Based on the work done in \cap (B \cap D\) \cap D\) (h) \((A \cap B) \cap (C \cap D)\) Based on the work done in Part A (2), does the placement of parentheses matter when determining the intersection (or) intersection of the four sets? Does this allow \(A \cup B \cup C \cup D\) and \(A \cap B \cap C \cap D\)? We've already seen that elements of a set can be sets themselves. For example, the performance set of a set \(T\), \(\mathcal{P}(T)\) is a set of all subsets of \(T\). The term inventory set sounds confusing, so we often use the terms collection and family when we want to emphasize that elements of a particular set are sets themselves. We would then say that the performance set \(T\) is a family (or collection) of subsets of \(T\). One of the goals of the work done so far in the preview was to show that it was possible to define the alliance and intersection of a group of members. Definition: Make it a \(\mathcal{C}\) pool family. Meseting \(\mathcal{C}\) is a set of all items in at least one set of \(\mathcal{C}\). \(\bigcup_{X \in \mathcal{C}}^{} X = \{x \in U\ |x \in X \text{ some } X \in \mathcal{C}\}\) The intersection intersection is defined as a set of all items in all sets of \(\mathcal{C}\). For that matter, \(\bigcap_{X \in \mathcal{C}}^{} X = \{x \in U\ |\ |x \in X \text{ some } X \in \mathcal{C}\}\) For example, For example, consider the four \(A\), \(B\), and the following presets \(S=\) {5, 7, 8, 9} and \(T=\) {6, 7, 8, 9} and \(T=\) {6, 7, 8, 9, 10} Then we can consider the following sets families. : \(\mathcal{A} = \{A, B, C, D\}\) and \(\mathcal{B} = \{A, B, C, D, S, T\}\) Explain why \(\\bigcup_{X \in \mathcal{A}{{{{} X = A \cup B \cup C \cup D\) and \(\bigcap_{X \in \mathcal{A}}^{} X = A \cap B \cap C \cap D\) and use the work to define (1), (2) and (3) \(\bigcup_{X \in \mathcal{A}}{{} X\) and \(\bigcap_{X \in \mathcal{A}}{{{X). Use the manifest method to specify \(\bigcup_{X \in \mathcal{B}}^{X\) and \(\bigcap_{X \in \mathcal{B}}^{X\) 6. Use the roster method to specify \(((\bigcup_{X \in \mathcal{A}}^{X)^c\) and \(\bigcap_{X \in \mathcal{A}}^{{ X^c\). Note that the universal set is \(\mathbb{N}\). View activity \(\PageIndex{2}\): An indexed set family that is often used by subscripts to identify sets. For example, in \(\PageIndex{1}\), you could have used \(A\), \(B\), \(C\), and \(D\) as the names of the presets, \(A_1\), \(A_2\), \(A_3\), and \(A_4\). When you do this, the subscript is used as an identifying tag or index for each set. This idea can also be used to provide an endless set of sets. For example, for each natural number, \(n\) is defined as \(C_n = \{n, n + 1, n + 2, n + 3, n + 4\}.