Probability class 12 ncert pdf

NcERT solutions to the questions in Chapter 13 The probability of class 12 NCERT textbooks is provided below to help students in their studies. Probability is the risk of an event occurrence. In this chapter, students learn about probability in detail. Solving all questions from this chapter of the NCERT book of class 12 math, repeatedly, will help students in understanding the problem solving method. Class 12 NCERT solutions in the Probability chapter are very easy to understand, as expert experts at BYJU'S ensure that solutions are given in the simplest form. These solutions cover all the training questions included in the book and are in line with the latest CBSE guidelines. Students can also avail of these NCERT solutions and download it for free to practice them offline as well. Download PDF of NCERT Solutions for Class 12 Math Chapter 13 - Probability Access All Exercises in Class 12 Math Chapter 12 Exercise 13.1 Solutions 17 Questions Exercise 13.2 Solutions 18 Question Exercise 13.3 Solutions 13.3 Solutions 114 Question Exercise 13.4 Solutions 17 Question Exercise 13.5 Solutions 15 Questions Miscellaneous Exercise On Chapter 13 Solutions 10 Question Access Answers to Mathematics NCERT Class 12 Chapter 12 Exercise 13.1 Page No. Since E and F are events that are of such a way that P (E) = 0.6, P (F) = 0.3 and P (E ∩ F) = 0.2, find P (E| F) and P (F| E) Solution: 2. Calculate P (A| B) if P (B) = 0.5 and P (A ∩ B) = 0.32 Resolution: 3. If P (A) = 0.8, P (B) = 0.5 and P (B| A) = 0,4, find (i) P (A ∩ B) (ii) P (A| B) (iii) P (A ∪ B) Solution: 4. Assess P (A ∪ B) if 2P (A) = P (B) = 5/13 and P (A| B) = 2/5. Solution: 5. If P (A) = 6/11, P (B) = 5/11 and P (A ∪ B) = 7/11, find (i) P (A∩B) (ii) P (A| B) (iii) P (B| A) Solution: Determine P (E| F) in Exercises 6 to 9. A coin is thrown three times, i) E : head of third, F : heads of the first two toes (ii) E : at least two heads, F : not more than two heads (iii) E : not more than two tails, F : at least one tail Solution: 7. Two coins are thrown once where (i) E: tail appears on a coin, F: a coin shows head (ii) E: no tail appears, F: no head appears Solution: (ii) Here, E: no tail appears And F: no head appears ⇒ E = {HH} and F ={TT} ⇒ E ∩ F = φ 8. A door is thrown three times, E: 4 appears on the third throw, F: 6 and 5 appear respectively on the first two throws. Solution: 9. Mother, father and son line up randomly for a family picture E: son at one end, F: father in the middle Solution: Let M denote mother, F denotes father and S denotes son. Then, the preview space for the given experiment will be: S = {MFS, SFM, FSM, MSF, SMF, FMS} Here, E: Son at one end and F: Father in the middle ⇒ E = {MFS, SFM, SMF, FMS} and F = {MFS, SFM} ⇒ E ∩ F = {MFS, SFM} 10. A black and a red cube is thrown. a) Find the conditional probability of obtaining 9 when the black door resulted in a 5. b) Find the conditional probability of obtaining the amount 8, as the red door resulted in a number less than 4. Solution: 11. A fair door is rolled. Consider events E = {1.3.5}, F = {2.3} and G = {2,3,4,5} Search (i) P (E| F) and P (F| E) (ii) P (E| G) and P (G| E) (iii) P ((E ∪ F)| G) and P ((E ∩ F)| G) Solution: (iii) Clear from (i), we have E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5} ⇒ E ∪ F = {1, 2, 3, 5} ⇒ (E ∪ F) ∩ G = {2, 3, 5} 12. Suppose that every child born is just as likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls, since (i) the youngest is a girl, (ii) at least one is a girl? Solution: Let B denote boy and G denot girl. Then the sample space in the given experiment is S = {GG, GB, BG, BB} Let E be the event that 'both are girls'. 13. An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice issues. If a question is chosen at random from the banking question, what is the likelihood that it will be an easy question as it is a multiple choice question? Solution: Here are two types of questions, True/False or Multiple Choice Questions (T/F or MCQ), and each of them is divided into Easy and Difficult Type, as shown below in the tree chart. 14. Since the two numbers appearing on the roll of two dice are different. Find the probability that the 'sum of numbers on the dice is 4' event. Solution: 15. Consider the experiment of throwing a door if a multiple of 3 comes up, tossing die again, and if another number comes, toss a coin. Find the conditional probability of the 'coin shows a tail' event because 'at least one ter shows a 3'. Solution: 16. If P (A) = 1/2, P (B) = 0, then P (A| B) A. 0 B. 1/2 C. is not defined D. 1 Solution: C. Undefined Explanation: 17. If A and B are events that are of such a way that P (A| B) = P (B| A), so A. A ⊂ B, but A ≠ B B. A = B C. A ∩ B = φ D. P (A) = P (B) Solution: D. P (A) = P (B) Explanation: Exercise 13.2 Page No: 546 1. If P (A) = 3/5 and P (B) = 1/5, find P (A ∩ B) if A and B are independent events. Solution: Given P (A) = 3/5 and P (B) = 1/5 As A and B are independent events. ⇒ P (A ∩ B) = P (A). P (B) = 3/5 × 1/5 = 3/25 2. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both cards are black. Solution: Given a package of 52 cards. As we know, there are a total of 26 cards that are black. Let A and B denote respectively the events that the first and second drawn cards are black. Now, P (A) = P (black card in first draw) = 26/52 = 1/2 Because the second card is drawn without replacement, so now the total number of black cards will be 25 and the total card will be 51, which is the conditional probability of B, as A has already taken place. Now P (B / A) = P (black card in second draw) = 25/51 Hence the probability that both cards are black ⇒ P (A ∩ B) = 1/2 × 25/51 = 25/102 Therefore the probability that both cards are black = 25/102. 3. A box of oranges is inspected by examining three randomly selected oranges that are pulled without replacement. If all three oranges are good, the box is approved for sale, otherwise it is rejected. Find the probability that a box containing 15 oranges, of which 12 are good and 3 are bad, will be approved for sale. Solution: Given a box of oranges. Let A, B and C denote respectively the events that the first, second and third drawn orange are good. Now P (A) = P (good orange in the first draw) = 12/15 Because the second orange is drawn without replacement, so now the total number of good oranges will be 11 and total oranges will be 14, which is the conditional probability of B, since A has already occurred. Now P (B / A) = P (good orange in the second draw) = 11/14 Because the third orange is drawn without replacement, so now the total number of good oranges will be 10 and total orangs will be 13, which is the conditional probability of C given that A and B have already occurred. Now, P (C / AB) = P (good orange in third draw) = 10/13 Thus the probability of, that all oranges are good ⇒ P (A ∩ B ∩ C) = 12/15 × 11/14 × 10/13 = 44/91 Therefore, the likelihood that a box will be approved for sale = 44/91 4. A fair currency and an impartial door have been twisted. Let A be the event 'head appears on the coin' and B be the event '3 on door'. Check whether A and B are independent events or not. Solution: Given a fair coin and an impartial door has been acidic. We know that test slot S: S = {(H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,1), (T.2), (T.3), (T.4), (T.5), (T,6)} Let A be the event head of the coin: ⇒ A = {(H), (T,6)} Let A be the event head of the coin: ⇒ A = {(H) 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)} ⇒ P (A) = 6/12 = 1/2 Now, Let B be the case 3 on door ⇒ B = {(H, 3), (T, 3)} ⇒ (B) = 2/12 = 1/6 As, A ∩ B = {(H, 3)} ⇒ P (A ∩ B) = 1/12...... (1) and P(A). P (B) = 1/2 × 1/6 = 1/12 ...... (2) From (1) and (2) P (A ∩ B) = P (A). P (B) A and B are therefore independent events. 5. A cube marked 1, 2, 3 in red and 4, 5, 6 in green is acidic. Let A be the event, 'the number is straight', and B be the event, 'the number is red'. Are A and B independent? Solution: The test space for the dice will be S = {1, 2, 3, 4, 5, 6} Let A be the event, the number is right: ⇒ A = {2, 4, 6} ⇒ P (A) = 3/6 = 1/2 Now, Let B be the case, the number is red: ⇒ B = {1, 2, 3} ⇒ P (B) = 3/6 = 1/2 As, A ∩ B = {2} ⇒ P (A ∩ B) = 1/6........ (1) and P(A). P (B) = 1/2 × 1/2 = 1/4 ..... (2) From (1) (2) P (A ∩ B) ≠ P (A). P (B) A and B are therefore not independent events. 6. Let E and F be events with P (E) = 3/5, P (F) = 3/10 and P (E ∩ F) = 1/5. Are E and F independent? Resolution: Given P (E) = 3/5, P (F) = 3/10 and P (E ∩ F) = 1/5 P (E). P (F) = 3/5 × 3/10 = 9/50 ≠ 1/5 ⇒ P (E ∩ F) ≠ P (E). P (F) Therefore, E and F are not independent events. 7. Since events A and B are of such a way that P (A) = 1/2, P (A ∪ B) = 3/5 and P (B) = s. Find p if they are (i) mutually exclusive (ii) independent. Solution: Given P (A) = 1/2, P (A ∪ B) = 1/5 and P (B) = p i) Mutually exclusive when A and B are mutually exclusive. Then (A ∩ B) = φ ⇒ P (A ∩ B) = 0 As we know, P (A ∪ B) = P (A) + P (B) – P (A ∩ B) ⇒ 3/5 = 1/2 + p -0 ⇒ P = 3/5 – 1/2 = 1/10 (ii) Independent when A and B are independent. ⇒ P (A ∩ B) = P (A). P (B) ⇒ P (A ∩ B) = 1/2 p As we know, P (A ∪ B) = P (A) + P (B) – P (A ∩ B) ⇒ 3/5 = 1/2 + 2 - p/2 ⇒ p/2 = 3/5 – 1/2 ⇒ p = 2 × 1/10 = 1/5 8. Let A and B be independent events with P (A) = 0.3 and P (B) = 0.4. Search in) P (A ∩ B) (ii) P (A ∪ B) (iii) P (A| B) (iv) P (B| A) Resolution: Given P (A) = 0,3 and P(B) = 0,4 i) P (A ∩ B) When A and B are independent. ⇒ P (A ∩ B) = P (A). P (B) ⇒ P (A ∩ B) = 0.3 × 0.4 ⇒ P (A ∩ B) = 0.12 (ii) P (A ∪ B) As we know, P (A ∪ B) = P (A) + P (B) – P (A ∩ B) ⇒ P (A ∪ B) = 0.3 + 0.4 – 0.12 ⇒ P (A ∪ B) = 0.58 (iv) P (B| A) 9. If A and B are two events so that P (A) = 1/4 , P (B) = 1/2 and P (A ∩ B) = 1/8, find P (not A and not B). Resolution: Given P (A) = 1/4, P (B) = 1/2 and P (A ∩ B) = 1/8 P (not A and not B) = P (A' ∩ B') As { A' ∩ B' = (A ∪ ∩ <5> B)'} ⇒ P (not A and not B) = P ((A ∪ B)') = 1 - P (A ∪ B) = 1- [P (A) + P (B) – P (A ∩ B)] 10. Events A and B are such that P (A) = 1/2, P (B) = 7/12 and P (not A or not B) = 1/4. Indicate whether A and B are independent? Resolution: Given P (A) = 1/2, P (B) =7/12 and P (not A or not B) = 1/4 ⇒ P (A'∪ B') = 1/4 ⇒ P (A ∩ B)' = 1/4 ⇒ P (A ∩ B)' = 1/4 ⇒ P (A ∩ B)' = 1/4 ⇒ P (A ∩ B)' = 1/4 ⇒ ⇒ 1 – P (A ∩ B) = 1/4 ⇒ P (A ∩ B) = 1 – 1/4 ⇒ P (A ∩ B) = 3/4........ 1) and P, point A). P (B) = 1/2 × 7/12 = 7/24 .... (2) From (1) and (2) P (A ∩ B) ≠ P (A). P (B) A and B are therefore not independent events. 11. In view of two independent events A and B, so that P (A) = 0,3, P(B) = 0,6. Find i) P (A and B) (ii) P (A and not B) (iii) P (A or B) (iv) P (neither A nor B) Solution: Given P (A) = 0.3, P (B) = 0.6. i) P (A and B) As A and B are independent events. ⇒ P (A and B) = P (A ∩ B) = P (A). P (B) = 0.3 × 0.6 = 0.18 (ii) P (A and not B) ⇒ P (A and not B) = P (A ∩ B') = P (A)) – P (A ∩ B) = 0.3 – 0.18 = 0,12 (iii) P (A or B) ⇒ P (A or B) = P (A ∪ B) As know, P (A ∪ B) = P (A) + P (B) – P (A ∩ B) ⇒ P (A ∪ B) = 0.3 + 0.6 – 0.6 – 0.6 – 0.6 0.18 ⇒ P (A ∪ B) = 0.72 (iv) P (neither A nor B) P (neither A nor B) = P (A' ∩ B') As { A' ∩ B' = (A ∪ B)'} ⇒ P (neither A nor B) = P ((A ∪ B)') = 1 – P (A ∪ B) = 1 – 0.72 = 0,28 12. A door is tossed three times. Find the probability of getting an odd number at least once. Solution: Given a door is tossed three times. Then test space S = {1, 2, 3, 4, 5, 6} Let P (A) = probability of getting an odd number in the first roll. ⇒ P (A) = 3/6 = 1/2. Let P (B) = the probability of getting an even number. ⇒ P (B) =3/6 = 1/2. Now the probability of getting an even number in three times = 1/2 × 1/2 × 1/2 = 1/8 So, the probability of getting an odd number at least once = 1 – the probability of getting an odd number without a roll = 1 – the probability of getting an even number in three times = 1 – 1/8 ∴ Probability of getting an odd number at least once = 7/8 13. Two balls are drawn randomly with replacement from a box containing 10 black and 8 red balls. Find the probability that (i) both balls are red. (ii) The first ball is black and the second is red. (iii) One of them is black and another is red. Solution: Given a box of 10 black and 8 red balls. Total number of balls in box = 18 (i) Both balls are red. Likelihood of getting a red ball in the first draw = 8/18 = 4/9 Since the ball is replaced after the first throw, Therefore, likely to get a red ball in the second draw = 8/18 = 4/9 Now the probability of getting both balls red = 4/9 × 4/9 = 16/81 (ii) First black ball is and second is red. Probability of getting a black ball in the first draw = 10/18 = 5/9 When the ball is replaced after the first throw, Therefore, the probability of getting a red ball in the second draw = 8/18 = 4/9 Now the probability of getting the first ball is black and the second is red = 5/9 × 5/9 = 20/81 (iii) One of them is black and the other is red. Probability of getting a black ball in the first draw = 10/18 = 5/9 When the ball is replaced after the first throw, Therefore, the probability of getting a red ball in the second draw = 8/18 = 4/9 Now the probability of getting the first ball is black and secondly is black = 5/9 × 4/9 = 20/81 The probability of getting a red ball in the first draw = 8/18 = 4/9 When the ball is replaced after the first throw, Probability of getting a black ball in the second draw = 10/18 = 5/9 Now the probability of getting the first ball is red and the second is black = 5/9 × 4/9 = 20/81 Therefore, the probability of getting one of them is black and others are red: = The probability of getting first ball is black and the second is red + The probability of getting first ball is red and the second is black = 20/81 + 20/81 = 40/81 14. The probability of resolving specific problems independently of A and B is 1/2 and 1/3 respectively. If both try to resolve the problem independently, find for that (i) the problem is resolved (ii) Exactly of them solve the problem. Solution: Given, P (A) = Probability of solving the problem of A = 1/2 P (B) = Probability of solving the problem with B = 1/3 Because A and B are both independent. ⇒ P (A ∩ B) = P (A). P (B) ⇒ P (A ∩ B) = 1/2 × 1/3 = 1/6 P (A') = 1 - P (A) = 1 - 1/2 = 1/1/1/1 2 P (B') = 1 – P (B) = 1 – 1/3 = 2/3 (i) The problem is solved it is either solved by A or it is solved by B. = P (A ∪ B) As we know P (A ∪ B) = P (A) + P (B) – P (A ∩ B) ⇒ P (A ∪ B) = 1/2 + 1/1/13 - 1 / 6 = 4 / 6 ⇒ P (A ∪ B) = 2 / 3 (ii) Exactly one of them solves the problem It is either the problem is solved by A, but not by B or vice versa It is P (A). P B') + P (A'). P (B) = 1/2 (2/3) + 1/2 (1/3) = 1/3 + 1/6 = 3/6 ⇒ P (A). P B') + P (A'). P (B) = 1/2 15. A card is drawn randomly from a well mixed deck with 52 cards. In which of the following cases are events E and F independent? (i) E: 'the drawn card is a spade' F: 'The drawn card is an ace' (ii) E: 'the drawn card is black' F: 'the drawn card is a king or a queen' F: 'The drawn card is a queen or a jack'. Solution: Given: A set of 52 cards. (i) In a game with 52 cards, 13 cards are spade, and 4 cards are ace, and there is only one card, which is spade and ace both. Therefore P (E) = the map drawn is a spade = 13/52 = 1/4 P (F) = the map drawn is an ace = 4/52 = 1/13 P (E ∩ F) = the card drawn is a spade and ace both = 1/52 ..... 1) and P(E). P (F) = 1/4 × 1/13 = 1/52.... (2) From (1) and 2) ⇒ P (E ∩ F) = P (E). P (F) Therefore, E and F are independent events. (ii) In a set of 52 cards, 26 cards are black, and 4 cards are king, and only two cards are there, which are black and king both. Therefore P (E) = the card drawn is of black = 26/52 = 1/2 P (F) = the card drawn is a king = 4/52 = 1/13 P (E ∩ F) = the card drawn is a black and king both = 2/52 = 1/26.... 1) and P(E). P (F) = 1/2 × 1/13 = 1/26.... (2) From (1) and 2) ⇒ P (E ∩ F) = P (E). P (F) Therefore, E and F are independent events. (iii) In a game with 52 cards is 4 cards queen, 4 cards are king and 4 cards are jack. Therefore P (E) = the card drawn is either king or queen = 8/52 = 2/13 P (F) = the card drawn is either queen or jack = 8/52 = 2/13 There are 4 cards which are either king or queen and either queen or jack. P (E ∩ F) = the drawn card is either king or queen and either queen or jack = 4/52 = 1/13 ... 1) and P(E). P (F) = 2/13 × 2/13 = 4/169.... (2) From (1) and 2) ⇒ P (E ∩ F) ≠ P (E). P (F) Therefore, E and F are not independent events. 