Banzhaf power index

They're equals! If you're not interested in mathematics, you can say. Well, although I included all the math, I also wrote this page to be accessible to those without a strong math background. I've also written to entertain myself, and I hope it entertains you too. I got some positive feedback, so go ahead and you can take a chance. Live a little! If you don't understand how the Electoral College works, we suggest you read a little more about it now. The National Archives and Records Administration holds a full page of the U.S. Electoral College of Federal Registration. Click on the Frequently Asked Questions picture and what is the Electoral College on the menu just below the eagle? link. Banzhaf and his previous methods were published in various legal journals, especially villanova law journal, Winter 1968 [1]. I learned of Banzhaf's work in an article by Ian Stewart in the July 1995 issue of Scientific American [6] and searched for other rescilious articles. Here's a list of relevant references I know. The basic principle of Banzhaf's method is that voting power is based on your ability to change--- or rather, to replace the outcome of the election with ---. Keep in mind that this is a bit difficult to think about because you don't vote directly for a candidate in a block voting system. Banzhaf's method then calculates the likelihood that a citizen's vote will change the bloc's decision, and then determines the likelihood that the bloc's votes will change the outcome of the election. Note: The Electoral College system does not actually require voters to give their electoral votes to candidates who are thrown out of the popular vote. In addition to showing the absurdity of the system, this failure also does not surprise any attempt at analysis. Rarely do these things happen. On NARA's Electoral College site, see the 1988, 1976 and 1972 elections to give a few new examples. So the two numbers we need to calculate the Banzhaf Power Index are the probability of an individual changing the outcome of a bloc's decision and the possibility of changing the bloc's election result. Banzhaf uses simple unifying analysis to reach the possibilities for each block. He claims (and quoted): In order for an individual voter to give a critical vote, other voters in the group must be evenly divided. M is the formula for the number of combinations that M people can create by dividing them into two equal groups! M 2! M 2! This expression comes from the standard combinatorial formula for the number of ways in which you can select p elements (in this case, voters for voting candidate #1) from n total elements (i.e. total voters). This is a combination of quantity (n,p) and = n ! P! (n - p) ! Banzhaf continues: ... When calculating how many times each person can vote a critical vote, the function must be multiplied by account 2 for voting for two different candidates. ... Think of a group of N+1 citizen voters where N is a double number. The total number of ways each citizen-elect can vote for one of the two main candidates is 2 x 2^N. Each will only be able to give a critical vote in which other N citizen-voters are evenly divided into two groups; N/2 votes yes and N/2 voting nay. This, as mentioned earlier, can be 2 N ! N 2 ! N 2 ! Way. Therefore, the following portion of individual citizen-voter combinations will be critical in determining the outcome of a vote. 2 N ! N 2 ! N 2 ! (2 ·2N) You might think this possibility has an inverse linear relationship with the block's population. However, as Banzhaf and others have shown, this is not true. Probability relates to the reverse square root of the block population. This is quite surprising at first glance, but you can either accept the following equations as evidence, or do it yourself and see. That's why Banzhaf continues to achieve the ultimate possibility of simplifying expression and using Stirling's approach to factorial large numbers that are critical in determining the decision of an individual voter bloc. This inverse frame-root-relationship gives. ... The factorial of large numbers can be very closely related to the following formula known as the Stirling formula. [For those who are not familiar with mathematical representation, wavy equal sign means approximately equal] The function that converts this value to the previous formula by ensuring that N/2 is equal to m is 22m e-m mm æÖ 2pm e-m mm æÖ, which is the first factor in Banzhaf's Power Index. (breath, breath) The next factor can't really be calculated directly with a simple (ahem) formula like Stirling's approximate. Or at least, I don't know one (and aren't satisfied). Looks like Banzhaf is, too. He said in a footnote (quite dry, if you can editorialize for a moment) ... Naturally there are some shortcuts that can be done in the application of calculations, when calculations are done manually or with the use of an electronic computer. However, they are beyond the scope of this article. [5] On another occasion, Banzhaf spoke a little more about what he had done. [1960] Calculations of [this] step of the Electoral College's analysis were actually done in two slightly