\) So if you have a family of sets \(\mathcal{C} = \{C_1, C_2, C_3, C_4\}\), use notation \(bigcup_{j = 1}^{4} C_j\) to mean the same as \(bigcup_ {x \in \math{C}}^{}. Define \(\bigcup_{j = 1}^{4} C_j\) and \(\bigcap_{j = 1}^{4} C_j\) We see that you don't even need to define the pool family by using subscripts \(\mathcal{A}\). You can work with the infinite pool family \[\mathcal{C} ^{\ast} = \{A_n\ |\ \n \in \mathbb{N}\}\] and use subscripts to indicate which sets to use in a merge or intersection. Use the roster method to specify the following pairs of sets. The universal set \(\mathbb{N}\). (a) \(\bigcup_{j = 1}^{6} C_j\) and \(\bigcap_{j = 1}^{6} C_j\) (b) \(\bigcup_{j = 1}^{8} C_j\) and \(\bigcap_{j = 1}^{8} C_j\) (c) \(\bigcup_ {j = 4}^{8} C_j\) and \(\bigcap_{j = 4}^{8} C_j\) (d) \(((\bigcap_{j = 1}^{4} C_j)^c\) and \(\bigcup_{j = 1}^{4} C_j ^c\) One of the goals of the preview tasks was to to show that we often encounter situations in which more than two sets can be defined as mergers and intersections of more than two sets. Preliminary activity \(\PageIndex{2}\) We also saw that it is often convenient to index stocks in a stock family. Especially if you are a natural natural and \(\mathcal{A} = \{A_1, A_2, ..., A_n\}\) is the family of \(n\) sets, then merge the following \(n\) sets that you can \(A_1 \cup A_2 \cup \cdot\cdot\cdot \cup A_n\) or \(\bigcup_{} j = 1 }^{n} A_j\) you can also specify the intersection of \[\bigcup_{j = 1}^{n} A_j = \{x \in U\ |\ \in A_j, \text{ some } j \text{ and } 1 \le j \le\n\}.\] These \(n\) presets can also be intersectioned. \(A_1 \cap A_2 \cap \cdot\cdot\cap A_n\) or \(\bigcap_{j = 1}^{n} A_j\). such as \[\bigcap_{j = 1}^{n} A_j = \{x \in U\ |\ x \A_j, \text{ some } j \text{ and } 1 \le j \le n\}.\] This idea can be extended to define the merge and intersection of a family of infinitely many sets. So if \(\mathcal{B} = \{B_1, B_2, ..., B_n, ...\}\), then \(\bigcup_{j = 1}^{\infty} B_j = \{x \in U\ |\ x \B_j, \text{ some } j \text{ with } j \ge 1\}\), and \(\bigcap_{j = 1}^{\infty} B_j = \{x \in U\ |\ x \in B_j, \text{ for all } j \text{ with } j \ge 1\}\). Progress Check 5.26 (Infinite Set Family) For all natural numbers \(n\), leave \(A_n = \{1, n, n^2\}\). For example\(A_1 = \{1\}\), \(A_2 = \{1, 2, 4\}\), \(A_3 = \{1, 3, 9\}\) and \(\bigcup_{j = 1}^{3} A_j = \{1, 2, 3, 4, 9\}\), \(\bigcap_{j = 1}{3} A_j = \{1\}\). Define each of the following sets: \(\bigcup_{j = 1}^{6} A_j\) \(\bigcap_{j = 1}^{6} A_j\) \(\bigcup_{j = 3}^{6} A_j\) \(\bigcap_ {j = 3}^{6} A_j\) \(\bigcup_{j = 1}{\\infty} A_j\) \(\bigcap_{j = 1}^{\infty} A_j\) Do not delete this text first. In all the examples we've studied so far, we've used some of \(\mathbb{N}\) or \(\mathbb{N}\) to index or tag inventory in a pool family. You can also use other sets to index or tag inventory in a set family. For example, for each real number, \(x\) \(B_x\) can be the closed interval [x, x + 2]. As a matter of fact, \(B_x = \{y \in \mathbb{R}\ |\ x \le y \le x + 2\}\). So we're going to make the following definition. In this definition,\(\wedge\) is the Greek capital lambda, and \(\alpha\) is the Small Greek alpha letter. Definition Let \(\Lambda\) is not an empty set, and let's say that for each \(\alpha \in \wedge\), there is an appropriate set \(A_{\alpha}\). The set family \(\{A_{\alpha}\ |\ \alpha \in \wedge\}\) is called the indexed pool family \(\wedge\). All \(\alpha \in \wedge\) are called indexes, \(\Lambda\) indexing