What does l'hopital's rule mean

Mathematical rule for assessing certain limits Example of the application of the Hospital rule to f(x) - sin(x) and g(x) - '0.5x': the function h(x) - f(x)/g(x) is not set to x -0, but can be supplemented to a continuous function on the R set by defining h(0) -f(0)/g(0) - 2. In mathematics, specifically in calculus, the Hospital rule or the Hospital rule (French: [lopital], English: /ˌloʊpiːˈtɑːl/, loh-pee-TAHL) provides a technique to assess the limits of indeterminate forms. The application (or repeated application) of the rule often converts an indeterminate form into an expression that can be easily evaluated by substitution. The rule is named after the 17th-century mathematician from the French Hospital. Although the rule is often attributed to The Hospital, the theorem was first introduced to it in 1694 by the Swiss mathematician Johann Bernoulli. The Hospital rule states that for f and g functions that are different on an open interval, I save perhaps at a point c contained in I, if lim x → c f (x) - lim x → c g (x) - 0 or ± ∞ , 'displaystyle' lim 'x' to c'f (x) -lim -x-to-c-g() ≠ x) for all x in I with x ≠ c, and lim x → c f (x) g - (x) 'displaystyle' lim 'x'to c'frac 'f'(x)'g' (x)' exists, then lim x → c f (x) g (x) - lim x → c f 'displaystyle 'lim': 'x'to c'frac 'f(x)'-g(x)'-l-x-to c-frac 'f'(x)'g' (x)' The differentiation of the numerator and denominator often simplifies the quotient or converts it into a limit that can be assessed directly. History Guillaume of the Hospital (also written the Hospital[a]) published this rule in his book Analysis of the Infinitely Small for the Intelligence of The Curve Lines (literal translation: Analysis of the infinitely small for the understanding of curved lines), the first manual on differential calculus. [1] [b] However, it is believed that the rule was discovered by the Swiss mathematician Johann Bernoulli. [3] [4] General Form The general form of the Hospital rule covers many cases. Whether c and L are extended actual numbers (i.e. actual numbers, positive infinity, or negative infinity). Whether I'm an open interval containing c (for a two-side limit) or an open interval with the endpoint c (for a one-sided limit, or a limit to infinity if it's infinite). The actual f and g value functions are supposed to be different on i except maybe to c, and in addition to g ' (x) ≠ 'displaystyle g' (x)eq 0' on I except maybe at c. It is also assumed that lim x → c f ' (x) g ' (x) ' 'displaystyle 'lim'to c'frac 'f'(x)'g' (x)-L.' Thus, the rule applies to situations in which the derivatives ratio has a or infinite, but not in situations in which this relationship fluctuates permanently as x approaches and at c. If lim x → c f (x) - lim x → c g (x) - '0'displaystyle 'lim 'x'to c'f (x)-lim -x-x-e-to-c-g (x) or lim x x → c f (x) 'lim x → c' g (x) ∞, ''displaystyle'' 'lim','x's (x)', → 'l'l' 'displaystyle': 'lm' to c'frac 'f(x)g(x)'L.' Although we have written x → c throughout, the boundaries can also be unilateral limits (x → c- or x → c), when it's a finite endpoint of I. In the second case, the assumption that f diverges infinitely is not used in the evidence (see note at the end of the section of evidence); thus, although the terms of the rule are normally stated as above, the second sufficient condition for the rule procedure to be valid may be more briefly indicated as lim x → c g (x) = ∞ . 