Isogeometric analysis essential boundary conditions

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Elsevier/Academic Press (2009)Google ScholarSokolnikoff, I.S.: Mathematical Theory of Elasticity. McGraw-Hill, New York (1956)zbMATHGoogle ScholarWriggers, P.: The method ofarly limited elements. Springer, Berlin (2008)zbMATHGoogle ScholarZienkiewicz, O.C., Taylor, R.L., Zhu, J.Z.: The Limited Elements Method: Its Base and Fundamentals, 6th EDN. Elsevier, Oxford (2005)zbMATHGoogle Scholar Shuohui Yin1*, Tiantang Yu2 and Crystal National Bui3 Received: 22 January 2018;   Published: January 29, 2018 Corresponding author: Shuohui Yin, School of Mechanical Engineering, Xiangtan University, Xiangtan, Hunan, 411105, Chinese PR DOI: 10.32474/TCEIA.2018.01.0001 Pdf Summary Also see in : Although the well-known standard lagrange (LMM) method can produce even greater accuracy and make it easier to implement than other common diagrams (e.g., modified transformation principles, Nitsche's method), it does possess many difficulties in solving discrete equations, mainly caused by new indimmed Lagrange systems. LMM naturally increases the problem size and leads to a poorly conditioned matrix equation. Singly points are also commonly encountered because of the inappropriate selection of internal space for the lagrange meast. In this paper, we propose an improved method, called the reduced Lagrange method of clemming, which can overcome such limitations raised by LMM in the handling of Dirichlet type boundary conditions for Isogeometric Analysis. By simply dividing the system equations into boundaries and interior groups, the size of the system equation derived from LMM is reduced; no additional unknowns were added to the result equation system; the lagrange is therefore disappearing; and more importantly, the singly issue in question is avoided. Accuracy and convergence rate of the proposed method research through three digital examples, exhibiting all the desired features of this method. Optimal convergence rates and high accuracy for the current method are found. Keywords: NURBS; Isogeometric analysis; Dirichlet boundary conditions; Lagrange human method; Phương pháp phần tử hữu hạn Abbrevations: LMM: Lagrange Multiplier Method; IGA: Iso-iso-photoo-analysis; CAD: COMPUTER-aided DBCs: Design boundary conditions; DM: Direct Method; LSCM: Less square collocation method; RLMM: Lagrange Cause Reduction Isogeometric Analysis (IGA)[1] uses the same computer-aided design (CAD) spline base function (e.g., NURBS, T-splines) to describe accurate metrics and appegeeed physical reactions. The IGA provides the ability to integrate limited elements analysis into CAD design tools and avoid the process of grid generation and follow-up communication with cad models during refining. The IGA has shown to be a highly accurate and robust approach to digital simulation with accurate picture representation even with rough mesh[2]. Furthermore, the high-order smoothness of nurbs base functions allows direct construction of free-spin/shell formulas[3,4] and is attractive to solve PDEs that combine the fourth (or higher) variations of field variables, such as the Hill-Cahnard equation [Gomez et al., 2008][5] and damage gradient models. The IGA has shown to be an effective approach in solving technical problems as a large number of theoretical and applied works of the IGA have been conducted and reported in literature, including mathematical properties[7,8], integrated methods[9,10], spline engineering[11], fluid mechanicality[12], fluid structure interactions[13], sheet/shell structure[14-17], electrolytes[18], feline-elastic electrolytes[19], shape optimization[20], mechanical fractures[21], and unsaturated flow problems[22]. Despite efforts dedicated to the IGA in recent years and significant progress has been made, some relevant technical aspects still require further research; and among others the imposing of essential Dirichlet type boundary conditions (DBCs) in the IGA is an important technical issue. Due to the non-inferent nature of the NURBS facility, similar to moving at least squares[23], the properties of the Kronecker Square are unhappy, directly imposing DBCs is difficult, and special techniques are therefore required. Hughes et al. [1] mentioned a direct method (DM) for enforcing DBCs to control points. However, Wang & Xuan[24] found that DM can cause significant errors with worsening convergence rates, and a special DBC treatment technique of calculating fewer grids for contact issues[25] is presented, which is done by separating internal points and boundaries on the basis of mixed conversion methods. Last-family No. However, the selection of boundary infererance points should be emphasized to avoid a singly conversion matrix. Koo et al. [26] recently expanded this method to ISO shape design. Costantini et al. [27] adopts quasi-endotopes to establish hethotopithized DBCs in the IGA on the basis of general B-splines to eliminate the solution of large global linear systems. Lu et al. [28] presents a technique for blending NURBS with Lagrangian endolysts within essential boundaries and allowing direct imposing of prescribed values. Other strategies based on the modified weak form equation, such as the Nitsche method[29] and the