different different methods, using two similar techniques on two fundamentals. The first performance was performed by Lloyd S. Shapley and Irwin Mann (both helped [1]) in the formula of matter. The words in the above excerpt [square paratheses] are my attempts to have clear references to Banzhaf's words in the context of this web page. With these changes, there is no change in what Banzhaf means. Now the question is, what did Shapley and Mann do to calculate their numbers? There was a slightly different definition of the power of a bloc in the bloc voting system but that doesn't matter for the current purpose. The important thing was that they used a very general approach to computational probabilities on large sets of possible outcomes, known as the Monte Carlo approach. This will surprise a few mathematicians and computer scientists. For those of you who don't know the Monte Carlo technique, the brief explanation is that you sample possible output several times and count the number of times that event occurred. You then assume that your sampling is neutral, so that the actual probability of the event is divided only by the number of instances you receive the number of events you see. Obviously, you need to cover a significant portion of the possible outcomes before this will give you the right results. Monte Carlo simulation is a big topic, and I'm not going here anymore. I searched for the Monte Carlo simulation in a search engine and came up with about 22,000 webpages. Although most subjects are applied to a particular topic, from economics to applied physics. If you want to know more, find one that you can relate to. On an interesting note, we can only hope that the so-called random number generator does not give us the same combination of multiple times in a particular simulation. So I wrote a code to form a coalition through the blocks at the entrance. It then determines which of the blocks is critical, according to Banzhaf's definition, which means they have power in this coalition. They got a point for the Monte Carlo simulation experiment. After a series of trials, each block will divide the points that have accumulated according to the number of attempts and give the Banzhaf Power Index the second factor. Banzhaf's definition of power was as follows: A particular bloc has power in a particular coalition only if the bloc can leave the coalition and turn it into a losing coalition from the winning coalition. (Obviously, leaving a losing coalition does not affect the outcome, because it has no negative weights.) Therefore, if the current number of available 'yes' votes is Y and at least M votes are required to win, a bloc with B votes is critical, and if there are other power definitions discussed below only under criticism of the Y - B < M Banzhaf Power Index, the probability of a citizen changing the outcome of the election with the votes of the voters is the product of the two above probability (i.e. According to probability laws, the possibility of two independent events occurring at the same time is a product of the possibility that each of them will happen, as both the individual's decision of the bloc and the bloc's votes change the election). That's why we multiply two numbers for each block to get the power attributed to each voter within the block. This is the Banzhaf Power Index. Banzhaf analyzed two proposed alternatives to the Electoral Council: He gave a bloc's electoral votes to each candidate in proportion to the percentage of popular votes received in that bloc. Assigning the electoral votes of each congressional district to the candidate who won the popular vote in the district. Banzhaf's analysis showed that the bloc voting system (i.e. the Electoral College) heavily supports the residents of large blocs, while both alternative plans support the residents of smaller blocs. Banzhaf also said direct popular elections would give equal authority to each voter. If the number of blocks in the system is relatively small, the Monte Carlo approach is not required. At another time, it can form all possible coalitions and don't worry about sampling the area of potential outcomes. This is a little easier conceptually, and thus Stewart's description [6] emphasizes counting more than my above explanation considering the Monte Carlo approach. Stewart then throws power to individual voters by simply dividing block power into the number of residents. Stewart also gives the example of Tompkins County, N.Y. In 1982, who was able to create a weighted system with power per person for each region within 1.8% of others. (The table is below, but read the paragraphs first between here and the table!) BUT WAIT! You have to say. Doesn't that assume that power within the bloc has an inverse linear relationship with the population? In other words, don't you want to be equal among districts divided by the population square-root-of-population of power, but power per person (power divided by population)? Ok, maybe that's not exactly what you were saying.... But the answer is actually 'Yes.' That's what Stewart left in his article. Apparently, the Tompkins County, New York board of