'displaystyle' 'lim' 'x'to c(x)'-infty .- The hypothesis that g ' (x) ≠ 0 'displaystyle g' (x)eq 0' most often appears in the literature, but some authors circumvent this hypothesis by adding other hypotheses elsewhere. One method[5] is to define the limit of a function with the additional requirement that the limiting function be defined all over the relevant interval I except perhaps c.[c] Another method[6] requires that the two f and g be differentiated everywhere over an interval containing c. Limitation Requirement The requirement that the limit is lim x → c f - (x) g - (x) 'displaystyle 'lim'to c'frac 'f'(x)'g' (x)' exists is essential. Without this condition, f's 'displaystyle f' or 'g' 'displaystyle' may exhibit unstamped oscillations such as x 'displaystyle x' approaches c 'displaystyle c' , in which case the Hospital rule does not apply. For example, if f (x) - 'sin (x) 'displaystyle f(x)', g (x) ' x 'x', 'x', 'x's', 'x's', 'x' and c -± ∞ 'displaystyle c'pm 'infty', then f (x) g 's 'displaystyle' 'frac'(x)'g' (x)'-frac '1'cos(x)'{1}; this expression does not approach a limit because x 'displaystyle x' goes to c'displaystyle c' , since the cosine function oscillates between 1 and 1. But by working with the original functions, lim x → ∞ f (x) g (x) 'displaystyle', 'l' lim', 'x'to 'frac', 'f(x)' can be shown to exist: lim x → ∞ f (x) g (x) - lim x → ∞ (1 - sin 'displaystyle 'lim', 'x'to 'frac',f(x)-g(x)'-lim_x'to 'infty' 'left'(1'frac'-sin (x)-x-right)-1. In a case like this, all that can be concluded is that lim inf x → c f (x) g (x) ≤ lim inf x → c f (x) g (x) ≤ lim sup x → c f (x) g (x) ≤ lim sup x → c f ' x ', ' 'Displaystyle', 'liminf', 'x'to c'frac 'f'(x)'g'(x)-leq 'liminf'to c-frac 'f(x)g(x)'-leq 'limsup' F(x) g(x) 'leq's 'limsup' to c'frac so that if the f/g limit exists, then it must be between the lower and upper limits of f/g. (In the example above, this is true, since 1 is indeed between 0 and 2.) Examples Here's a basic example involving exponential function, which involves the indeterminate form 0/0 to x -0: lim x → 0 e x '1 x 2' x 'l lim x → 0 d x x '1 ' d x ' d x (x 2 'x) 'l x →' 'displaystyle's 'begin'aligned'l'e-frac 'e'x-1'{2} 'x' 'x'and 'to'to 0'frac'-dx (e-1)-frac 'd {2} This is a more elaborate example of which is a more elaborate example of which is involved in 0/0. The application of the Hospital rule only once always results in an indeterminate form. In this case, the limit can be assessed by applying the rule three times: lim x → 0 2 sin (x) - sin (2 x) x - sin (x) - lim x → 0 2 cos (x) - 2 cos (2 x) 1 - cos (x x) - lim x → 0 - 2 sin (x) - 4 sin (2 x) sin (x) - lim x → 0 - 2 cos (x) - 8 cos (2 x) cos (x) - 2 - 8 1 - 6. 