lagrange method (LMM) have been applied in NURBS-based IGA. Recently, De Luycker et al. [30] proposed a square collocation method (LSCM) to impose arbitrary DBCs in the extended IGA. Govindjee et al. [31] then improved LSCM by providing a local algorithm that at least squares effectively for the least global squares. However, LSCM is affected by the score of endo]. Therefore, an important issue of how to choose the correct location and score of endo-failure arises. LMM is widely used in weak form methods for dbcs implementation. LMM provides solutions with greater accuracy than the revised variant principle[33]. The implementation of LMM is simpler and simpler than Nitsche's method. However, LMM introduced a new indimmed Lagrange meath, making it difficult to select infererural space for the Lagrange cyge to avoid singly problems in solving the equation system. In this article, we present a new method of improvement, reducing the Lagrange human method (RLMM) for the effective treatment of DBCs on IGA. Given that NURBS control points can be clearly divided into boundary and interior groups and nurbs base functions of interior control points disappear at boundaries[24], we divide LMM's system equation into boundary and interior groups. Therefore, the size of the equation system is now reduced and the shortcomings of LMM can therefore be overcome. Moreover, the selection of infer infer inner space for the Lagrange meath is studied in numbers and discussed in the number results. The rest of this article is organized as follows. Part 2 presents a summary of NURBS based on the IGA. Part 3 details RLMM for boundary treatment treatments required in the IGA. For example numbers are considered in Section 4 to verify the accuracy and effectiveness of the current method. Conclusions are drawn in Part 5. In a 1D ε[0.1] function space, a κ() node vector for building B-spline base functions is a set of indeplar numbers written where and which is the number basic functions and order of basic functions, respectively. With a κ (node vector), the B-spline base function of p degrees, written ni,p (), is defined recursively as follows[34] and the base function NURBS Ri,p () within the framework of the partition of unity built by a weighted average of B-spline base functions [35]: where is the weight ith. Similarly, the two-variable NURBS base functions are expressed in the following form: The case represents a 2D weight; Ni,p () and Nj,q (η) are B-sp line base functions defined on the κ() and κ(η) button vectors, respectively, according to the recursive formula displayed in Eqs. (1) and (2). The NURBS base function is non-inferent in the interior parameters space. Therefore, essential boundary conditions cannot be applied simply by enforcing individual values for control points. By using the NURBS base function, a NURBS surface of order in the direction and order in the direction can be constructed as follows: where Pij is the coordinate of the control point in two dimensions. Without losing generality, we look at a 2D isothongthlym elasticity problem with a boundary (and), the corresponding adjustment equations are expressed as follows: where ∇ deity gradients; It's stressful. b and t are body and surface force vectors, respectively; n is the normal vector on the Neumann Γt;g boundary which is the known shift on Γu. The total energy potential of the system can be built as follows: In the NURBS-based IGA, an isoqual concept is adopted and nurbs base functions are used for modeling and analysis of the carast: After the control points are rearranged with a sub-unified index , approximately the field can then be written as follows: where Î is the coordinates of parameters; RI (Î) shows the NURBS base function at control point I; NP is the total number of control points. On the basis of the principle of minimal potential energy and by replacing the approximate shift of Eq. (10) to Eq. (7), a discrete almolycies equation system can be obtained as follows: KU =f, (11) vectors of variables identified at control points; and is the global hardness matrix and external vector at the control points, respectively. The contributing factors are as follows: where D is Hooke's elastic matrix. The NURBS base functions commonly used in IGA do not meet the Kronecker uniform functions, and direct imposing of uI (x) = gI (x) on control points can cause significant errors with worsening convergence rates. The novel RLMM is presented in the following. LMM is understandable and implementable because it has been widely used to enforce DBCs. To overcome these shortcomings, we modify the standard LMM so that the size of the equation system is reduced. We named the modified method RLMM. By applying LMM, the limited weak form obtained is as follows: The matrix form of LMM originates as follows [Ghorashi et al., 2012]: Lagrange factor approximated by NURBS base functions as follows: The level of freedom (DOF) u for discrete control points in the structure can be classified into two groups : uB DOF restrictions are sets of control points () for boundary description, and dof uA unlimited that is the collection of control points ( ) for interior domain description. NURBS control points can be clearly divided into groups of boundaries and interiors because the corresponding NURBS base functions involve interior control points disappearing at the boundary. Therefore, approximately the field can be divided as follows: Given that the basic functions of the interior control points (NA) disappear identically at the boundary Γu , CA = 0, Eqs. (22a) and (22b) can then be written accordingly as follows: The Eqs result system. (23a) and (23b) suggest that additional unknowns, lagrange variations disappear, singly problems can be avoided, and the size of the equation system is actually reduced to an (n-NB) × (n-NB) matrix problem. In addition, since nurbs base functions are used as endomic functions of the analysis of the limited and lagrange (Eq.