elections also left out this section of their distribution of votes between the blocks. Where is John Allen Paulos (author of Innumeracy - Mathematical Ignorance and Consequences) when you need him? Here's the table. Note that the numbers in the fifth column are almost the same for each region. Stewart says this means that every election in all elections has almost the same power. But look at the last column. It gives power-split-by-the-square-root-of-the-population ratios. (That is, for each block, divide the column four columns into two square roots. Then divide, for each block by the smallest of such numbers for each block.) Because the Banzhaf Power Index cancels square root ratios of 2 over p, these numbers are equal to the Banzhaf Power Ratios for each region. The other thing is, divide the Banzhaf Power Index for each region into the Power Index for the smallest region and measure relative strength. Keep in mind that Banzhaf Power Ratios are not that close together. As Banzhaf noted in general, voters in larger regions have more power than voters in smaller areas. For example, voters in Lansing county have about 1.33 times more power than voters in the Dryden West area. The only exception is between two regions with an equal number of numbers of numbers. Then voters in the district with 10 additional people have slightly less power, which makes sense. Block Pop. Weight Power Power/Pop Norm. Power/Sqrt(Pop) = (Votes) Banzhaf Power Ratio -------- ---- ------- ----- --------- ---------------------- Lansing 8317 404 4734 0.569 1.335714 Dryden_East 7604 333 4402 0.579 1.298965 Enfield_Newfield 6776 306 3934 0.581 1.229748 Ithaca_3 6550 298 3806 0.581 1.210087 Ithaca_4 6002 274 3474 0.579 1.153852 Ithaca_SE 5932 270 3418 0.576 1.141931 Ithaca_1 5630 261 3218 0.572 1.103571 Ithaca_2 5378 246 3094 0.575 1.085621 Ithaca_NE 5235 241 3022 0.577 1.074743 Groton 5213 240 3006 0.577 1.071306 Caroline_Danby 5203 240 3006 0.578 1.072335 Ithaca_5 5172 238 2978 0.576 1.065526 Ithaca_West 4855 224 2798 0.576 1.033288 Ulysses 4666 214 2666 0.571 1.004283 Dryden_West 4552 210 2622 0.576 1.000000 Robert J. Sickels objected to Banzhaf's method , Shapley-Shubik yöntemi olarak bilinen daha önce yayınlanmış bir yöntemi tercih eder. Sickels claims banzhaf counts the block effect twice because of the two-step process and taunts his (evaluated) advantage. In step one, Banzhaf measures how a state can influence the outcome by changing the voting bloc. In step 2, Banzhaf measures how an individual can influence the choice of a state, and Banzhaf ignores combinations with a simple majority. That Banzhaf should not be able to give critical status to more than one majority member at the same time. The Shapley-Shubik method empowers only one member of the majority and calculates the likelihood that each voter will be the voter who puts the candidate at the top of all permutations. Now, on behalf of Banzhaf, I'm going to try to answer Sickels in part by quoting Banzhaf. First, the fact that there are two stages of the Banzhaf method is no reason to criticize him. After all, a block voting system is inhernally two-stage. The block effect doesn't count twice in Banzhaf's equations; is one of two factors. Second, I don't understand what's wrong with assigning critical status to multiple members of the majority coalition. It is true that if the coalition, say, has a majority by 18-15, then everyone The two votes are critical to the coalition conglomerate. Sickels wants to define the critical coalition as the person who placed it in a simple majority and give each of them a chance to be that person, while Banzhaf describes the critical coalition as having enough votes from winner to loser, or on the contrary. I prefer Banzhaf's description. I don't think Banzhaf ignores combinations with simple majorities as well as critical of extra votes, gives every voter in the bloc the opportunity to become a critical voter in that bloc, and defines each bloc as a critical bloc that has enough votes to change elections. This shapley-Shubik method (above) strikes me as almost the same as Sickels' description. In fact, Banzhaf noted his method and the Shapley-Shubik method ... significantly similar results for a large number of voting units. Indeed, the differences between these two sets of calculations were no more than a few percent. The difference really only comes to play when the choice is a landslide (or at least not close). Banzhaf can't give anyone critical status when Shapley and Shubik's election is overwhelming. I think Banzhaf's definition is more accurate, since a landslide does not have a fundamentally intuitive sense of power. So, in an election decided by 525-13 (e.g. the 1984 U.S. Presidential election), should everyone be considered critical (i.e. having power)? You can also see why the gap is up to a few percent, since by definition only a few percent of coalitions make landslides. It would be relatively easy to change my schedule to calculate the Shapley-Shubik index (see below), but since Shapley-Shubik has only assigned power to a region that puts the coalition above the majority, I think the program will