'displaystyle'- 'begin-aligned' -lim 'x'to'0 'frac'-2-sin (x)-sin (2x)-'x-sin (x)' 'cos(2x)'1-cos(x)'[4pt]'s '0'frac '-02'sin (x)-4-sin (2x)-sin (x)[4pt] 2x -4pt-frac - {1}2-cos (x)8-cos (2x)4pt Here's an example involving ∞/∞: lim x → ∞ x n e - x -1 e x → ∞ x n x x x x x → ∞ x x no x n x n - 1 e x ' n ' n ' n' lim x → ∞ x n ' 'displaystyle 'lim'x'to 'infty' 'x'n'cdot e'x'l'l 'x'to 'frac 'x'n'x'l'x'x'- infty 'frac 'nx-1'1'n'1'n'n'n'l'l 'x'to 'frac 'x-n-1' Repeatedly apply the Hospital rule until the exhibitor is zero (if n is a whole) or negative (if not split) to conclude that the limit is zero. Here's an example involving the indeterminate form 0 ∞ (see below), which is rewritten as the ∞/∞ form: lim x → 0 - x ln x - lim x → 0 - ln x 1 x - lim x → 0 - 1 x 1 x 2 - lim x → 0 - x - 0. 'displaystyle 'lim', 'x'to 0', 'x'ln x'l'l'l'x-to-frac 'ln x'frac {1}'x-x-l'x-x at 0-frac {1} x-e-frac {1} {2} -1-x-to-0. Here is an example of the mortgage repayment formula and 0/0. Whether P is the main (loan amount), are the interest rate per period and n the number of periods. when no, the amount of repayment per period is P n'displaystyle 'frac' (since only the principal is refunded); this is in line with the formula for non-zero interest rates: lim r → 0 P r (1 - r) n (1 - N) 1 - P lim r → 0 (1) n - n) n - 1 n (1 - n) n - 1 - P n . 'displaystyle' 'begin'aligned'lim (1)n-n-n(1)n (1)n (1)n(1)n-1-[4pt] -frac -P-n-rn(end-aligned) The Hospital rule can also be used to prove theor. If f is twice different in a district of x, then lim h → 0 f (x 'h) ' f (x 'h) ' 2 f (x) h 2 ' lim h → 0 f ' (x 'h) ' f ' (x 'h) 2 h ' lim h → 0 f (x 'h) ' f (x) lim 'h'to 0'frac 'f(x-h)-f (x-h)-2f (x)'{2} 'l'4'to 0'frac 'f'(x-h)-f' ((x-h)'; 4pt] and lim 'h'to 0'frac'(x-h)''(x-h) {2}'[4pt] and 'f''(x).'-end-aligned' The Hospital rule is invoked in a delicate way: suppose that f(x) - f(x) converges as x → ∞ and that e x 'f ' (x) 'displaystyle e'x's f(x) - converges towards positive or negative infinity. Then: lim x → ∞ f (x) - lim x → ∞ e x x - x) e x ' lim x → ∞ e x (f (x → ∞) ' x-cdot f(x)e-x-lm 'x'to 'frac'-e-x'2-bigl (f(x)'f'(x)'-bigr) , lim x → ∞ f (x) 'displaystyle', 'l'x'to 'infty', and lim x → ∞ f' (x) - 0. 'displaystyle' 'lim' 'x' to 'infty' 'f'(x)'0. The result remains true without the added assumption that e x f (x) 'displaystyle e'x's f(x) converges to positive or negative infinity, but the justification is incomplete. Complications Sometimes the Hospital rule does not lead to a response in a limited number of steps unless certain additional steps are applied. Here are a few examples: Two applications can lead to a return to the original expression that was to be evaluated: lim x → ∞ e x -e -x -e -x - e → ∞ x -e x - e x x - x - x - x -x -e → ∞ x -e x - e x - e - e - x - x - x - x - x - . 'displaystyle': 'x'to 'frac' 'e'x'-x'e'x--x'-x'-x'- 'x' x-x-x-x-x-x-x-x-x-to-frac 'e'x'-x'-x'-e-x-x-x-x'-1st'-cots .- This situation can be dealt with by replacing y'e x'displaystyle y'e'x' and noting that there goes endlessly that x goes to infinity; with this substitution, this problem can be solved by a single application of the rule: lim x → ∞ e x - e - x e - x - x - lim y → ∞ y - 1 y - y - 1 - 1 - 1 → ∞ - 1. 'displaystyle': 'x'to 'frac' 'e'x' -x'-x'-e-x'-lm'-to -infty 'e' 1-y-y-1 -lim -y-to -frac -frac -1-y-2 -1-2- frac {1}{1}.- 1. Alternatively, both the numerator and the denominator can be multiplied by e x, 'displaystyle e'x', at which point the Hospital rule can be immediately applied successfully:[7] lim x → ∞ e x e - x e x - x - lim x → ∞ e 2 x 1 e 2 x ' 1 ' lim x → ∞ 2 e 2 x 2 e 2 x '1. 