sets. Progress Check 5.27 (Indexed stock families) In each of the indexed stocks seen so far, if the indexes were different, the inventories were different. This means that if \(\Lambda\) indexes the set of sets \(\mathcal{A} = \{A_{\alpha}\ |\ \alpha \in \wedge\}\), then if \(\alpha, \beta \in \wedge\) and \(alpha\e \beta\), then \(A_{\alpha} e A_{\beta}\). (Note: \(\beta\) is the Greek small font.) Leave \(\Lambda = \{1, 2, 3, 4\}\) and for all \(n \in \Lambda\), \(A_n = \{2n + 6, 16 - 3n\}\) and leave \(\mathcal{A} = \{A_1, A_2, A_3, A_4\}\). Specify \(A_1\), \(A_2\), \(A_3\), and \(A_4\). Is the following statement true or false in indexed <a1>\(\mathcal{A}\) in (1)? Now be \(\Lambda = \mathbb{R}\). For each \(x \in \mathbb{R}\), define \(B_x = \{0, x^2, x^4\}\). Is the following statement true for the indexed family of the pool \(\mathcal{B} = \{B_x\ |\ |\ x \in \mathbb{R}\}\)? All \(x, y \in \mathbb{R}\), if \(x e y\), \(B_x for B_y\). Answer: Add text here. Don't delete this text first. We now re-define the definitions of the union and the intersection of a family of stocks with an indexed family of stocks. Definition Let \(\Lambda\) is a non-empty indexing set and should be the indexed pool family \(\mathcal{A} = \{A_{\alpha}\ \\ \alpha \in \Lambda\}).\ 10.A_ 2.2.2.2.2.1.1.1.1.1.1.2.2.1.2.1.2.1.2.1.2.2.1.2.2.2.1.2.1. Type \(\bigcup_\alpha \in \Lambda}^{} A_{\alpha} = \{x \in U\ |\ \text{ from } \alpha \in \Lambda \text{ and } x A_{\alpha}\}\). Above the intersection (\mathcal{A}\) are all the items that all sets \(A_{\alpha}\) all \(\alpha\\) all \(\alpha \a \Lambda\). As a matter of fact, \(\\bigcap_\\alpha \in \wedge}^{} A_{\alpha} = \{x \in U\ |\ \text{ all } \alpha \in \wedge, x \in A_{\alpha}\}\). Example 5.28 (Set family indexed by positive real numbers) For each positive real numbe \(\alpha\), leave \(A_{\alpha}\) as the interval (-1, \(\alpha\)). As a matter of fact, \(A_{\alpha} = \{x \in \mathbb{R}\ |\ -1 < x \le \alpha\}.\) If you leave \(\mathbb{R} ^{+}\) to be the set of positive real numbers, we have a set family that has been indexed by \(\mathbb{R} ^{=}\). First, we determine the union of this team family. Notice that all \(\alpha \in mathbb{R} ^{+}\), \(\alpha \in A_{\alpha}\), and if \(y\) is a real number \(-1 < y \le 0\), \(y \in A_{\alpha}\). Also note that if \(y \in \mathbb{R}\) and \(y < -1\), all \(\alpha \in mathbb{R} ^{+}\), \(y otin A_{\alpha}\). With these observations, we have concluded that \(\bigcup_{\alpha \in \mathbb{R} ^{+}}^{{% A_{\alpha} = (-1, \infty) = \{x \in \mathbb{R}\ |\ -1 < x\}.\) To determine the intersection of the family, observe that if \(y \in \mathbb{R}\) and \(y < -1\), all \(\alpha \in \mathbb{R} ^{+}\), \(y otin A_{\alpha}\); if \(y \in \mathbb{R}\) and \(-1 < y \le 0\) , then \(y \in A_{\alpha}\) all \(\alpha \in mathbb{R} ^{+}\), and if \(y \in \mathbb{R}\) and \(y > 0\), leave \(\beta = \dfrac{y}{2}\), \(y > \beta\) and \(y otin A_{\beta}\). From these observations, we concluded that \(\bigcap_\\alpha \in \mathbb{R} ^{+}^{{{ A_{\alpha} = (-1, 0] = \in \mathbb{R}\ |\ -1 < x \le 0\}.