not be so efficient. This means that the order of voting is important at every hearing, and then every bloc is given the health of being a vital bloc. However, much more coalitions need to be made for this strategy, as there are now not only N coalitions, but also R orders that this coalition can be provided for. And R is a big number for systems as big as the U.S. Electoral College. R = B! (B factorial) is the number of blocks B. 51! approximately 2.6 x 10^(66) - yes, this is a 67-digit number. In Monte Carlo, there is a minimum percentage of the possible output field you need to sample (e.g. minimum number of attempts), such as the method I use to calculate the solution for a system as large as the Electoral College. Despite Banzhaf's description, I probably don't sample enough. And now I have to multiply the number of instances (and time length) by a 67-digit number?!? No, thank you! None of these two indexes (Banzhaf and They were the only measures that were in place in the 1960s, and there have been proposed extensions and alternatives ever since. See references for a list of several new articles. Most of the recent work is in the field of game theory, which is not a topic I am familiar with closely. If Stewart's introduction is very accessible to those without a mathematical background, this means that recent articles are very mathematical and academic. Of course, Stewart's article description is incomplete and very misleading about what to do. Moreover, the debate up to this point assumed that a winning coalition only required an outsumned majority. However, voting assemblies sometimes need more than just a majority to pass a measure. The Banzhaf Power Index can be easily adjusted by changing the number of votes needed to form a winning coalition. All other mechanics of calculations remain the same, as is the critical definition. I changed my code so that the user could enter the number of votes needed to form a winning coalition. I want to thank Elaine Garcia for showing me this case. Even if it is because the code is required to calculate the case .... I have written a C program to calculate the Banzhaf Power Index for mounting any size. It takes a description of the assembly and the relevant population of the blocks and outputs the Banzhaf Power Index for each. The input format is quite simple. The first line of entry contains a single number that gives the number of voting blocks. It is a line for each voting bloc with three fields following this line: the name of the block, the population of the bloc, and the number of votes the bloc has. Files tested.dat and TompkinsCountyNY.dat Stewart from the article, while the U.S.dat Electoral College and the U.S. 1960.dat 1960 Electoral College correspond to the 1990 census configuration. My code uses the brute force method (that is, counting the total number of coalitions and the number of which each block is critical) when the number of blocks is below a predefined constant. There's nothing magical about the number, but I would not recommend raising it high enough to try a great example brute force method like the U.S. Electoral College (51 blocks). It may not end in your life. You should be able to run the program on several instances that are not too small for scheduling data, then select a suitable constant for your machine using the fact that there are 2^n n object combinations when each has a yes/no option. For large numbers of blocks, the program uses the Monte Carlo method, which means banzhafNextCoalition() function in Banzhaf.c basically has an if statement that divides into two different functions. Equations listed above for large or small block numbers The mathematics of the Banzhaf Power Index is used to calculate this factor to change the power of each elective. These two factors are put together in banzhafComputeRatios(). If you find an error in my code or have your own app, please email me. Feel free to use the code here to help you get started. Please give me a loan. I ran my application to the Electoral College of 1990 on the U.S. census distribution. I used 4.29 billion samples in Monte Carlo. The program took just under 25 hours to complete. Here are the results. State PopulationEV PR Population PR CA 29760021 54 3344 LA 4219973 9 1308 NY 17990455 33 2394 MS 2573216 7 1302 TX 1698 6510 32 2384 SC 3486703 8 1278 FL 12937926 25 2108 IA 2776755 7 1253 PA 11881643 23 2.018 AZ 3665228 8 1247 IL 11430602 22 1965 KYK 3685296 8 1.243 YT 10847115 21 1923 OR 2842321 7 1 .239 MI 9295297 18 1775 NM 1515069 5 1211 NC 6628637 14 1629 AK 550043 3 1.205 NJ 7730188 15 1617 VT 562758 3 1192 VA 6187358 13 13 1564 RI 1003464 4 1190 GA 6478216 13 13 1529 ID 1006749 4 1188 5 544159 12 1524 NE 1578385 5 1186 WA 4866692 11 1490 AR 2350725 6 1.167 TN 4877185 11 1489 DC 606900 3 1148 WI 4891769 11 1486 KS 2477574 6 1137 MA 6 6016425 12 12 1463 UT 1722850 5 1,135 MO 51 17073 11 1453 HI 1108229 4 1.132 MN 4375099 10 1428 NH 1109252 4 1132 MD 4781468 10 1366 ND 638800 3 1118 OK 3145585 8 1346 WV 1793477 5 1113 AL 4040587 9 1337 DE 666168 3 1095 WY 453588 3 1327 NV 1201833 4 1087 CT 3287116 8 1317 ME 1227928 4 1.5076 CO 3294394 8 1315 SD 