16. In a hostel, 60% of students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random. (a) Find the probability that she will read neither Hindi nor English newspapers. (b) she reads Hindi newspaper, find the likelihood that she reads English newspaper. (c) If she reads English newspaper, find the probability that she read Hindi newspaper Solution: Given: Let H and E denote the number of students reading Hindi and English newspaper respectively. reading Hindi newspaper = 60/100 = 3/5 P (E) = Probability of students reading English newspaper = 40/100 = 2/100 5 P (H ∩ E) = Probability of students reading neither Hindi and English newspaper = 20/100 = 1/5 (a) Find the likelihood that she reads neither Hindi nor English newspapers. P (neither H nor E) P (neither H nor E) = P (H' ∩ E') As { H' ∩ E' = (H ∪ E)'} ⇒ P (neither A or B) = P ((H ∪ E)') = 1 – P (H ∪ E) = 1- [P (H) + P (E) – P (H ∩ E)] 17. The probability of achieving an even prime number on each cube when a pair of dice is tossed is A. 0 B. 1/3 C. 1/12 D. 1/36 Solution: D. 1/36 Explanation: Given a few dice tossed. Therefore, the number of results = 36 Let P (E) be the probability of getting an even prime on each door. As we know, the only even prime number 2. So E = {2, 2} ⇒ P € = 1/36 18. Two events A and B will be independent, if (A) A and B mutually exclusive (B) P (A'B') = [1 - P (A)] [1 – P (B)] (C) P (A) = P (B) (D) P (A) + P (B) = 1 Solution: (B) P (A'B'B).) = [1 - P (A)] [1 - P (B)] Explanation: P (A'B') = [1 - P (A)] [1 - P (B)] ⇒ P (A'∩ B') = 1 – P (A) – P (P) (P) B) + P (A) P (B) ⇒ 1 - P (A ∪ B) =1 - P (A) – P (B) + P (A) P (B) = [P (A) + P (B) – P (A ∩ B)] = - P (P (P) (A) – P (B) + P (A) P (B) = – P (A) – P (B) + P (A ∩ B) = - P (A) – P (B) + P (A) P (B) ⇒ P (A ∩ B) = P (A). P (B) Therefore, it shows A and B are independent events. Exercise 13.3 Page No: 555 1. An ant contains 5 red and 5 black balls. A ball is drawn at random, its color is noted and returned to the urn. In addition, 2 extra balls of the drawn color are put in the urn and then a ball is drawn randomly. What is the probability that the second ball is red? Solution: Given the owls contain 5 red and 5 black balls. Let at the first attempt the ball drawn is of red color. ⇒ P (probability of drawing a red ball) = 5/10 = 1/2 Now the two balls of the same color (red) are added to the urn, then the urn contains 7 red and 5 black balls. ⇒ P (probability of drawing a red ball) = 7/12 Now let the first attempt ball drawn is of black color. ⇒ P (probability of drawing a black ball) = 5/10 = 1/2 Now the two balls of the same color (black) are added to the urn, where the urn contains 5 red and 7 black balls. ⇒ P (probability of drawing a red ball) = 5/12 Therefore, the probability of drawing the second ball with red color: 2. One bag contains 4 red and 4 black balls, another bag contains 2 reds and 6 Balls. One of the two bags is selected at random, and a ball is pulled from the bag, which turns out to be red. Find the probability that the ball has been pulled from the first bag. Solution: Let E1 be the case to choose the bag I, E2 be the case to choose the bag say bag II and A be case of drawing a red ball. Then P (E1) = P (E2) = 1/2 Also P (A| E1) = P (drawing a red ball from bag I) = 4/8 = 1/2 and P (A| E2) = P (drawing a red ball from bag II) = 2/8 = 1/4 Now the probability of pulling a ball from bag I, as it is red, is P (E1| A). By using Bayes' phrase, we have: 3. Of the students at a college, it is known that 60% live in the hostel and 40% are day researchers (do not live in hostel). Last year results show that 30% of all students living in the hostel reach A class and 20% of day researchers reach A class in their annual exam. At the end of the year, a student is chosen at random from college and he has an A-grade, what is the likelihood that the student is a hostlier? Solution: 4. When answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability 1/4. What is the probability that the student knows the answer when he answered correctly? Solution: Let E1 be the event that the student knows the answer, E2 be the event that the student guess the answer and A be the event that the answer is correct. Then P (E1) = 3/4 and P (E2) = 1/4 Also P (A| E1) = P (correct answer as he knows) = 1 And P (A| E2) = P (correct answer as he guesses) = 1/4 Now the probability that he knows the answer, as the answer is correct, is P (E1| A). By using Bayes' phrase, we have: 5. A laboratory blood test is 99% effective at detecting a particular disease, when it is, in fact, present. But the test also gives a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then with probability 0.005, the test will imply that he has the disease). If 0.1 percent of the population actually has the disease, what is the likelihood that a person has the disease given that his test result is positive? Solution: Let E1 be the event that the person has a disease, E2 be the case that the person does not have a disease, and A be the case that the blood test is positive. Like E1 and E2, the events are free for each other. Then P (E1) + P (E2) = 1 ⇒ P (E2) = 1 – P (E1) Then P (E1) = 0.1% = 0.1/100 = 0.001 and P (E2) = 1 – 0.001 = 0.999 Also P (A| E1) = P (result is positive as person has disease) = 99% = 0.99 and P (A| E2) = P (the result is positive as the person has no disease) = 0.5% = 0.005 Now the probability of that person an illness, give that his his the result is positive, is P (E1| A). By using Bayes' phrase, we have 6. There are three coins. One is a two-headed coin (with head on both faces), another is a biased coin that comes up heads 75% of the time and third is an impartial coin. One of the three coins is chosen at random and cut, it shows heads, what is the probability that it was the two heading coin? Solution: Let E1 be the case to choose a two-headed coin, E2 be the case to choose a biased coin and E3 be the case to choose an impartial coin. Then P (E1) = P (E2) = P (E3) = 1/3 Since we have a headed coin has its head on both sides, so it will show the head. Also P (A| E1) = P (correct answer as he knows) = 1 And P (A| E2) = P (coin shows head as coin is biased) = 75% = 75/100 = 3/4 And P (A| E3) = P (coin shows head as the coin is impartial) = 1/2 Now the probability that the coin is two headings, as it shows head, is P (E1| A). By using Bayes' phrase, we have 7. An insurance company insured 2000 scooter drivers, 4000 drivers and 6000 lorry drivers. The probability of an accident is 0.01, 0.03 and 0.15, respectively. One of the insured encounters an accident. What is the likelihood that he is a scooter driver? Solution: Let E1 be the event that the driver is a scooter driver, E2 be the case that the driver is a motorist and E3 be the event that the driver is a truck driver. Let A be the event that the person meets with an accident. Total number of drivers = 2000 + 4000 + 6000 = 12000 Then P (E1) = 2000/12000 = 1/6 P (E2) = 4000/12000 = 1/3 P (E3) = 6000/12000 = 1/2 Since we have a headed coin head on both sides, so it will show the head. Also P (A| E1) = P (accident of a scooter driver) = 0.01 = 1/100 And P (A| E2) = P (accident of a motorist) = 0,03 = 3/100 And P (A| E3) = P (accident of a truck driver) = 0.15 = 15/100 = 3/20 Now the probability that the driver is a scooter driver when he met with an accident is P (E1| A). By using Bayes' phrase, we have 8. A factory has two machines A and B. Previous record shows that machine A produced 60% of the elements of output and machine B produced 40% of the goods. In addition, 2% of the goods produced by machine A and 1% produced by machine B were defective. All items are put in one warehouse and then an item is selected at random from this and is found to be defective. What is the probability that it was produced by machine B? Solution: Let E1 be the event that the product is produced by A, E2 be the event that the item is produced by B and X be the event produced product turns out to be defective. Then P (E1) = 60% = 60/100 = 3/5 P (E1) = 40% = 40/100 = 2/5 Also P (X| E1) = P (item is defective, since it is produced by machine A) = 2% = 2/100 = 1/50 And P (X| E2) = P (item is defective, it is produced by machine B) = 1% = 1/100 Now Now the probability that the product has been produced by B because the product concerned is defective is P (E2| A). By using Bayes' phrase, we have 9. Two groups compete for the position on the board of directors of a company. The probability of the first and second groups