'displaystyle', 'l'x'to 'frac', 'e'x', 'x'-e-x', 'x' 2x -2x-1 -1 -1-x-to-frac '2e'2e'2e -2e-2x-1. An arbitrarily high number of requests can never lead to an answer even without repeating: lim x → ∞ x 1 2 x - 1 2 x 1 2 - x - 1 2 - lim x → ∞ 1 2 x 1 2 - 1 2 x 3 2 1 2 x 1 2 - 1 2 x - 3 2 - lim x → ∞ - 1 4 x - 3 2 - 3 4 x - 5 2 - 1 4 x 3 2 - 3 4 x - 5 2 . 'displaystyle 'lim 'x'to 'frac'x'frac {1}{2} 'ex' {1}{2} 'x'e-frac {1}{2}-x-e-frac {1}{2}-lim 'x'to 'frac' {1}{2} 'ex' {1}{2} 'frac {1}{2}'xe-1st {3}{2} {3}{2} frac {1}{2} x-1st {1}{2} {1}{2} e-frac {3}{2} lim -x-to-frac {1}{4} -x-e-frac {3}{2} {3}{4} {5}{2} {5}{2} {1}{4}-e-frac {3}{2} {3}{4}-x-frac {5}{2}-{5}{2}-15 {3}{2}-frac {3}{4}-frac {5}{2}. This situation can also be dealt with by a transformation of variables, in this case y 'x'x'displaystyle y'sqrt 'x': lim x → ∞ x 1 2 x ' 1 2 x '1 2' x '1 2 ' lim y → ∞ y 'y ' 1 y ' '1 ' lim y → ∞ 1 ' y ' 2 'y' ' '2' ' '2' ' 2 ' 1 ' 1 ' 1' 'displaystyle 'lim 'x'to 'frac'x'frac {1}{2} 'e-3{1}{2}'e-e-frac {1}{2}-x-e-frac {1}{2} infty 'frac'y'y-1-y-1 -lim 'y'to 'frac '1-y'2 -1-2-2-2-frac {1}{1} -1.1. Again, another approach is to multiply the numerator and denominator by x 1 / 2 'displaystyle x'1/2' before applying the Hospital rule: lim x → ∞ x 1 2 x ' 1 2 ' 1 2 ' x '1 2 ' → ∞ '1 x ' 1 ' lim x → ∞ 1 1 1. 'displaystyle', 'l' x'to 'infty' 'frac' {1}{2} 'x'-e-frac {1}{2} 'x'-frac {1}{2}-x-x-e c {1}{2} 'lim' 'x'to 'frac'-x-1-1-lim 'x'to 'infty' {1}{1} '1.' A common trap is to use the Hospital rule with circular reasoning to calculate a derivative via a difference quotient. For example, consider the task of proving the derivative formula for the powers of x: lim h → 0 (x 'h)n' x n h ' n x no x No 1 . 'displaystyle 'lim'h'to 0 'frac' (x-h)'n-x-no'h'nx-1.' By applying the Hospital rule and finding derivatives in relation to numerator h and denominator, yields n xn-1 as expected. However, the differentiation of the numerator necessitated the use of the very fact that is proven. This is an example of begging for the issue, because it cannot be assumed that being proven in the course of the evidence. Counter-example when the derived denotorator is zero The need for the condition that g ' (x) ≠ 0 'displaystyle g' (x)eq 0' near c'displaystyle c' can be seen by the following because of Otto Stolz. [8] Let f (x) - 'sin x cos x 'displaystyle f(x)'s 'ix' x-cos x' and g (x) f (x) e sin x . 'displaystyle g(x)f(x)e-sin x.' Then, there is no limit to f (x) / g (x) 'displaystyle f(x)/g(x)' as x → ∞ . 'displaystyle x' to 'infty 'infty 'infty 'infty 'infty 'infty 'infty 'infty 'inft However, f - (x) g - (x) - 2 cos 2 x (2 cos 2 x) e sin x - (x -sin x cos x) e sin x cos x cos x 2 cos x ' x's sin x cos x e ' sin x ', 'displaystyle' 'begin'aligned 'f'(x)'g' (x) 'frac'2 'cos '{2}x'(2-cos '{2}x)e-sin x'(x-sin x-cos x)e-sin x-cos x-cos x 'frac' '2'cos x'2'cos x-x-sin x'e'sin x', 'end-aligned', which tends to 0 as x → ∞ 'displaystyle x'to 'infty' . Other examples of this type have been found by Ralph P. Boas Jr.