696004 3 1071 MT 799065 3 1,000 States (plus Washington, DC, has its own Electoral Votes) is listed with two-letter mail abbreviations. Population 1990 and 1992 Information Please For Almanac. The number of electoral votes (EV) is based on the redistrining of votes in 1990. Pr power ratio, defined by Banzhaf. This is the strength of every voters in the given state based on the power of voters with the weakest voting power. Again, as defined by Banzhaf and shown above, voting power is likely to change the outcome of the election by vote. With the current distribution and the final population, a montana voter is the least likely to change the outcome of the election, and so Montana's Power Ratio is 1,000. A voter in California is 3,344 times more likely to change the outcome of the election. Note that the real possibilities (not given) are small in either case (not surprising given that there are some 260 million U.S. citizens, although I don't know how many of these are eligible or registered to vote), but this is still a good measure of a priori voting power. I have 1964 data (1960 census data) which run the program published figures. You can click on the results file of the simulation below. The numbers don't exactly agree; Both are probably a bit off the actual answers according to the above definitions. Banzhaf's original numbers would be somely off because of what he did in the 1960s to avoid long computational transactions (a wise decision). Maybe one day I will update this page with a description of what I did; I talked to him about it. My numbers will be off because monte carlo numbers are probably always a bit off, and I'm not really able to create enough samples with the standard numerical precision supported in this app. But it works again really, really close, so I doubt the mistakes are great. Some details, for cows between you: I calculated 4.29 billion coalitions using an SGI Onyx ^ 2 with an R10000 processor. It took 24 hours and 49 minutes to find the 1990 data. Let me take this opportunity to climb my soapbox for a moment and beg everyone to vote in every election (especially in the US, where voter turnout is very low). It's never hard to register (at least not in the U.S.) and although you can't see it in your daily life, it's important to you who's chosen. These numbers may not seem important, but they don't matter. Also, keep in mind that if you don't vote, you have ZERO voting power. Don't throw away what you can use to influence your government. That's enough soap cans for today. Gzip'ed Unix tar file archive of everything. Banzhaf calculations C code module and header file. There's also a main drive and a GNU Makefile. NOTE I've made a few minor changes to the code in the past. I hope the current version supports all the claims you make on this page. Look at the code for an option to be faster, but less accurate. Additionally, the program now accepts an argument that requires more than a simple majority. Pc-compatible executable. A test entry file from the Scientific American article and the output of the code on it. My output of the SciAm test entry is shown above in this Tompkins County, NY login data. This is data that Stewart's Banzhaf Power Index statement is misleading. My sample output file was not fully created by my program. I manually entered the power-by-square-root-population column to show the difference between what Stewart and Tompkins County did and Banzhaf's intentions. I also have more decimal places in my program that will print at the moment, but I can easily change it enough. The extra data from the test entry in Tompkins County, N.Y., came out with the output of my code along with two United States census/electoral college configurations. These numbers were obtained from the Monte Carlo simulation. each time you run the program you can get slightly different numbers for the first few digits Deci/deci point. See the analysis below on why this is unlikely and how much difference there may be. These numbers were obtained with examples of 4.29 billion (yes, 'b'. The figures in Banzhaf's article, of course, don't agree, but they're close. The differences stem from the Monte Carlo approach based on so-called random numbers (and, of course, different algorithms for such production since the late 1960s, when Banzhaf did his job), and possibly changes in division and square root algorithms. (So, modern machines probably have more digits and approaches to these processes supposedly more accurately.) The 1990 U.S. Electoral College and my exit from the 1960 U.S. Electoral College, my exit and Banzhaf's published numbers always have more! -Of course, every time my mother present such an analysis, I really need to calculate the varience of the numbers. (For non-mathematical, varience tells you how far the answer might be and indirectly suppose you know what the probability distribution is between the specified estimate and possible values. Hmmm, maybe it wasn't the best explanation for non-mathematical either. Sning.) Here's what's very much a back-of-the-envelope calculation here, but this should give some idea of the accuracy of the numbers in the