(17) elements, so the same 1D volume matrix as the boundary, and the full quarter (the number of Gauss points in each direction is p +1) is used to ensure that it is inverse. A number of possibilities are available for the selection of inferent space for the lagrange meast, such as (1) lagrange shape functions or (2) similar shape functions used in the inferent of U (i.e., NURBS base functions in the IGA). In order to verify babuska-Brezzi stability and impose the necessary boundary conditions precisely, the inferoth selection of the Lagrange factor is very important. In the IGA, the matrix form of the LMM equation is shown as follows: The combination of Eqs. (24a) and (24b) made gaussian block removal wise to get additional Schur pressure as follows: The question now is whether CK-1CT can be reversed. The K-1 was often irreversible for Laplace and Poisson-style problems; therefore, we just need to consider whether CCT may be irreversible. Babuska-Brezzi stable conditions also ensure irreversible CCT. In IGA, CA = 0; therefore, CCT was reformed into becBcTB. If nurbs base functions are used as endolyst functions of the lagrange (Eq.(17) system, then similar to the 1D and CBCTB mass matrix may be irreversible. Therefore, nurbs base function is a stable space for Lagrange to impose essential boundary conditions in the IGA. Nurbs base functions can get an acceptable solution on digital examples (Part 4), and the system results of the equation will not be singly even if the number of Lagrange personnel is large. In this section, three 2D examples are considered to show the accuracy and performance of the RLMM approach. The number of Gauss points in each direction is p+1 in all studies, with p being the order of the NURBS base function. For convergence analysis, the H-tuning strategy is applied and knots are added between the knots. Furthermore, three different error measurements are calculated to assess the accuracy of the current method. The L2 error level is expressed as follows: where and where is the correct and appe% shift, respectively. The H1 error line is expressed as follows: The stress energy error level is expressed as follows: where and where are the correct and appe% correct strains, respectively; and are precise and appe% precise stresses, respectively. The exact stress energy level is defined as follows: Laplace issue on a square domain The first example considers the Laplace issue. The adjustment equation and boundary conditions are shown as follows: Figure 1: Original carothology: physical grid (a) and control grid (b). As shown in Figure 1, the original visuality was constructed by the NURBS surface of the order p=3 for both directions with unit weight. The initial space parameters are given by the κ button vectors = {0,0,0,0,0,0,5,1,1,1}and κθ = {0.0 ,0,0,0.5 ,1,1,1,1}. For convergence analysis, h tweaking is used to refine the grid by inserting a button. The refined meshes have 16, 64, 256 and 1024 elements (Figure 1). Convergence results of RLMM, LMM and DM are shown in Figure 2 and 3, respectively. Optimal convergence rates are obtained for current methods and LMM while the results based on RLMM are identical to those from LMM. A tier two convergence rate in the L2 error limit and a block convergence rate in the obtained H1 error limit are comparable to the second-tier convergence rate in both cases for the direct application of boundary conditions. We can also predict that RLMM can be calculated faster than traditional LMM because only one matrix problem (n-NB) x (n-NB) (Eq.23) needs to be addressed in the matrix issue (n+NB) without conditions (n+NB) [Eq. (16)] later. Table 1: Compare the DOFs and the number of conditions used to calculate between different methods. The comparison of methods differs in the number of degrees of freedom and the number of matrix conditions presented in Table 1. Dividing the total DOF n into DOF NB boundaries and DOF NA interiors, it can be seen from Eq.