[9] Other undetermined forms Other indeterminate forms, such as 1∞, 00, ∞0, 0 ∞, and ∞ - ∞, can sometimes be assessed according to the Hospital's rule. For example, to evaluate a limit involving ∞ - ∞, convert the difference of two functions to a quotient: lim x → 1 (x x - 1 ln x) - lim x → 1 x 'ln x ' x '1 ' '1' - ln x (1) - lim x → 1 ln x x 2) - lim x → 1 x ln x x - 1 - x ln x (3) - lim x → 1 1 - ln x 1 - 1 - ln x (4) - lim x → 1 1 - ln x 2 - ln x 1 2 , ''displaystyle' 'begin'aligned 'lim'x-to 1 'left'(left' x-x-1 --frac {1}-ln x-right) 'ln x' and quad (1)'[6pt] '1'1 'frac'x 'frac'x-1 x'l'ln x '2)'[6pt] 'e-frac' x-frac 'x-1' x-cdot 'ln x'x-1' x-cdot 'ln x'quad (3)[6pt] '1'1'ln 1'ln x '1'1'ln x'amp'quad (4)lim The Hospital's rule is applied at the step from (1) to (2) and again at the step from ({1}{2} 3) to (4). The Hospital rule can be used on indeterminate forms involving exhibitors using logarithms to move the exhibitor down. Here's an example involving the indeterminate form 00: lim x → 0 - x - lim x → 0 - e ln (x x) - lim x → 0 - e x -ln x -e lim x → 0 'displaystyle': 'x'to 0'x'l'x'- 'x'to 0'ln (x-x)-lim x-to 0'e'x-cdot 'ln x 'e'lim 'limits ' 'x'to 0' (x-cdot 'ln x)'. It is valid to move the limit within the exponential function because the exponential function is continuous. Now, the exhibitor x 'displaystyle x' has been 'moved down'. The limit lim x → 0 - x 'ln x 'displaystyle', 'l'x-to 0' is from the indeterminate form 0 ∞, but as shown in an example above, the Hospital rule can be used to determine that lim x → 0 x 'ln x 0.' 'displaystyle' 'lim' 'ln x'0.' So lim x → → x x - e 0 - 1. 'displaystyle' 'lim' 'x'to 0'x'{0}'1. Theorem stolz-Cesàro Main article: Theorem stolz-cesàro Theorem stolz-cesàro is a similar result involving sequence limits, but it uses finite difference operators rather than derivatives. Geometric interpretation Consider the curve of the plane whose coordinate x is given by g(t) and whose coordinate is given by f(t), with the two continuous functions, i.e. the location of the points of the shape [g(t), f(t)]. Suppose f(c) - g(c) - 0. The limit of the ratio f(t)/g(t) as t → it is the slope from the tangent to the curve to the point [g(c), f(c)] - [0.0]. The tangent at the curve at the point [g(t), f(t)] is given by [g(t), f(t)] The Hospital rule then states that the slope of the curve when t - it is the limit of the slope of the tangent to the curve as the curve approaches the origin, provided that this is defined. Proof of the Hospital Rule Special Case The proof of the Hospital rule is simple in cases where f and g are continually differentiated at point c and where a finite limit is found after the first round of differentiation. This is not evidence of the Hospital's general rule because it is stricter in its definition, requiring both differentiation and that it be a real number. Since many common functions have continuous derivatives (e.g. polynomials, sinus and cosine, exponential functions), this is a special case worthy of attention. Suppose f and g are continually different to a real number c, that f (c) - g (c) - 0 -displaystyle f(c) g(c)-0) and that g -c) ≠ 0 'displaystyle g' (c)eq 0' . Then lim x → c f (x) g (x) - lim x → c f (x) - 0 g (x) - 0 - lim x → c f (x) - f (c) g (x) - g (c) - lim x → c (f (x) - f (c) (g x) - g (c) x - c) - lim x → c (f) - f (c) x) lim x → c (g (x) - g (c) x) - f -c) g) - lim x → c f (x) g (x) . 