winning is 0.6 and 0.4, respectively. In addition, if the first group wins, the probability of introducing a new product is 0.7, and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product that was introduced was by the other group. Solution: Let E1 be the case that the first group wins the competition, E2 be the case that the second group wins the competition and A be the case to introduce a new product. Then P (E1) = 0.6 and P (E2) = 0.4 Also P (A| E1) = P (introduction of a new product as first group wins) = 0.7 and P (A| E2) = P (introduction of a new product as second group wins) = 0.3 Now the probability that the new product was introduced is by the second group, when a new product was introduced is P (E2| A). By using Bayes' phrase, we have P (E2| A) = 2/9 10. Suppose a girl dies. If she gets a 5 or 6, she sres a coin three times and notices the number of heads. If she gets 1, 2, 3 or 4, she sews a coin once and notices whether a head or tail is achieved. If she got exactly a head, what is the probability that she threw 1, 2, 3 or 4 with the door? Solution: let E1 be the event that the result on the door is 5 or 6, E2 be case that the result on the door is 1, 2, 3 or 4 and A be the event to get exactly the head. So P (E1) = 2/6 = 1/3 P (E2) = 4/6 = 2/3 As by tossing a coin three times we get 8 options. {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT} ⇒ P (A| E1) = P (achieving exactly one head by tossing the coin three times if she gets 5 or 6) = 3/8 And P (A| E2) = P (achieve exactly a head by tossing the coin three times if she gets 1, 2, 3 or 4) = 1/ 2 Now the probability that the girl threw 1, 2, 3 or 4 with a door when she achieved exactly one head is P (E2| A). By using Bayes' phrase, we have P (E2| A) = 8/11 11. One manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, while the other two operators B and C produce 5% and 7% defective items respectively. A is at work for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A? Solution: Let E1 be the case of time consumed by machine A, E2 be case of time consumed by machine B and E3 be cases of time consumed by machine C. Let X be the case with the manufacture of defective items. Then P (E1) = 50% = 50/100 = 1/2 P (E2) = 30% = 30/100 = 3/10 P (E3) = 20% = 20/100 = 1/5 Since we a headed coin has its head on both sides, so it will show Also P (X| E1) = P (defective (defective produced by A) = 1% = 1/100 And P (X| E2) = P (defective product produced by B) = 5% = 5/100 And P (X| E3) = P (defective product produced by C) = 7% = 7/100 Now the probability that the product produced by machine A, as defective goods are produced, is P (E1| A). By using Bayes' phrase, we have P (E1| A) = 5/34 12. A card from a pack of 52 cards is lost. From the remaining cards in the package, two cards are drawn and found to be both diamonds. Find the probability that the lost card is a diamond. Solution: Let E1 be the case that the drawn card is a diamond, E2 be the case that the drawn card is not a diamond and A be the case that the card is lost. As we know, out of 52 cards, 13 cards are diamond and 39 cards are not diamond. So P (E1) = 13/52 and P (E2) = 39/52 Now when a diamond card is lost then there are 12 diamond cards out of a total of 51 cards. Two diamond cards can be pulled out of 12 diamond cards in 12C2 ways. Similarly, two diamond cards can be pulled out of a total of 51 cards in 51C2 ways. So the probability of getting two cards when a diamond card is lost is P (A| E1). Also P (A| E1) = 12C2 / 51C2 Now, when not a diamond card is lost then there are 13 diamond cards out of a total of 51 cards. Two diamond cards can be pulled out of 13 diamond cards in 13C2 ways. Similarly, two diamond cards can be pulled out of a total of 51 cards in 51C2 ways. So the probability of getting two cards when the card is lost, which is not diamond, is P (A| E2). Also P (A| E2) =13C2 / 51C2 13. The probability that A is telling the truth is 4/5. A coin is acidic. A report that a header is displayed. The probability that there was actually head is A. 4/5 B. 1/2 C. 1/5 D. 2/5 Solution: A. 4/5 Explanation: Let E1 be the event A is telling the truth, E2 be the event that A lies and X be the event that it looks head. Therefore, P (E1) = 4/5 As E1 and E2 are the events that are free for each other. Then P (E1) + P (E2) = 1 ⇒ P (E2) = 1 – P (E1) ⇒ P (E2) = 1 – 4/5 = 1/5 If a coin is thrown, it may show the head or tail. Therefore, the probability of getting the head is 1/2, whether A speaks a truth or A lies. P (X| E1) = P (X| E2) = 1/2 Now the probability that there was actually head, give that A speaker a truth is P (E1| X) By using Bayes' phrase, we have 14. If A and B are two events so that A ⊂ B and P(B) ≠ 0, which of the following is correct? A. P (A| B) = P (B)/ P (A) B. P(A| B) < P(A) C.P(A| B) ≥ P(A) D. None of these solutions: C. P (A| B) ≥ P (A) Explanation: Exercise 13,4 Page No: 569 1. indicate which of the following are not the probability distributions for a random variable. Give a justification for your answer. Solution: Here we have table with values for X and P(X). As we know the sum of all probabilities in a probability distribution of a random variable must be one. Therefore, the sum of probabilities of given table = 0.4 + 0.4 + 0.2 = 1 Therefore the given table probability distributions for a random variable. Solution: Here we have table with values for X and P(X). As we see from the table, P(X) = -0.1 for X = 3. It is known that the probability of any observation must always be positive, that it cannot be negative. Therefore, the specified table is not the probability distributions for a random variable. Solution: Here we have table with values for X and P(X). As we know the sum of all probabilities in a probability distribution of a random variable must be one. Therefore, the sum of probabilities for given table = 0.6 + 0.1 + 0.2 = 0.9 ≠ 1 Therefore, the given table is not the probability distributions for a random variable. Solution: Here we have table with values for X and P(X). As we know the sum of all probabilities in a probability distribution of a random variable must be one. Therefore, the sum of probabilities for given table = 0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 ≠ 1 Therefore, the given table is not the probability distributions for a random variable. 2. An ant contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Solution: Given the nod containing 5 red and 2 black balls. Let R represent red ball and B represents black ball. Two balls are drawn at random. Therefore, the sample space of the experiment S = {BB, BR, RB, RR} X represents the number of black balls. ⇒ X (BB) = 2 X (BR) = 1 X (RB) = 1 X (RR) = 0 Therefore, X is a test range function whose range is {0, 1, 2}. Thus, X is a random variable that can take the values 0, 1 or 2. 3. Let X represent the difference between the number of heads and the number of tails obtained when a coin is played 6 times. What are possible values of X? Solution: Given a coin is tossed 6 times. X represents the difference between the number of heads and the number of tails. ⇒ X (6H, 0T) = |6-0| = 6 X (5H, 1T) = |5-1| = 4 X (4H, 2T) = |4-2| = 2 X (3H, 3T) = |3-3| = 0 X (2H, 4T) = |2-4| = 2 X (1H, 5T) = |1-5| = 4 X (0H, 6T) = |0-6| = 6 Therefore, X is a test range function whose range is {0, 2, 4, 6}. Thus, X is a random variable that can take the values 0, 2, 4 or 6. 