(23) that RLMM owns the same DOF number as DM, but the DOF NB boundary is applied directly in the DM while it is obtained solve the equation system with a smaller size than Eq. (23a) in RLMM. The total number of unknowns in LMM will be n+NB because the Lagrange number equals the DOF NB boundary. On the other side, the number of matrix conditions of RLMM and DM is the same as shown in Eq. (23b). As shown in Table 1, LMM's matrix condition number is three orders of greater intensity than RLMM and DM is even worse for complex issues. In conclusion, RLMM has the same matrix condition number as DM but needs to solve an Eq. (23a) to get the unknown NB boundary. Compared to Eq. (23b), Eq. (23a) is smaller and won't waste too much time. However, compared to DM and LMM, additional unknowns and bad matrix conditions were found for LMM. Figure 2: Compare L2 error targets for Laplace issues on a square domain. RLMM delivers the same accuracy as LMM while avoiding the number of bad matrix conditions such as DM and is also a common singfulst in LMM for the large number of degrees of freedom no longer encountered (Figure 2 & 3). Absolute error distributions use the 256 elements described in Figure 4 to show the accuracy of the proposed approach. The following can be observed from the results: (1) domain errors derived from RLMM are small, while less accuracy at the boundary can be found for DM; (2) according to the error distribution, although the maximum value of the error given by DM is smaller by RLMM, the error area of DM is greater than those of RLMM (Table 1) (Figure 4). Figure 3: Compare the H1 error targets for the Laplace issue on a square domain. Figure 4: Distribution of absolute errors obtained by (a) DM, (b) RLMM. A cantilever beam with a length of L = 48 units and a height of H = 12 units is subjected to a parabolic traction at the free end as shown in Figure 5 being considered. The material properties of the beam are made as follows: Young E module = 3.0x107 units, poisson ratio v = 0.3. Aircraft stress conditions are assumed. The imposing parabolic traction is expressed by the following: where I = H3 / 12 shows the moment of inerness, the applied sheath is P = 1000 units, and the left edge of the beam is clamped. The analytical shift of this issue is given by the following [35.36]: The order of nurbs base functions is p=2, and the parameters space is originally defined by the node vectors. κ=κη,={0'0'0'0'0.5'U'1} The weights associated with all basic functions are set to one; therefore, nurbs base functions are identical to the corresponding B-spline functions. The meshes used for the convergence study were 4 x 4, 8 x 8, 16 x 16 and 32 x 32 elements. The results of L2 and stress energy error targets shown in Figure 6 and 7 clearly confirm that the proposed method has a solution error less than DM and maintains an optimal convergence rate error criteria: 3.016, error target H1: 2.014) (Figure 5-7). Figure 5: Describe the cantilever beam problem. Figure 6: Compare L2 error targets for cantilever beam problems. Figure 7: Comparison of stress energy error targets for cantilever beam problems. The final example refers to an infinite plate tension plane with a central circular hole under traction x-direction Tk = 1 unit (Figure 8a). Due to the symmetry of themetry, a quarter of the domain name is simulated (Figure 8b). To avoid singly points in the photogenic performance, we continue to look at a limited array of admonitions in the digital model (Figure 8c). Precise shifts using Eq. (36) are imposed on the inner and outer edges of the piercing plate. The analytical shifting solutions are shown as follows[35]: Figure 8: Describes the diagram of (a) an infinite plate with a circular hole, (b) a quarter of the square, and (c) an annually limited plate used for the digital performance model. The geological and material parameters are the radius of the round hole a = 1 unit, the outer radius b = 4 units, the Young E module = 105 units, and the ratio of Poisson v = 0.3. The initial form of this problem is shown in Figure 9. The refined meshes used in convergence analysis are elements 4 x 4, 8 x 8, 16 x 16 and 32 x 32. The results of L2 and stress energy error indicators are analyzed and drawn accordingly in Figures 10 & 11. The proposed method has fewer solution errors and achieves an optimal convergence rate, thus outperforming DM (Figure 9-11). Figure 9: The original picture of (a) the physical grid and (b) the control grid. Figure 10: Compare L2 error targets for infinite plates with a round hole. Figure 11: Compare stress energy error limits for infinite plates with a circular hole. The Dirichlet Boundary Conditions (DBCs) in the IGA were studied by developing an accurate treatment, RLMM, to overcome the difficulties in implementing these DBCs. Compared to standard LMM, the size of the equation system is reduced as the system results in no additional unknowns, the Lagrange cause thus disappears and singly problems in LMM are avoided. The benefits of the proposed approach can be achieved even greater if the number of degrees of freedom of discrete systems is relatively large. The choice of infer inner space for the lagrange system is solved in detail. As a result, the NURBS base function is found to be a stable space for the Lagrange meaent that the DBCs in the IGA are imposed. However, the implementation of improved methods of treating dbcs in IGA is simple, and can be integrated into existing IGA code with less effort. So improved methods can be the ideal candidate for practical issues. Based on the investigation of the number of convergences, the new method provides high accuracy on optimal convergence solution and rate, confirming the effectiveness of advanced treatment techniques of DBCs in IGA. The degree of accuracy and convergence rate achieved by RLMM is better than DM.

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