\) 0\}.\) ^{+}\), we see that \(A_{\alpha} ^c = (-\infty, 1] \cup (\alpha, \infty).\) In step 5.28, click the following: For each set, use the interval mark or the editor markup. \(/\bigcup_{\alpha \in \mathbb{R} ^{}}{{A_{\alpha}})^c\) \((\bigcap_{\alpha \in \mathbb{R} ^{}}^{A_{\alpha}})^c\) \\c (\c \bigcap_{\alpha \in \mathbb{R} ^{+}}^{{} A_{\alpha} ^c\) \(\bigcup_{\alpha \in \mathbb{R} ^{}}^{A_{\alpha} ^c\) Add text here. Don't delete this text first. At Theorem 5.30, we demonstrate some of the properties of specific operations indexed to families' stocks. Some of these properties are a direct extension of the corresponding properties to two sets. For example, we've proven De Morgan's laws with two sets of theorem 5.20. Activities in preview and Progress Check 5.29 work recommend that you get similar results using actions with an indexed inventory family. For example, in \(\PageIndex{2}\), we saw in preview that \(\bigcap_{j = 1}^{4} A_j)^c = \bigcup_{j = 1}^{4} A_j ^c.\) Theorem 5.30. \(\Lambda\) should not be an empty indexing set, and \(\mathcal{A} = \{A_{\alpha}\ \\ \alpha \in \wedge\}) should be an indexed pool family. Then all \(\beta \in \Lambda\), \(\bigcap_\\alpha \in \Lambda} A_{\alpha} \subseteq A_{\beta}\) all \(\beta \in \Lambda \(A_{\beta} \subseteq \bigcup_{\alpha \in \Lambda} A_{\alpha}\) \(\bigcap_\alpha \in \Lambda}^{} } A_{\alpha})^c = \bigcup_{\alpha \in \Lambda}^{} A_{\alpha} ^c\) \((\bigcup_{\alpha \in \Lambda}^{} }A_{\alpha})^c = \bigcap_{\alpha \in \Lambda}^{A_{\alpha} ^c\) Parts (3) and (4) (De Morgan' 's Laws. Evidence We will prove Components (1) and (3). The test prints of parts 2 and 4 shall be as referred to in Article 4(2). Therefore, allow \(\Lambda\) to be a non-empty indexing set and for the indexed pool family \(mathcal{A} = \{A_{\alpha}\ \\ \alpha \in \Lambda\}\). Article 1(2) of the A_ A_ bigcap_ shall be replaced by the following: Because \(\beta\) is an item in the \(\Lambda\folder), it is \(x \in A_{\beta}\). This proves that \(\bigcap_{\alpha \in \Lambda}^{} A_{\alpha} \subseteq A_{\beta}\). To prove Part (3), you can prove that each set is part of the other set. First, leave \(x \in (\bigcap_\\alpha \in \Lambda}^{} A_{\alpha})^c\). This means \(x otin (\bigcap_\\alpha \in \Lambda}^{} A_{\alpha})\), and this means that there is a \(\beta \Lambda\) that is ,(x otin A_{\beta}\). Therefore, \(x \in A_{\beta} ^c\), which means \(x \in \bigcup_\\alpha \in \Lambda}^{} A_{\alpha} ^c\). Therefore, we have demonstrated that \[\\bigcap_{\alpha \in \Lambda}^{{} A_{\alpha})^c \subseteq \bigcup_{\alpha \in \Lambda}^{} A_{\alpha} ^c.\] Now \(y \in \bigcup_\\alpha \in \Lambda}^{{A_{\alpha} ^c\). This means that \(\beta \in \Lambda\), whether \(y \in A_{\beta} ^c\) or \(y otin A_{\beta}\). However, because \(y otin A_{\beta}\), we can conclude that \(y otin \bigcap_\\alpha \in \Lambda}^{} A_{\alpha}\) and thus \(y \in (\bigcap_{\alpha \in \Lambda^{} A_{\alpha}^c\). This demonstrates that \[\bigcup_{\alpha \in \Lambda}^{} A_{\alpha} ^c \subseteq (\bigcap_{\alpha \in \Lambda}^{} A_{\alpha})^c.