4. Find the probability distribution of (i) number of heads in two toss of a coin. Therefore, the sample space of the experiment S = {HH, HT, TH, TT} X represents the number of heads. ⇒ X (HH) = 2 X (HT) = 1 X (TH) = 1 X (TT) = 0 Therefore, X is a test range function whose range is {0, 1, 2}. Thus, X is a random variable that can take the values 0, 1 or 2. As we know, P (HH) = P (HT) = P (TH) = P (TT) = 1/4 P (X = 0) = P (TT) = 1/4 P (X = 1) = P (HH) + P (TH) = 1/4 + 1/4 = 1/2 P (X = 2) = P (HH) = 1/4, Therefore, the required probability distribution is X 0 1 2 P (X) 1/4 1/2 1/4 (ii) Number of tails in the simultaneous crass of three coins. Given three coins are slaunched simultaneously. Therefore, the trial space of the experiment S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT} X represents the number of tails. As we see, X is a function in the preview area whose range is {0, 1, 2, 3}. Thus, X is a random variable that can take the values 0, 1, 2 or 3. P (X = 0) = P (HHH) = 1/8 P (X = 1) = P (HHT) + P (HTH) + P (THH) = 1/8 + 1/8 + 1/8 = 3/8 P (X = 1 2) = P (HTT) + P (THT) + P (TTH) = 1/8 + 1/8 + 1/8 = 3/8 P (X = 3) = P (TTT) = 1/8 Therefore, the required probability distribution X 0 1 2 3 P (X) 1/8 3/8 3/8 1/8 (iii) Number of heads in four crass of a coin. Therefore, the trial room of the experiment S = {HHHH, HHHT, HHTH, HTHH, HTHTH, HTHT, HHTT, HTTT, THHH, TTHH, THTH, THHT, THTT, TTHT, TTTH, TTTT} X represents the number of heads. As we see, X is a function in the preview area whose range is {0, 1, 2, 3, 4}. Thus, X is a random variable that can take the values 0, 1, 2, 3 or 4. P (X = 0) = P (TTTT) = 1/16 P (X = 1) = P (HTTT) + P (TTTH) + P (THTT) + P (TTHT) = 1/16 + 1/16 + 1/16 + 1/16 = 1/4 P(X = 2) = P (HHTT) + P (THHT) + P (TTHH) + P (THTH) + P (HTHT) + P(HTTH)= 1/16 + 1/16 + 1/16 + 1/16 + 11/16 + 1/16 = 6/16 = 3/8 P(X = 3) = P (THHH) + P (HHHT) + P (HTHH) + P (HHTH) = 1/16 + 1/16 + 1/16 + 1/16 16 = 1/4 P(X = 4) = P (HHHH) = 1/16 Therefore, the required probability distribution, X 0 1 2 3 4 P (X) 1/16 1/4 3/8 1/4 1/16 5. Find the probability distribution of the number of successes in two crass of a door, where a success is defined as (in) numbers greater than 4 (ii) six appear on at least one door Solution: Since a door is twice. When a cube is twice the number of observations will be (6 × 6) = 36. Now let X be a random variable that represents success. (i) Here success is given as the number greater than 4. Now P (X = 0) = P (number ≤ 4 in both eyes) = 4/6 × 4/6 = 4/9 P (X = 1) = P (number ≤ 4 in first tree ≥ 4 in second case) + P (number ≥ 4 in first case and number ≤ 4 in second case) is = (4/6 × 2/6) + (2/6 × 4/6) = 4/9 P (X = 2) = P (number ≥ 4 in both sexes) = 2/6 × 2/6 = 1/9 Therefore, the required probability distribution X 0 1 2 P (X) is 4/9 4/9 1/9 (ii) Here is success as six are shown on at least one door. Now P (X = 0) = P (six does not appear on any of die) = 5/6 × 5/6 = 25/36 P (X = 1) = P (six appears at least once of the cube) = (1/6 × 5/6) + (5/6 × 1/6) = 10/36 = 5/18 Therefore, the required probability distribution is, 6. From a lot of 30 bulbs, which include 6 defects, a sample of 4 bulbs is drawn randomly with replacement. Find the probability distribution of the number of defective bulbs. Solution: Given a lot of 30 bulbs, which include 6 defects. Then subtract the number of non-defective bulbs = 30 – 6 = 24 Since 4 bulbs are drawn randomly with replacement. Let X denote defective bulbs from the selected bulbs. Clearly, X can take the value of 0, 1, 2, 3 or 4. P (X = 0) = P (4 are non-defective and 0 defective) P (X = 1) = P (3 is non-defective and 1 defective) P (X = 2) = P (2 are defective and 2 defective) P (X = 3) = P (1 is non-defective and 3 defective and 3 defective) P (X = 4) = P (0 is non-defective and defective) the required probability distribution is X 0 1 2 3 4 P (X) 256/625 256/625 96/625 16/625 1/625 7. A coin is biased, so the head is 3 times as likely to occur as tail. If the coin is twice, find the probability distribution of the number of tails. Solution: Given head is 3 times as likely to occur as tail. Now let the probability of getting a tail in the biased coin be x. ⇒ P (T) = x And P (H) = 3x For a biased coin, P (T) + P (H) = 1 ⇒ x + 3x = 1 ⇒ 4x = 1 ⇒ x = 1/4 Therefore P (T) = 1/4 and P (H) = 3/4 Since the coin is twice, so the test site is {HH, HT, TH, TT} Let be a random variable representing the number of tails. Obviously, X can take the value of 0, 1 or 2. P(X = 0) = P (no tail) = P (H) × P (H) = 3/4 × 3/4 = 9/16 P(X = 1) = P (one tail) = P (HT) × P (TH) = 3/4. 1/4 × 1/4. 3/4 = 3/8 P(X = 2) = P (two tails) = P (T) × P (T) = 1/4 × 1/4 = 1/16 Therefore, the required probability distribution X 0 1 2 P (x) 9/16 3/8 1/16 8. A random variable X has the following probability distribution: Determine (i) k (ii) P (X < 3) (iii) P (X > 6) (iv) P (0 < X < 3) Solution: Given a random variable X with probability distribution. (i) As we know the sum of all probabilities in a probability distribution of a random variable must be one. Therefore, the sum of probabilities for given table: ⇒ 0 + k + 2k + 2k + 3k + k2 + 2k2 + 7K2 + k = 1 ⇒ 10K2 + 9k = 1 ⇒ 10K2 + 9k – 1 = 0 ⇒ (10K-1) (k + 1) = 0 k = -1, 1/10 It is known, the likelihood of any observation must always be positive, that it cannot be negative. Så k = 1/10 (ii) Nu skal vi finde P(X < 3) P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = 0 + k + 2k = 3k P (X < 3) = 3 × 1/10 = 3/10 (iii) Nu har vi finde P(X > 6) P(X > 6) = P(X = 7) = 7K2 + k = 7 × (1/10)2 + 1/10 = 7/100 + 1/10 P (X > 6) = 17/100 (iv) Overvej P (0 < X < 3) P (0 < < 0 < < < 0 < P (0 < < < p (0 < <1> <3> X < 3) = P(X = 1) + P(X = 2) = k + 2k = 3k P (0 < X < 3) = 3 × 1/10 = 3/10 9. The random variable X has probability distribution P(X) in the following form, with k being a display number: (a) Determine the value of k. (b) Find P (X < 2), P (X ≤ 2), P(X ≥ 2). Solution: Given: A random variable X with its probability distribution. (a) As we know the sum of all probabilities in a probability distribution of a random variable must be one. Therefore, the sum of probabilities for given table: ⇒ k + 2k + 3k + 0 = 1 ⇒ 6k = 1 k = 1/6 (b) Now we must find P(X < 2) P (X < 2) = P (X = 0) + P (X = 1) k + 2k = 3k P (X < 2) = 3 × 1/6 = 1/2 Overvej P (X ≤ 2) P (X ≤ 2) = P (X (X(X) 0) + P (X = 1) + P (X = 2) = k + 2k + 3k = 6k P (X ≤ 2) = 6 × 1/6 = 1 Now we must find P(X ≥ 2) P (X ≥ 2) = P(X = 2) + P(X> 2) = 3k + 0 = 3k P (X ≥ 2) = 3 × 1/6 = 1/2 10. Find the average number of heads in three tosses of a fair coin. Three coins are launched simultaneously. Therefore, the trial space of the experiment S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT} X represents the number of heads. As we see, X is a function in the preview area whose range is {0, 1, 2, 3}. Thus, X is a random variable that can take the values 0, 1, 2 or 3. P (X = 0) = P (TTT) = 1/8 P (X = 1) = P (TTH) + P (THT) + P (HTT) = 1/8 +1/8 + 1/8 = 3/8 P (X = 1 2) = P (THH) + P (HTH) + P (HHT) = 1/8 + 1/8 + 1/8 = 3/8 P (X = 3) = P (HHH) = 1/8 Therefore, the required probability distribution X is 0 1 2 3 P (X) 1/8 3/8 3/8 1/8 Therefore, μ is: 11. Two dice are thrown simultaneously. If X indicates the number of sixes, you must find the expectation of X. Solution: When a door is thrown twice. When a cube is twice the number of observations will be (6 × 6) = 36. Now, let X is a random variable that represents success and is given as six appear on at least one door. Now P (X = 0) = P (six does not appear on any of the die) = 5/6 × 5/6 = 25/36 P (X = 1) = P (six appear at least once of the cube) = (1/6 × 5/6) + (5/6 × 1/6) = 10/10/10 36 = 5/18 P (X = 2) = P (six appears on both doors) = 1/6 × 1/6 = 1/36 Therefore, the required probability distribution, X 0 1 2 P (X) 25/36 5/18 1/36 Therefore, expectation of X E (X): 12. Two numbers are randomly selected (without replacement) from the first six positive integers. Let X denote the largest of the two figures obtained. Find E(X). Solution: Given first six positive integers. Two numbers can be selected at random (without replacement) from the first six positive integers in 6 × 5 = 30 ways. X indicates the largest of the two numbers obtained. Therefore, X can take any value of 2, 3, 4, 5 or 6. For X = 2, the possible observations (1, 2) and (2, 1) P (X) = 2/30 = 1/15 For X = 3 are the possible observations (1, 3), (3, 1), (2, 3) and (3,2). P (X) = 4/30 = 2/15 For X = 4 are the possible observations (1, 4), (4, 1), (2,4), (4.2), (3,4) and (4.3). P (X) = 6/30 = 1/5 For X = 5 are the possible observations (1, 5), (5, 1), (2.5), (5.2), (3.5), (5.3) (5, 4) and (4.5). P (X) = 8/30 = 4/15 For X = 6 are the possible observations (1, 6), (6, 1), (2,6), (6.2), (3.6), (6.3) (6,4), (4.6), (5.6) and (6.5). P (X) = 10/30 = 1/3 Therefore, the required probability distribution X 2 3 4 5 6 P (X) 1/15 2/15 1/5 4/15 is 1/3 Therefore, E(X): 13. Let X indicate the sum of the numbers obtained when two reasonable dice are thrown. Find the variance and standard deviation for X. Solution: When two reasonable dice are thrown when two reasonable dice are thrown, the number of observations will be 6 × 