'displaystyle'-begin-aligned'- 'x'to c-frac 'f(x)'-lim 'x'-to c-frac 'f(x)-0-g(x)-0-lim 'x'to c'frac 'f (x)-f(c)g(x)-g (c)-[6pt]-e-l-x-to c-frac -left (f(x)-f (c)-x-c-right)-left (o'e c-g(x))-g (c)-x-c-right) - frac -lim -limits -x-to c-left (frac -f(x)-f (c)-x-c-right)-lim 'limits ' x'to c'left ('frac' 'g(x)) This is a result of the definition of the difference quotient. The last tie stems from the continuity of the derivatives to c. The limit of the conclusion is not indeterminate because g 'c) ≠ 0 'displaystyle g' (c)eq 0 . The evidence of a more general version of the Hospital rule is given below. General Evidence The following evidence is due to (1952), where unified evidence for 0/0 and ±∞/±∞ ±∞/±∞ forms are given. Taylor notes that different evidence can be found in Lettenmeyer (1936) and Wazewski (1949). Let f and g be functions satisfying the assumptions in the general form section. That I 'displaystyle' -mathcal 'I' is the open interval in the assumption with the endpoint c. Considering that g ' (x) ≠ 0 'displaystyle g' (x)eq 0' on this interval and g is continuous, I 'displaystyle' -mathcal 'I' can be chosen smaller so that g is nonzero on I 'displaystyle 'math'. [d] For each x in the interval, define m (x) - inf f - (? g) (? g) Ot (') g ' ('displaystyle M(x)') 'sup'' 'frac'('o'o'g' ('xi)' as a 'displaystyle' -xi' varies on all values between x and c. (Inf and sup symbols denote infimum and supremum.) From the f and g differentiation on I 'displaystyle', Cauchy's average value theorem guarantees that for two separate x and y points in I 'displaystyle', 'I' there is a 'displaystyle', 'x' between x and y such as f (x) 'f (y) g (x) 'g (y) 'f' (?) g ' (?) 'displaystyle'-frac 'f(x)-f (y)-g(y)-g (y)'-frac 'f' (xi) ' 'g' (' As a result, m (x) ≤ f (x) - f (y) g (x) - g (y) ≤ M (x) -displaystyle m(x)-leq -frac -f(x)-f(y)-g(y)-g(y)- 'leq M(x)' for all distinct x and y choices in the interval. The value g(x)-g(y) is always nonzero for x and y distinct in the meantime, because if this were not the case, the theorem of the average value would imply the existence of a p between x and y such as g' (p)-0. The definition of m(x) and M(x) will result in an extended real number, so it is possible for them to take on the values ±∞. In the following two cases, m(x) and M(x) will set limits on the f/g ratio. Case 1: lim x → c f (x) - lim x → c g (x) - 0 'displaystyle', 'x' to c'f (x)-l-lim -x-to-c-0- For any x in the interval I 'displaystyle 'mathcal 'I' , and dot y between x and c, m (x) ≤ f (x) - f (y) g (x) - g (y) - f (x) g (x) - f (y) g (x) 1 'g (y) g (x) ≤ M (x) 'displaystyle m(x)-fraq 'f(x)-f (y)-g (y)-g (y)-frac 'frac'f (x))E-e/e-f(y)g(x)--1-frac 'g(y)'-leq M(x)' and therefore as you approach c, f (y) g (x) 'displaystyle'- 'frac',f (y)-g(x)' and g (y) g (x) 'displaystyle' 'frac'- 'g(y)' become zero , and therefore m (x) ≤ f (x) g (x) ≤ M (x) . 'displaystyle m(x)'leq 'frac 'f(x)'- 'leq M(x)) Case 2: lim x → c g (x) For each x of the interval I 'displaystyle 'I' , define S x 'y |'y is between x and c 'displaystyle S_'y mid y'text is between 'X' and c 'displaystyle' S_ 'x' ∞ For each point y between x and c, m (x) ≤ f (y)) f (x) g (y) - g (x) - f (y) g (y) - f (x) g (y) 1 - g (x) g (y) ≤ M (x) . 