above outlets without undertaking heroic effort like the average person looking at a math book. He's out of breath! For the Monte Carlo approach, the error is limited to 3v/(sqrt(N)), the number of tries of N and the varience of v for each attempt. N is 4.29 billion for sample outputs that have above. I didn't actually calculate v, but something like this goes away. Varience is defined as the expected value of (x - u)^2 with a u average. In this case, the average probability for a particular block. Of course, this has an unknown priori. However, they are limited to 0<=u<=1. I think in the worst case, we're looking for a difference for every case of 1.0. This is a difference that is no worse than 4.48e-05 for the probability calculated for each block. Stirling has approximately a large number of varieties. I used the banzhaf he was given upstairs. This approximate error is limited to 1/(12 *N). N here is the number of people on the block, so we're looking at numbers ranging from 450,000 to 29.76 million for 1990 data and 226,000 to 16.78 million for 1960 data. So, let's approximate all these numbers with an error around 1/220000 or 4.55e-06. On the other hand, approaches that use the approximate value of Stirling should not be more than this number, and in fact it is better for all data from 1990, and larger states have more accurate values than smaller states. Then, you can make this error in each approach to a large number of factors in the equation given above for the probability of an individual being Vote in the block. Remember that this equation is theoretically three factorial factors that should be approximately predicted (or scroll up and see). In fact, Banzhaf symbolically plug about and then simplified. Note, however, that the two factors in the denominator, the factor in the denominator, are half the size of the factor in the denominator. So the worst thing that can happen is a mistake that looks like 1 2 + 1 6 N 1 2 + 1 6 N after some algebraic manipulation and some eyeballing, I'll bet it's limited to 4. (The denominator and denominator are both quadratic in N due to the constant terms and quadratic denominator of the fraction nested within the denominator of the outer fraction. Then follow the L'H pital rule. Rinse. Repeat.) So I'm guessing there's nothing less than error 1.82e-05, even in the worst case for instances. In all this conclusion I expect that the calculated power for each block (unless the so-called random number generator did a really bad thing) is both factor 4 important figures (i.e. significant decimal places to begin answering, not including decimal points and consecutive zeros between important digits). In other words, I expect that if you run the simulation again (or run your own, the numbers will accept over four important digits. But that's calculated raw power. The ratios between the smallest calculated power and other blocks I showed you (and banzhaf published). This division with a fairly small number has the potential to introduce error, given that the highest rate is only 3,345, you should introduce a minimum amount of 3.6e-03 error by a change that does not affect the third decimal place (the fourth significant digit after the rates are calculated) when the answers are rounded up as I do here (and in my program , i.e. I'm not looking at more numbers than I've shown you here). So this fourth important step can fit three or four variations. Such a large variation is of course unlikely. I've been generous with the error predictions I'm giving here, so the actual values are a little less and so the variation is almost certainly less than four in the fourth major rung. So I expect another simulation will really give you exactly the same numbers for power ratios within three decimal places on the show. A few days later .... I actually wrote this claim on this page and re-executed the simulations. (Of course, I immediately reran them, so I could only be an idiot for a short time. Fortunately, however, my calculations were quite accurate.) For 1960 data, all but four of the 51 blocks had the same power ratio within four key figures; The other four were th out of the game by one margin in the fourth major. For 1990 data, all Six of the 51 blocks had the same power ratio in four key figures; the other six were closed by one margin in the fourth major rung. Some of these ten were high and some were low. I don't know how close they are in the two cases. I noticed that those off tend to be larger blocks; The difference between the two attempts may be due to a small change in the raw power of the smallest block. I would classify this as a successful test, and I believe you won't see more than four variations in the final decimal place from the numbers here. You probably won't see more than one variation in the fourth decimal place. Tell me if you do, and I'll put it on this page. This should include any changes to increase the accuracy of stirling's approximate number or the increase in the number of samples. Obviously, if you reduce the number of samples or the approximate accuracy of Stirling, then this claim does not apply. Apparently, if you try