6 = 36. X indicates the sum of the numbers obtained when two reasonable dice are thrown. Therefore, X can take any value of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12. For X = 2 are the possible observations (1, 1). P (X) = 1/36 For X = 3 are the possible observations (1, 2) and (2, 1) P (X) = 2/36 = 1/18 For X = 4 are the possible observations (1,3), (2, 2) and (3,1). P (X) = 3/36 = 1/12 for X = 5 the possible observations are (1, 4), (4, 1), (2, 3) and (3, 2) P (X) = 4/39 = 1/9 For X = 6, the possible observations are (1.5), (5, 1), (2, 4), (4, 2) and (3,3). P (X) = 5/36 For X = 7 are the possible observations (1, 6), (6, 1), (2.5), (5.2),(3,4) and (4.3). P (X) = 6/36 = 1/6 For X = 8 are the possible observations (2, 6), (6, 2), (3, 5), (5, 3) and (4,4). P (X) = 5/36 For X = 9 are the possible observations (5,4), (4, 5), (3, 6) and (6.3) P (X) = 4/36 = 1/9 For X = 10, the possible observations are (5.5), (4.6) and (6,4). P (X) = 3/36 = 1/12 For X = 11 are the possible observations (6, 5) and (5.6) P (X) = 2/36 = 1/18 For X = 12 are the possible observations (6, 6). P (X) = 1/36 Therefore, the required probability distribution, X 2 3 4 5 6 7 9 10 11 12 P (X) 1/36 is 1/18 1/12 1/9 5//12 36 1/6 5/36 1/1/1/12 1/18 1/36 Therefore E(X): E (X) = 7 = √5,833 = 2,415 14. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20. A student is selected in such a way that each one has the same chance of being selected and age X for the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance, and standard deviation of X. Solution: Considering the class of 15 students with their age. Form the given data we can draw a table X 14 15 16 17 18 19 20 21 f 2 1 2 3 1 2 3 1 P(X = 14) = 2/15 P(X = 15) = 1/15 P(X = 16) = 16) = 16 2/15 P(X = 17) = 3/15 P(X = 18) = 1/15 P(X = 19) = 2/15 P(X = 20) = 3/15 P(X = 21) = 1/15 Therefore, the required probability distribution X 14 15 16 17 18 19 19 19 20 21 P (X) 2/15 1/15 2/15 3/15 1/15 2/15 3/15 1/15 Therefore, E(X): 15. At one meeting, 70% of Members are in favour and 30% are against a particular proposal. A member is selected at random and we take X = 0 if he was against, and X = 1 if he is in favor. Find E(X) and Var (X). Solution: Given: X = 0 if members are against, and X = 1 if members are in favour. P(X = 0) = 30% = 30/100 = 0.3 P(X = 1) = 70% = 70/100 = 0.7 Therefore, the required probability distribution is, Therefore E(X) is: = 0 × 0.3 + 1 × 0.7 ⇒ E(X) = 0.7 and E(X2) are: = (0)2 × 0.3 + (1)2 × 0.7 ⇒ E(X2) = 0.7 Then Variance, Var(X) = E(X2) – (E(X))2 = 0.7 – (0.7)2 = 0.7 – 0.49 = 0.21 16. The average of the numbers obtained by throwing a door that has written 1 on three faces, 2 on two faces and 5 on one face is A. 1 B. 2 C. 5 D. 8/3 Solution: B. 2 Explanation: Given a cube that has written 1 on three faces, 2 on two faces and 5 on one side. Leave X alone variable representing a number on given door. Then, X can take a value of 1, 2 or 5. The total number is six. Now P(X = 1) = 3/6 = 1/2 P(X = 2) = 1/3 P(X = 5) = 1/6 Therefore, the required probability distribution X 1 2 5 P (X) 1/2 1/3 1/6 Therefore, expectation of X E(X): 17. Suppose that two cards are drawn randomly from a game of cards. Let X be the number of aces obtained. Then the value of E(X) A. 37/221 B. 5/13 C. 1/13 D. 2/13 Solution: D. 2/13 Explanation: Given a game of cards. X be the number of aces obtained. Therefore, X can take a value of 0, 1 or 2. As we know, in a game with 52 cards, 4 cards are aces. Therefore, 48 cards are non-ace cards. = 6/1326 Therefore, the required probability distribution X 0 1 2 P (X) 1128/1326 192/1326 is 6/1326 Therefore Expectation of X E(X): Exercise 13.5 Page No. One door is cast 6 times. If 'getting an odd number' is a success, what is the probability of (i) 5 successes? (ii) At least 5 successes? (iii) Maximum 5 successes? Solution: We know that the repeated roll of a dice is known as bernoulli trials. Let the number of successes to get an odd number in a 6-trial experiment be x. The probability of getting an odd number in a single roll with a cube (p) Thus q = 1 - p = 1/2 Now, here x has a binomial distribution. Thus P(X = x) = nCx qn-x px, where x = 0, 1, 2 ... n = 6Cx (1/2)6-x (1/2)x = 6Cx (1/2)6 (i) Probability of 5 successes = P(X = 5) = 6C5 (1/2)6 = 6 × <5>1/64 = 3/32 (ii) Probability of having at least 5 successes = P(X ≥ 5) = P(X = 5) + P(X = 6) = 6C5 (1/2)6 + 6C5 (1/2)6 = 6×1/64 + 6 ×1/64 = 6/64 + 1/64 = 7/64 (iii) Probability of achieving no more than 5 successes = P(X ≤ <5> 5) We can also write it as: 1 – P(X>5) = 1 – P(X = 6) = 1 – 6C6 (1/2)6 = 1 – 1/64 = 63/64 2. A few dice are thrown 4 times. If getting a double is considered a success, you need to find the probability of two successes. Solution: We know that the repeated toss of a few dice is known as bernoulli trials. Let the number of times to get doubles in an experiment of rolling two dice simultaneously four times be x. Thus q = 1 - p = 1 - 1 / 6 = 5 / 6 Now, here x has a binomial distribution where n = 4, p = 1 / 6, q = 5 / 6 Thus, P (X = x) = nCx qn-x px, where x = 0, 1, 2, ... n = 4Cx (5/6)4-x (1/6)x = 4Cx (54-x/66) Therefore The probability of having 2 successes = P(X = 2) = 4C2 (54-2/64) = 6 (52/64) = 6 × (25/1296) = 25/216 3. There are 5% defective goods in a large part of the goods. What is the probability that a sample of 10 items does not contain more than one defective item? Solution: Leave x number of defective items in a sample of ten items that are pulled successively. Now, as we can see, the drawing of the goods is finished with replacement. Thus, the trials Are Bernoulli trials. Now the probability of getting a defective item, p = 5/100 = 1/20 Thus q = 1 - 1/20 = 19/20 ∴ We can that x has a binomial distribution where n = 10 and p = 1/20 Thus P(X = x) = nCx qn-x px, where x = 0, 1, 2 ... n Probability of not receiving more than one defective item = P(X ≤1) = P(X= 0) + P(X = 1) = 10C0 (19/20)10(1/20)0 +10C1 (19/20)9(1/20)1 4. Five cards are drawn successively with the replacement from a well-mixed game with 52 cards. What is the probability that (i) All the five cards are spades? (ii) Only 3 cards are spades? (iii) No one is a spade? Solution: Leave the number of spade cards among the five cards drawn x. As we can see that the drawing of cards is with replacement, thus, the trials will be Bernoulli's attempt. Now we know that in a game with 52 cards there are a total of 13 spade cards. Thus, probability of drawing a spade from a deck of 52 cards = 13/52 = 1/4 q = 1 – 1 /4 = 3/4 Thus x has a binomial distribution with n = 5 and p = 1 / 4 Thus P (X = x) = nCx qn-x px , where x = 0, 1, 2, ... n = 1/1024 5. The probability that a bulb produced by a factory will melt after 150 days of use is 0.05. Find the probability that out of 5 such bulbs (i) none (ii) no more than one (iii) more than one (iv) at least one will melt after 150 days of use. Solution: Let's assume that the number of bulbs that will melt after 150 days of use in an experiment with 5 trials be x. As we can see that the experiment is made with replacement, thus, the trials will be Bernoulli trials. It is already mentioned in the question that p = 0.05 Thus q = 1 - p = 1 - 0.05 = 0.95 Here we can clearly note that x has a binomial representation with n = 5 and p = 0.05 Thus P (X = x) = nCx qn-x px, where x = 0, 1, 2 ... n = 5Cx (0.95)5-x(0.05)x (i) Probability of no such bulb in a random drawing of 5 bulbs = P(X = 0) = 5C0 (0.95)5-0(0.05)0 = 1× 0.955 = (0.95)5 (ii) Probability of not more than one such bulb in a random drawing of 5 bulbs = P (X≤ 1) = P(X = 0) + P(X = 1) = 5C0 (0.95)5-0(0.05)0+ 5C1(0.95)5-1(0.05)1 = 1× 0.955 + 5 × (0.95)4 × 0.05 = (0.95)4 (0.95 +0.25) = (0.95)4 × 1.2 (iii) Probability of more than one such bulb in a random drawing of 5 bulbs = P (X>1) = 1 – P(X ≤ 1) = 1 – [(0.95)4 × 1.2] (iv) Probability of at least one such bulb in a random drawing of 5 bulbs = P (X ≥ 1) = 1 – P(X < 1) = 1 – P(X = 0) = 1 – (0.95)5 6. A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are pulled successively with the replacement from the bag, what is the probability that no one is marked with digit 0? Solution: Let's assume that the number of balls with digit marked as zero among the experiment with 4 balls drawn simultaneously be x. As we can see that the balls are drawn with