'displaystyle m(x)-leq 'frac'f(y)-f(x)g(y)-g(x)-frac 'frac 'f (y)g (y)---frac 'f (x)'g (y)') As it approaches c, the two f (x) g (y) 'displaystyle 'frac 'f(x)g(y)' and g (x) g (y) 'displaystyle', 'frac'g (x)-g (y)' become zero, and therefore m (x) ≤ lim inf y ∈ S x f (y) g (y) ≤ lim sup y ∈ S x f (y) g (y) ≤ M (x) . 'displaystyle m(x)'leq 'liminf'S_ 'x'e-frac 'f(y)'-leq 'lq 'y'in S_ '1e-frac 'f (y)g (y)'-leq M(x).' The upper and lower limits are necessary as the f/g limit has not yet been established. It is also the case that lim x → c m (x) - lim x → c M (x) - lim x → c f - (x) g - X . 'displaystyle 'lim 'x'to c(x)'l 'x'to c(x)'l 'x'to c'frac 'f'(x)'g' (x)-L [e] and lim x → c (lim inf y ∈ S x f (y) g (y)) - lim inf x → c f (x) g (x) ''displaystyle' lim 'x'to c'left ('liminf'y'in S_ x'frac 'f (y)g(y)-right) -liminf -x-e-frac -f (x)g (x)- and lim x → c (lim sup y ∈ S x f (y) g (y) - sup lim x → c f (x) g (x) . 'displaystyle 'lim'x-to c-left ('limsup'in S_ 'ex'frac 'f(y)'g(y)-right)'-limsup 'x'to c'frac 'f (x)g (x)' In case 1, the compression theorem establishes that lim x → c f (x) g (x) 'displaystyle', 'lim'x-to c'frac 'f(x)g(x)' exists and is equal to L. In case 2, and the compression theorem again states that lim inf x → c f (x) g (x) - lim sup x → c f (x) g (x) -L'displaystyle 'liminf' to c(e-frac) tupids - x-to-frac -f(x)g(x)-L, and thus the limit lim x → c f (x) g (x) 'displaystyle lim'x to c'frac 'f (x)g(x)' exists and is equal to L. This is the result that had to be proven. In case 2, the assumption that f(x) diverges infinitely was not used in the evidence. This means that if 'g(x)' diverges infinitely as x approach c and f and g satisfy the assumptions of the Hospital rule, then no additional assumptions are needed as to the f(x) limit: It might even be the case that the f(x) limit does not exist. In this case, the Hospital theorem is actually a consequence of Cesàro-Stolz. [10] In the event that 'g(x)' diverges infinitely as x approach c and f(x) converges to a finite limit at c, then the Hospital rule would be applicable, but not absolutely necessary, since the basic limit calculation will show that the limit of f(x)/g(x) as x approaches c must be zero. Corollary A simple but very useful consequence of the Horital rule is a well-known criterion of differentiation. It states the following: suppose that f is continuous to a, and that f ' (x) 'displaystyle f' (x) exists all x in some open open container has, except perhaps for x 'a'displaystyle x'a' . Suppose, moreover, that lim x → a f - (x) 'displaystyle', 'x'to a-f'(x)', exists. Then f ' (a) 'displaystyle f' (a) also exists and f ' (a) - lim x → a f ' (x) . 'displaystyle f'a)'l 'x'to a'f' (x). In particular, f' is also continuous to a. Proof Consider the functions h (x) - f (x) - f (a) -displaystyle h(x)-f(a)- and g (x) x - a 'displaystyle g(x)-x-a' The continuity of f at a tells us that lim x → a h (x) - 0 'displaystyle' 'lim'to a'h (x)-0' . In addition, lim x → a g (x) - 0 'displaystyle', since a polynomial function is always continuous everywhere. The application of the Hospital rule shows that f - (a) : lim x → a f (x) - f (a) a - a - a - lim x → a h (x) g (x) - lim x → a f (x) - 'displaystyle