\] The results are used in (5.5.4) and (5.5.5.5.5), that \(\\bigcap_\\alpha \in \Lambda}^{} A_{\alpha})^c = \bigcup_{\alpha \in \Lambda}^{} A_{\alpha} ^c.\) Many other properties of inventory operations are also true for indexed rows in sets. Item 5.31 states the distribution laws set out in the operations. Item 5.31 \(\Lambda\) should not be an empty indexing set, and \(\mathcal{A} = \{A_{\alpha}\ |\ \alpha \in \Lambda\}\) should be an indexed pool family, and \(B\) should be a pool. Then \(B \/bigcup_\alpha \in \Lambda}^{} A_{\alpha}) = \bigcup_{\alpha \in \Lambda}^{} (B \cap A_{\alpha})\), and \(B \bigcap_\\alpha \in \Lambda}^{} A_{\alpha}) = \bigcap_\\alpha \in \Lambda}^{} (B \cup A_{\alpha})\). Evidence from theorem 5.31 is evidence of practice (5). In section 5.2, the even families of sets have two sets \(A\) and \(B\) not properly defined, assuming that \(A \cap B = \emptyset\). Similarly, if \(\Lambda\) is not an empty indexing set, and \(\mathcal{A} = \{A_{\alpha}\ |\ \alpha \in \Lambda\}\\alpha\) is an indexed pool family, we can say that this indexed pool family is separate, assuming \\bigcap_{\alpha \in \Lambda}{A_{alpha} = \emptyset\). However, we can use the concept of two separate sets to determine a slightly more interesting type of disjointness in an indexed family of kits. Definition Let \(\Lambda\) is a non-empty indexing set and should be the indexed pool family \(\mathcal{A} = \{A_{\alpha}\ |\ \alpha \in \Lambda\}\). We say that \(\mathcal{A}\) are some wise separated, assuming that \(\alpha\) and \(\beta\) are all \(\beta\) if \(A_{\alpha} e A_{\beta}\) and then \(A_{alpha} \cap A_{\beta} = \emptyset\). Figure 5.32 (5.B_4 B_3 B_2 B_1 A_4 A_3 A_2 A_1 32) Stocks 5.7. Is the set \(\mathcal{A}\) a separate set family? A couple of you in the set family? Is the set \(\mathcal{B}\) a separate pool family? A couple of you in the set family? Now make it universal \(\mathbb{R}\). For all \(n \mathbb{N}\), leave \(C_n =\ (n, \infty)\) and leave \(\mathcal{C} = \{C_n\ |\ \n \in \mathbb{N}\}\). Is the set \(\mathcal{C}\) a separate pool family? A couple of you in the set family? Answer: Add text here. Don't delete this text first. Practice \(\PageIndex{1}\) For all natural numbers \(n\), leave \(A_n = \{n, n + 1, n + 2, n + 3\}\). Use the to specify all of the following sets: (a) \(\bigcap_{j = 1}^{3} A_j\) (b) \(\bigcup_{j = 1}^{3} A_j\) (c) \(\bigcap_{j = 3}^{7} A_j\) (d) \(\\j bigcup_{j = 3}^{7} A_j\) (e) \(A_9 \cap (\bigcup_{j = 3}^{7} A_j)\) (f) \(\bigcup_{j = 3}^{7} (A_9 \cap A_j)\) For all natural numbers \(n\) , leave \(A_n = \{k \in \mathbb{N}\ |\ k \ge n\}\). Use the roster method or the editor to specify each of the following presets: (a) \(\bigcap_{j = 1}^{5} A_j\) (b) \(\bigcap_{j = 1}^{5} A_j A_j)^c\) (c) \(bigcap_{j = 1}^{5} A_j ^c\) (d) \(\bigcup_{j = 1}{5} A_j ^c\c\) (e) \(\bigcup_{j = 1}^{5} A_j\) (f) \((\bigcup_{j = 1}^{5} A_j)^c\) (g) \(\bigcap_ {j \in \mathbb{N}^{} A_j\) (h) \(\bigcup_{j \in \mathbb{N}{{} A_j\) All positives are \(r\) real number, \(T_r\) is defined as the closed interval \([-r^2, r^2]\). This \[T_r = \{x \in \mathbb{R}\ |\ -r^2 \le x \le r^2\}.\] Be \(\wedges = \{m \in \mathbb{N}\ |\ 1 \le m \le 10\}\). To specify the following presets, use the interval markup or the editor markup: (a) \(\bigcup_\k \in \wedge}^{} T_k\) (b) \(\bigcap_{k \wedge}^. {} T_k\) (c) \(\bigcup_{r \in \mathbb{R}^}{%\T_k\) (d) \(d) \((d) \(d) \(d) \(\\) \bigcap_{r \in \mathbb{R}^+}^{T_k\) (e) \(\bigcup_{r \in \mathbb{N}}^{T_k\) (f) \(\bigcap_{r \in \mathbb{N}}^{T_k\) Prove Parts (2) and (4) of Theorem 5.30. \(\Lambda\) should not be an empty indexing set, and \(\mathcal{A} = \{A_{\alpha}\ \\ \alpha \in \Lambda\}) should be an indexed pool family. (a) All \(\beta \in \Lambda\), \(A_{\beta} \subseteq \bigcup_\\alpha \in \Lambda}^{} A_{\alpha}\). (b) \(\\bigcup_{\alpha \in \Lambda}^{} A_{\alpha})^c = \bigcap_{\alpha \in \Lambda}^{} A_{\alpha} ^c\) Prove Theorem 5.31. Make \(\Lambda\) a non-empty indexing set, make \(\mathcal{A} = \{A_{\alpha}\ |\ \alpha \in \Lambda\}\) an indexed pool family, and let \(B\) be a pool. Then (a) \(B \cap (\bigcup_\\alpha \in \Lambda}^{} A_{\alpha}) = \bigcup_{\alpha \in \Lambda}^{} (B \cap A_{\alpha})\, and (b) \(B \cup (\bigcap_\\alpha \in \Lambda}^{} A_{\alpha}) = \bigcap_\\alpha \in \Lambda}^{} (B \cup A_{\alpha})\). Make \(\Lambda\) and \(\Gamma\) non-empty indexing presets, and \(\mathcal{A} = \{A_{\alpha}\ |\ \alpha \\alpha \in \Lambda\}\) and \(\mathcal{B} = \{B_{\beta}\ |\ \beta \in Gamma\}\). Using the distribution laws of Practice 5: (a) Type \(((\bigcup_{\alpha \in \Lambda}^{} A_{{alpha}) \{alpha}) \cap (\bigcup_{\beta \in \Gamma}^{} B_{\beta}) as the intersection of two sets. (b) Type \((\bigcap_{\alpha \in \Lambda}^{} A_{\alpha}) \cup (\bigcap_{\beta \in \Gamma}^{} B_{\beta}) to merge the intersections of two sets. \(\Lambda\) should not be an empty indexing set, and \(\mathcal{A} = \{A_{\alpha}\ \\ \alpha \in \Lambda\}) should be an indexed pool family. Also, suppose \(\Gamma \subseteq \Lambda\) and \(\Gamma e (Note: \(\Gamma\) is the Great Greek gamma letter.) To prove it (a) \(\bigcup_\\alpha \in \Gamma}^{} A_{\alpha} \\alpha} \bigcup_\\alpha \in \Lambda}^{} A_{\alpha}\) (b) \(bigcap_{\alpha \in \Lambda}^{} A_{\alpha} \subsete q \bigcap_{\alpha \in \Gamma}^{} A_{\alpha}\) Be \(\Lambda\) is not an empty indexing set and let \(\mathcal{A} = \{A_{\alpha}\ |\ \alpha \in \Lambda\}\) be an indexed pool family. (a) Prove that \(B\) is a set that is \(B \subseteq A_{\alpha}\) all \(\alpha \in \Lambda\), and then \(B \subseteq \bigcap_\\alpha \in \Lambda}^{{A_{\alpha\\). (b) Prove that \(C\) is a set that is </a0> \(A_{\alpha} \subseteq C\) for each set of \(\alpha \in \Lambda\) values \(\bigcap_\\alpha \a\Lambda}^{} A_ {alpha} \subseteq C\) For all natural numbers \ (n\) leave \(A_n = \{x \in \mathbb{R}\ |n - 1 < x < n\}\). Prove that \(\{A_n\ |\ \in \mathbb{N}\}\) is an even set family, and that \(\bigcup_{n \in \mathbb{N}}^{%A_n = (\mathbb{R} ^{+} - \mathbb{N})\). For each natural number, \(n\) leave \(A_n = \{k \in \mathbb{N}\ |\ k \ge n\}\). Determine whether the following statements are true or false. Determine if the following statements are true or false. All conclusions must be justified. (a) For all \(j, k \in \mathbb{N}\), if \(j e k\), \(A_j \\cap A_k e \emptyset\). (b) \(\bigcap_{k \in \mathbb{N}}^{{ A_k = \emptyset\). Give an example of an indexed pool family \(\{A_n\ |\ \n \in \mathbb{N}\}\) all of the following conditions are true: (i) All \(m \in \mathbb{N}\), \(A_m \subseteq (0.1)\); (ii) All \(j, k \in \mathbb{N}\), if \(j e k\), and then \(A_j \\cap A_k e \emptyset\); and (iii) \(\bigcap_{k \in \mathbb{N}}^{} A_k = \emptyset\). Make \(\Lambda\) a non-empty indexing set, make \(\mathcal{A} = \{A_{\alpha}\ |\ \alpha \in \Lambda\}\) an indexed pool family, and let \(B\) be a pool. Use the results of Theorem 5.30 and Theorem 5.31 to prove all of the following: (a) \(\bigcup_{\alpha \in \Lambda}^{{} A_{\alpha}) - B = \bigcup_{\alpha \in \Lamb (A_{\alpha} - B)\) (b) \(\bigcap_{\alpha \in \Lambda}^{} A_{\alpha}) - B = \bigcap_{\alpha \in \Lambda}^{} (A_{\alpha} - B)\) (c) \(B - (\bigcup_{\alpha \in \Lambda}^{} A_{\alpha}) = \bigcap_{\alpha \in \Lambda}^{} B - (A_{\alpha})\) (d) \(B - ((\\bigcap_\\alpha \in \Lambda}^{} A_{\alpha}) = \bigcup_\\alpha \in \Lambda}^{} B - (A_{\alpha})\) An indexed family of subsets of the Descartes plane. Let \(\mathbb{R} ^{\ast}\) be a non-negative set of real numbers, and for all \(r \in \mathbb{R} ^{\ast}\), it should be \[\begin{array} {rcl} {C_r} {C_r} &{ {(x, y) \mathbb{R} \times \mathbb{R}\ |\x^2 + y^2 = r^2\}} \\ {D_r} &{={(x, y) \in \mathbb{R} \times \mathbb{R}\ |\ x^2 + y^2 \le\le r^2\}} \\ {T_r} & {\{(x, y) \mathbb{R} \times \mathbb{R}\ |\ x 2^+ y^+ 2 > r^2\} = D_r ^c.} \end{array}\] If \(r > 0\), \(C_r\) a circle with a radius of \(r\) centred on the 5.D_r 8.8. Shown in Figure 2. (a) Define \(\\bigcup_{r \in \mathbb{R} ^{\ast}}^{C_r\) and \(\bigcap_{r \in \mathbb{R} ^{ast bigcup_ C_r}}{/D_r\.) and \\\bigcap_\\r in \\mathbb{R{{ast}{{{} D_r\) (c) Define \(\bigcup_{r \in \mathbb{R} ^{\ast}}}{{{T_r \(\bigcap_{r \in \mathbb{R} ^{\ast}}^{/T_r\) (d) Let \\mathcal{C} = \{C_r\ |\ \\\\in \{bb R ^{} , \(\\ mathcal{D} = \{D_r\ |\ r \in \mathbb{R} ^\ast\}\) and \(\mathcal{T} = \{T_r\ |\ r \in \mathbb{R} ^{\ast}\}\). Are any of these indexed families stocks even? Explain. Now \(I\) should be the closed interval [0, 2] and \(J\) the closed interval [1, 2]. (e) Define \(\\bigcup_{r \in I}^{} C_r\), \(\bigcap_{r \in I}^{} C_r\), \(\bigcup_{r \in J}^{} C_r\), and \(\bigcap_{r \in J}^{{ C_r\) (f) Define \(\bigcup_ C_r\) {r \in I \}^{} D_r\), \(\bigcap_{r \in I}^{} D_r\), \(\bigcup_{r \in J}^{} D_r\), and \(\bigcap_{r \in J}^{} D_r\) (g) Define \((\bigcup_{r \in I }^{} D_r)^c\) \((/\bigcap_{r \in I}^{} D_r)^c\), \((\bigcup_{r \in J}^{} D_r)^c\), and \(((bigcap_{r \in J}^{} D_r)^c\) (h T_r bigcup_) (h) \(\bigcap_{r \in I}^{} T_r\), \(\bigcup_{r \in J}^{} T_r\), and \(\bigcap_{r \in J}^{} T_r\) (i) Use De Morgan's laws to explain the relationship between answers in Parts (13g) and (13h). Answer: Add text here. Don't delete this text first. First.