replacement, thus the trial is a Bernoulli trial. The probability of a ball from the bag being marked as digit 0 = 1/10 It can be clearly noted that X has a binomial distribution of n = 4 and p = 1/10 Thus q = 1 – p = 1 – 1 – 1 – = 9/10 Thus P(X = x) = nCx qn-x px, where x = 0, 1, 2, ... n 7. In one study, 20 questions are asked of a genuinely false type. Suppose a student chooses a fair coin to determine his or her answer to each question. If the coin falls heads, he replies 'true'; if it drops tails, he replies 'false'. Find the probability that he answers at least 12 questions correctly. Solution: Let's assume that the number of correctly answered questions out of twenty questions be x. Since the 'head' on the coin shows the true answer and the 'tail' of the coin shows the false answers. Thus, the repeated crazy or the correctly answered questions are Bernoulli trails. Thus, p = 1/2 and q = 1 – p = 1 – 1/2 = 1/2 Here it can be clearly noted that x has binomial distribution where n = 20 and p = 1/2 Thus P(X = x) = nCx qn-x px, where x = 0, 1, 2 ... n 8. Suppose X has a binomial distribution B (6, 1/2). Show that X = 3 is the most likely result. (Tip: P(X = 3) is the maximum among all P(xi), xi = 0,1,2,3,4,5,6) Solution: Given X is any random variable whose binomial distribution is B (6, 1/2) Thus, n = 6 and p = 1/2 q = 1 – p = 1 – 1/2 = 1/2 Thus P(X = x) = nCx qn-x px, where x = 0, 1, 2 ... n It can be clearly noted that P(X = x) will be maximum if 6cx will be maximum. ∴6cx = 6c6 = 1 6c1 = 6c5 = 6 6c2 = 6c4 = 15 6c3 = 20 Therefore, we can clearly see that 6c3 is the maximum. ∴ for x = 3, P(X = x) is maximum. Therefore, proven that the most likely outcome is x = 3. 9. What is the probability of a candidate getting four or more correct answers simply by guessing at a multiple choice survey with three possible answers for each of the five questions? Solution: In this question we have the repeated correct answer guessing form given multiple choice questions is Bernoulli trial Let's now assume, X represents the number of correct answers by guessing in the multiple choice set Now the probability of getting a correct answer, p = 1/3 Thus q = 1 – p = 1 – 1/3 = 2/3 Clearly we have X is a binomial distribution where n = 5 and P = 1/3 10. A person buys a lottery ticket in 50 lotteries where each his chance of winning a prize is 1/100. What is the probability that he will win a prize (a) At least once (b) Exactly once (c) At least twice? Solution: (a) Let X represents the number of prizes winning in 50 lotteries and trials are Bernoulli trials Here clearly we have X is a binomial distribution where n = 50 and p = 1 / 100 Thus q = 1 - p = 1 - 1 - 1 / 100 = 99/100 11. Find the probability of getting 5 exactly twice in 7 throws of a door. Solution: Let's assume X represents the number of times to get 5 in 7 throws of door Also the repeated throw of a door is Bernoulli attempt Thus the probability of getting 5 in a single throw, p = 1/6 And q = 1 - p = 1 - 1 - 1 / 6 = 5 / 6 It is clear that we have X has binomial distribution n = 7 and p = 1/6 12. Find the probability of throwing a maximum of 2 sixes in 6 throws of a single door. Solution: Let's assume X represents the number of times to get sixes in 6 throws of a door too, the repeated throw of door selection is Bernoulli trials Thus the probability of getting six in a single throw with door, p = 1 / 6 Clear, we have X has binomial distribution where n = 6 and p = 1 / 6 And q = 1 - p = 1 - 1 - 1 / 1 / 6 = 5 / 6 13. It is known that 10% of certain manufactured goods are defective. What is the probability that in a random sample of 12 such articles, 9 are defective? Solution: Let's assume X represents the number of times select the infected articles in a random sample space given 12 articles Also the repeating articles in a random sample space is Bernoulli attempt Clearly, we have X has binomial distribution where n = 12 and p = 10% = 1 / 10 And q = 1 - p = 1 - 1 - 1 - 1 / 10 = 9 / 10 14. In a box of 100 bulbs are 10 defective. The probability that none of a sample of 5 bulbs is defective is A. 10-1 B. (1/2)5 C. (9/10)5 D. 9/10 Solution: C. (9/10)5 Explanation: Let's assume X represents the number of times select of selected selected bulbs in a random sample of given 5 bulbs Also the repeated selection of defective bulbs from a box is Bernoulli trials Clearly, we have X has binomial distribution where n = 5 and p = 1/10 And q= 1 - p = 1 - 1 - 1 - 1 / 10 15. The probability that a student is not a swimmer is 1/5. So the likelihood that out of five students, four are swimmers are A. 5C4 1/5 (4/5) 4 B. (4/5) 4 (1/5) C. 5C1 1/5 (4/5) 4 D. None of these solution: A. 5C4 1/5 (4/5)4 Explanation: Let's assume X represents the number of students out of 5, there are swimmers also, the repeated selection of students who are swimmers is Bernoulli trials Thus the probability of students not swimmers = q = 1 / 5 Clear, we have X has binomial distribution where n = 5 And p = 1 - q = 1 - 1 - 1 / 5 = 4 / 5 Various Exercise Page No: 582 1. A and B are two events so that P (A) ≠ 0. Find P (B| A) if: (i) A is a subset of B (ii) A ∩ B = φ Solution: 2. A couple has two children, (i) Find the likelihood that both children are men if it is known that at least one of the children is male. (ii) Find the likelihood that both children are women if it is known that the older child is a woman. Solution: (i) If the couple has two children, the test site is according to the question: S = {b, b), b), g), g), g), g)) Suppose that a denotes the case of, that both children have male and B, indicates cases of having at least one of the male children Thus we have: (ii) Assume that C denotes the event with both children females and D denotes cases of having older child is female ∴ C = {g, g)} P (C) = 1/4 And D = {(g, b), (g, g)} P (D) = (2/ 4) 3. Suppose that 5% of men and 0.25% of women grey hair. A gray-haired person is chosen at random. What is the likelihood of this person becoming a man? Suppose there are equal numbers of men and women. Solution: Considering that 5% of men and 0.25% of women have grey hair ∴ Total % of people with grey hair = 5 + 0.25 = 5.25% is therefore likely to have a selected man with gray hair, P = 5/25 = 20/21 4. Suppose that 90% of the population is right-handed. What is the probability that no more than 6 of a random sample of 10 people are right-handed? Solution: Given that 90% of the population is right-handed Let p denotes the likelihood of humans, there are right-handed and q denotes the probability of people being left-handed p= 9/10 and q=1 - 9/10 = 1/10 Now using binomial distribution probability of having more than 6 right-handed people can be given as: 5. An owl contains 25 balls, 10 of which bear a mark 'X', and the remaining 15 bearing a mark 'Y'. A ball is drawn randomly from the urn, its mark is noted down and it is replaced. If 6 bullets are drawn in this way, you should find the probability that: (i) Everyone will wear the 'X' mark. (ii) A maximum of 2 shall bear the 'Y' mark. (iii) At least one ball will bear the 'Y' mark. (iv) The number of balls with the 'X' mark and the 'Y' mark will be the same. Solution: (i) It is stated in the question, total number of balls in the urn = 25 Number of balls marked 'X' = 10 Number of balls marked 'Y' = 15 Let p indicate the probability of balls marked 'X' and q indicates the probability of balls with mark 'Y' p = 10/25 = 2/5 and q = 15/25 = 3/5 Now 6 balls are withdrawn with replacement. Therefore, the number of attempts is Bernoulli triangle. Suppose Z be the random variable representing the number of balls bearing the 'Y' mark in the trials ∴ Z has a binomial distribution where n = 6 and p = 2/5 6. In a hurdle race, a player has to cross 10 obstacles. The probability that he will clear each hurdle is 5/6. What is the likelihood that he will overturn fewer than 2 hurdles? Solution: Suppose that p be the probability of player who will clear the hurdle, while q be the probability of player who will knock down the hurdle ∴ p = 5/6 and q= 1 - 5 / 6 = 1 / 6 Let's also assume X be the random variable that represents the number of times the player will knock down the hurdle 7. A door is thrown again and again until three sixes are obtained. Find the probability of getting the third six in the sixth throw of death. Solution: From the given question, it is clear that the probability of getting a six in a roll of die = 1/6 And the probability of not getting a six = 5 / 6 Let's assume, p = 1/6 and q= 5/6 Now we are likely that the 2 sixes will come in the first five throws of door 8. If a leap year is chosen at random, what is the chance that it will contain 53 Tuesdays? Solution: We know that in spring spring there are a total of 366 days, 52 weeks and 2 days Now for 52 weeks there are a total of 52 Tuesdays ∴ The probability that the leap year will contain 53 Tuesdays is equal to the probability that the remaining 2 days will be Tuesdays Thus, the remaining two days can be (Monday and Tuesday), (Tuesday and Wednesday), (Wednesday and Thursday), (Thursday and Friday), (Friday and Saturday), (Saturday and Sunday) and (Sunday and Monday) ∴ Total number of cases = 7 Cases where Tuesday can come = 2 Therefore, the probability (leap year with 53 Tuesdays) = 2/7 9. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes. Solution: Considering that probability of failure = x And the probability of success = 2x ∴ x + 2x = 1 3x = 1 X = 1/3 2x = 2/3 Assume p = 1/3 and q = 2/3 Also X be the random variable representing the number of Therefore trials, by binomial we have distribution we have: 10. How many times must a man size. Solution: Let's assume that man sews the coin n times. Thus, n tosses is Bernoulli's attempt ∴ The probability of getting head on the toss of the coin = 1/2 Let's assume p=1/2 and q=1/2 11. In one game, a man wins a rupee for a six and loses a rupee for any other number when a fair door is thrown. The man decided to throw a door three times, but to quit when and if he gets a six. Find the expected value of the amount he wins/loses. Solution: For the situation specified in the equation, Also, we are likely to get a six in a roll with a door = 1/6 Also the probability of not getting a 6 = 5/6 Now there are three cases where the expected value of the amount that he wins can be calculated: (i) First case is that, if he gets a six on his first through then the required probability will be 1/6 ∴ Amount received by him = Rs. 1 (ii) Second, if he gets six on his second throw then the probability = (5/ 6 × 1/6) = 5/ 36 ∴ Amount received by him = - Rs. 1 + Rs. 1 = 0 (iii) Finally if he does not get six in the first two throws and gets six in his third throw then the probability = 5/6 × 5/6 × 1/6 = 25/216 ∴ Amount received by him = - Rs. 1 - Rs. 1 + Rs. 1 = - 1 Therefore , expected value that he can win = 1/6 – 25/216 = (36 – 25)/216 = 11/216 12. Suppose we have four boxes A, B, C and D containing colored balls as listed below: One of the boxes has been selected at random and a single marble is drawn from it. If marble is red, what is the probability that it was pulled from box A?, box B?, box C? Solution: Let's assume R be the case to draw the red balls Let's also assume EA, EB and EC denote the boxes A, B and C respectively Given that Total number of balls = 40 Also the total number of red balls = 15 P (R) = 15/40 = 3/8 Probability of taking the red marble out from box A, 13. that the chances of a patient having a heart attack are 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescribing certain drugs reduces its chances by 25%. At some point a patient can choose one of the two options with the same probabilities. It is given that after undergoing one of the two options the patient chosen at random suffers a heart attack. Find the likelihood that the patient followed a course of meditation and yoga? Solution: Let's assume X denotes the events that have a person heart attack A1 denotes events that have the selected person followed the course of yoga and meditation And A2 denotes the events that have the person having the person who adopted the prescription It is given in the question that, P (X) = 0.40 14. If each element of another order determinant is either zero or one, what is the probability that the value of the essential is positive? (Suppose that each record in the determinant is selected independently, and each value is probably assumed 1/2.) Solution: From the question we have: Total number of determinants for the second order, where the item is or 1 = (2)4 = 16 Now we have the value of determinants is positive in the following cases: ∴ Required probability = 3/16 15. An electronic unit consists of two subsystems, e.g. Based on previous test procedures, the following probabilities are assumed to be known: P (A fails) = 0.2 P (B fails alone) = 0.15 P (A and B fail) = 0.15 Evaluate the following probabilities: (i) P (A fails | B has failed) (ii) P (A fails alone) Solution: (i) Let's assume, that the event failed by A is specified by EA And event that is failed by B is specified by EB It is indicated in the question that, Event failed by A, P (EA) = 0.2 (ii) We have, probability where A fails only = 0.2 – 0.15 = 0.05 16. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. A ball is transferred from Behind I to Bag II, and then a ball is drawn from Bag II. The ball saw the sign is found to be red in color. Find the probability that the transferred ball is black. Select the correct answer in each of the following: Solution: Let's first assume A1 denotes the events that a red ball is transferred from bag I to II And A2 denotes the event that a black ball is transferred from bag I to II ∴ P (A1) = 3/7 And P (A2) = 4/7 X Let be the case that the drawn ball is red ∴ when red ball is transferred from bag I to II, 17. If A and B are two events so that P(A) ≠ 0 and P(B| | A) = 1, then A. A ⊂ B B. B ⊂ A C. B = φ D. A = φ Solution: A. A ⊂ B Explanation: 18. If P (A| B) > P (A), which is correct: A. P (B| A) < P (B) B. P (A ∩ B) < P (A). P (B) C. P (B| A) > P (B) D. P (B| A) = P (B) Solution: C. P (B| A) > P (B) Explanation: 19. If A and B are two so that P(A) + + – P(A and B) = P(A), then A. P(B| A) = 1 B. P(A| B) = 1 C. P(B| A) = 0 D. P(A| B) = 0 Resolution: B. P (A| B) = 1 Explanation: NCERT solutions for Class 12 math chapter 13- Probability The main concepts of mathematics, covered by Chapter 13- Probability of NCERT solutions for Class 12 includes: 13.1 Initiation 13.2 Conditional probability 13.2.1 Characteristics with conditional probability 13.3 MultiplicationTeoreme on probability 13.4 Independent events 13.5 Bayes' Teorem 13.5.1 Partition of a sample space 13.5.2 Statement of the total probability 13.6 Random variables and its probability distributions 13.6.1 Probability distribution of a random variable 13.6.2 Average of a random variable 13.6.3 Variance of a random variable 13.7 Bernoulli Trials and Binomial Distribution 13.7.1 Bernoulli trials 13.7.2 Binomial distribution The chapter Probability itself represents up to an entire unit, Unit Six- Probability, bearing 8 marks of the total of 80 brands. There are 2 exercises along with a various exercise in this chapter to help students understand the concepts related to probability thoroughly. Some of the topics discussed in Chapter 13 of NCERT Solutions for Class 12 Maths are as follows: 0 ≤ P (E| F) ≤ 1, P (E′| F) = 1 - P (E| F)P ((E ∪ F)| G) = P (E| G) + P (F| G) – P ((E ∩ F)| G) P (E ∩ F) = P (E) P (F| E), P (E) ≠ 0P (E ∩ F) = P (F) P (E| F), P (F) ≠ 0 If E and F are independent, thenP (E ∩ F) = P (E) P (F)P (E| F) = P (E), P (F) ≠ 0P (F| E) = P (F), P(E) ≠ 0 The statement of total probability Bayes' phrase A random variable is a real valued feature whose domain is the sample space for a random experiment. Var (X) = E (X2) – [E(X)]2 Experimenting is called Bernoulli trials if they meet the following conditions : there should be a limited number of trials. The trials must be independent. Each trial has exactly two results : success or failure. The probability of success remains the same in each trial. Review Chapter 13, Probability, in the NCERT textbook for mathematics to know more about subjects related to probability. Key features of NCERT Solutions for Class 12 Math Chapter 13- Probability Study chapter Probability in Class 12 allows students to understand the conditional probability, multiplication phrase on probability, independent events, total probability, Bayes' phrase, Random variable and its probability distribution, mean and variance of random variable. Variable.