f' (a): i.e.) -f(x)-f (a)-x-a-lim -x-to a-frac 'h(x)-g (x)-lim 'x'to a-f'(x)' . See also The Hospital Controversy Notes - In the 17th and 18th centuries, the name was commonly spelled 'Hospital', and he himself spelled his name that way. However, French spellings were modified: the silent were removed and replaced by the circumflex compared to the previous vowel. The old spelling is always used in English where there is no circumflex. Proposal I. Problem. Either an AMD curve line (AP -x, PM- y, AB - a [see figure 130]) such that the value of the applied is expressed there by a fraction, don't the numerator - the count that more each x x has, ie when the P point falls on the given point B. On request which of them the value of the BD application. [Solution:]... If we take the difference in the numerator, and divide it by the difference of the denominator, after having made x -a-Ab or AB, the value will be applied bd or BD. Translation: There is an AMD curve (where AP - X, PM - y, AB - a) so that the value of the order is expressed by a fraction whose numerator and denominator each become zero when x i.e. when point P falls on point B. One wonders what the value of the coordinated comics will be then. [Solution:]... if one takes the numerator differential and divides it by the denominator differential, after defining x -a-Ab or AB, one will have the value [that has been] sought in the bd or BD order. [2] - The definition of functional analysis of the limit of a function does not require the existence of such an interval. Since g' is nonzero and g is continuous over the interval, it is impossible for g to be zero more than once over the interval. If it had two zeroes, the average value theorem would affirm the existence of a p point in the interval between zeroes such as g' (p) - 0. So or the other g is already nonzero on the or the interval can be reduced in size so as not to contain the single zero of g. The limits are lim x → c m (x) and lim x → c M (x) both exist because they have non-decrepit and non-incinerating x functions. Consider a sequence x i → c'displaystyle x_'i-to c' . Then lim i m (x i) ≤ lim i f ' (x i) g ' (x i) ≤ lim i M (x i x_) 'frac'(x_-i-g'(x_-i-) - leq 'lim'i'm (x_'i), as inequality continues for each i; this gives inequality lim x → c m (x) ≤ lim x → c f ' (x) g ' (x) ≤ lim x → c M (x) ''displaystyle' lim 'x'to c'm(x)'lq 'l'x-x-to-c-frac 'f'(x)g' The next step is to display lim x x → c M (x) ≤ lim x → c f - (x) g - x) 'displaystyle'- 'l'x-x-to-c-m(x)-leq 'lim'x-to-frac f'(x) g'(x) . In fact, correct a sequence of numbers ε i '0'displaystyle 'varepsilon' so that lim i ε i '0'displaystyle 'lim'i'i'er-varepsilon 'i'0', and a sequence x i → 'c'display x_'i'. For each i, choose x i 'lt; y i'lt; c 'displaystyle x_'y_ i'i'st]so that f' (y i) g - (y i) - ε i ≥ sup x i'lt; (y_ -i-g' (y_-i-x_) Thus lim i M (x i) ≤ lim i f - (y i) g - (y i) - ε i -lim i f - (y i) g - (y i) - lim i ε i -lim i f y i) g (y i) 'displaystyle', 'i'm(x_-i-)-leq 'i'frac 'f'(y_') (y_-i-i y_ y_-) I-varepsilon y_ y_ as desired. The argument that lim x → c m (x) ≥ lim x → c f - (x) g - (x) 'displaystyle 'l'x'to c'm(x)-geq 'x'to c'frac 'f'(x)' is similar.

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