to use every bit of the so-called random number as an argument does not apply either, a change that the calculations introduced into the code to squeeze a little more speed. Better speed (24:49 hours:minutes against 38:46) yielded, but changed some numbers in the third key digit. Thus, it has brought some inaccuracies to potentially problematic calculations. I think the problem would be the most serious for voting systems which exceed the threshold at which only full calculations are made. But I've tested it. I left both blocks of code in program C, and you can use both by coming the version you want. (There is a preprocessor directive that you must define or define.) Faster, less accurate version at a default time, but slower now, more accurate version. First of all, let me give you my deep congratulations and thank you for stroking your ego. As long as you're this far away, send me mail and tell me! And if you jumped down the hill, stop cheating! Second, if I do this again, let me think about things I can try differently. Yes, that's right! Using the Shapley-Shubik definition of power to make any difference to these most likely alternative calculated numbers so far, and so (I think) the most interesting of the ideas I list here. I gave Banzhaf's description the definition in the reviews section. I noted that Banzhaf said the two will basically give the same results and gave at least one intuitive reason why. From what I understand banzhaf said, Shapley helped him prepare his numbers, so I assume Shapley is not shy about the claim that these two methods produce similar results. But we can only assume .... Either way, you can make the change. I strongly recommend if you are going to do this, you reduce the sample size and apply a parallelization scheme. Read the note about the number of instances you need under Responses to Criticism and the idea of a clean parallelization of code in the following 3 items. I'm sampling a very small percentage of possible outcomes for coalitions still in increased sample size. There is 2.2518 x 1015 possible, I sample only 4.29 x 1012, or less 0.2%. That's a much smaller percentage than the one normally used in the Monte Carlo simulation. But more arithmetic requires some fancier processing and/or parallelization of code, I believe. Code parallelization also thought about splitting the sample area by allowing the user to enter a pre pre pre pre pre pre pre preceding to be prepared for the random bit stream. For example, using three preection bits also presents the code's eight-way parallel (8=23) by running eight processes with only eight possible three-bit pre pre pre pre pre pre pre pre preections. This can make it more toerable to create more instances. Assuming that each of the sub-processes uses the same (large) number of instances, you only need to average the power ratios from each process to get the final values. This suppose that the raw power calculated for the smallest block is always the same, and that each operation decides that the same block has the least power. With large sample sizes in each process, this seems quite unlikely. If you don't want to make this assumption, you can have the program always print power indices based on average. The beauty of the preem strategy for parallelization is that it guarantees that the two processes will form the same coalition, as blocks are guaranteed to have a difference in at least one (inside or outside) of the first three assignments. Of course, he can't guarantee that a single process can't form a coalition twice. Improving Stirling's approximate condition may be interesting to see how a better version of Stirling's approximate state will affect the recently calculated power ratios. I said I expected the answers not to change more than four times in the fourth major. I can't see that it would mean anything in more deciable places to increase sensitivity, but arbitrarily sensitive libraries' net is available. I used double precision. I would like to hear something interesting if I do with this code or link to this page. Please email me. He applied the Banzhaf Power Index to the German elections in 1994. The original page has a broken link, but I also found a mirror site for this page. Cal State-Fresno has a Mathematica code for the Banzhaf Power Index and the Shapley-Shubik Power Index, written by Peter Tannenbaum. I'd like to thank Ofer. From Brandeis University, he noticed the missing numbers in my original code, which caused me to re-read the original sources and correct them correctly. I would also like to thank other people who have taken the time to comment on this page (in particular, Prof Chase and math students at Christ College, Elaine and Jayne Garcia did some analysis of the European Union with my code, and especially Melissa Penn State's math great - but i already knew I was shaken!). It's good to know that people read, enjoy and learn a little bit. Mark Livingston Department of Computer Science Univ. North Carolina Last modified: April 14 <http: www.cs.unc.edu/~livingst/banzhaf/index.html=> <http: www.cs.unc.edu/~livingst/banzhaf/index.html=></http:> </http:> 2003 This document: