Uniform distribution worksheet

Uniform distribution is a continuous distribution of probability and is associated with events that are equally likely to occur. When working out problems that have a uniform distribution, be careful to note if the data is inclusive or exclusive. Example 5.3.1 Data in table \(\PageIndex{1}\) 55 smiling times, in seconds, of an eight-week-old baby. Table \(\PageIndex{1}\) 10.4 19.6 18.8 13.9 17.8 16.8 21.6 17.9 12.5 11.1 4.9 12.8 14 8 22.8 20.0 15.9 16.3 13.4 17.1 14.5 19.0 22.8 1.3 0.7 8.9 11.9 10.9 7 .0 3 5.9 3.7 17.9 19.2 9.8 5.8 6.9 2.6 5.8 21.7 11.8 3.4 2.1 4.5 6.3 10.1 7 8.9 9.4 9.4 7.6 10.0 3.3 6.7 7.8 11.6 13.8 18.6 Average sample = 11.49 and standard sample deviation = 6.23. We will assume that smiling times, in seconds, follow a uniform distribution between zero and 23 seconds inclusive. This means that any smiling time from zero to 23 seconds is equally probable. The histogram, which can be constructed from a sample, is an empirical distribution that closely matches the theoretical uniform distribution. Let \(X=\) the length, in seconds, of an eight-week-old baby's smile. The designation for uniform distribution is \(X \sim U(a, b)\), where \(a =\) is the smallest value \(x\) and \(b=\) the highest value is \(x\). Probability density function: \(f(x) = \frac{1}{b-a}\) for \(a\leq x \leq b\). For this example, \(X \sim U(0, 23)\) and \(f(x) = \frac{1}{23-0}\) for \(0 \leq X \leq 23\). Formulas for theoretical mean and standard deviation: \[\mu = \frac{a+b+b}{2} onumber\] and \[\sigma = \sqrt{\frac{(b-a)^{2}}{12}} onumber\] For this problem, the theoretical average and standard deviation \[\mu = \frac{0+23}{2} = 11.50 \, seconds onumber\] and \[\sigma = \frac{(23-0)^{2}}{12} = 6.64\, seconds. onumber\] Note that theoretical mean and standard deviation are close to the middle and standard sample deviation in this example. Exercise \(\PageIndex{1}\) The data that follows is the number of passengers on 35 different charter fishing boats. The sample average is 7.9 and the standard sample deviation is 4.33. The data follows a uniform distribution where all values between zero and 14 are equally likely. Status of values a and \(b\). Write the distribution in the correct designation, and calculate the theoretical average and standard deviation. Table \(\PageIndex{2}\) 1 12 4 10 4 14 11 7 11 4 13 2 4 6 3 10 0 0 0 0 12 6 9 10 5 13 4 10 14 12 11 6 10 11 0 11 13 2 Response \(a\) is zero; \(b\) — \(14\); \(X \sim U (0, 14)\); \(\mu = 7\) passengers; \(\sigma = 4.04\) passengers Example 5.3.2A a. Refer to example 5.3.1. What is the likelihood of an accidentally selected eight-week-old baby smiling from two to 18 seconds? Answer a. Find \(P(2 < x < 18)\). \(P(2 < x < 18) = (\text{base})(\text{height}) = (18 – 2)\left(\frac{1}{23}\right) = \left(\frac{16}{23}\right)\). Picture \(\PageIndex{1}\) Exercise \(\PageIndex{2}\)B Find the 90th percentile for the smiling time of an eight-week-old baby. The answer b. Ninety percent of smiling times fall below the 90th percentile, \(k\), so \(P(x < k) = 0.90\) \[P(x < k)= 0.90\] \[(\text{base})(\text{height}) = 0.90\] \[(k−0)\left(\frac{1}{23}\right) = 0.90\] \[k = (23)(0.90) = 20.7\] Figure \(\PageIndex{2}\) Exercise \(\PageIndex{3}\)C. Find the likelihood of the occasional eight-week-old baby smiling for more than 12 seconds knowing the toddler is smiling for more than eight seconds. Answer c. This probability question is conditional. You are asked to find the likelihood of an eight-week-old baby smiling for more than 12 seconds when you already know the baby was smiling for more than eight seconds. Find \(P(x > 12 | x > 8)\) There are two ways to make this problem. For the first method, use the fact that it is conditional and changes the space of the sample. The graph illustrates the sample's new space. You already know that the child smiled for more than eight seconds. Write New \(f(x): f(x) = \frac{1}{23-8} = \frac{1}{15}\) for \(8 < x < 23\) \(P(x > 12 | x > 8) = (((23 − 12)\left(\frac{1}{15}\right) = \left(\frac{11}{15}\right)\) Picture \(\PageIndex{3}\) For the second method, use a conditional formula from probability themes with original distribution \(X\sim U(0,23)\): \(P(\text{A| B}) = \frac{P(\text{A AND B})}{P(\text{B})}\) For this problem \(\text{A}\) is (\(x > 12\)) and \(\text{B}\) is (\(x > 8\)). So \(P(x > 12|x > 8) = \frac{(x > 12 \text{ AND } x > 8)}{P(x > 8)} = \frac{P(x > 12)}{P(x > 8)} = \frac{\frac{11}{23}}{\frac{15}{23}} = \frac{11}{15}\) Picture \(\PageIndex{4}\): Darker shaded area denotes \(P(x > 12)\). Displays the entire shaded area \(P(x > 8)\). The \(\PageIndex{2}\) distribution is provided as \(X \sim U(0,20)\). What is \(P(2 < x < 18)\)? Find the 90th iper centile. Reply \(P(2 < x < 18) = 0.8\); 90th percentile \(= 18\) Example 5.3.3 The amount of time in a matter of minutes that a person has to wait for a bus is evenly distributed between zero and 15 minutes inclusive. Exercise \(\PageIndex{3}\).1 a. What is the probability that a person is waiting less than 12.5 minutes? Answer a. Let \(X =\) the number of minutes a person has to wait for the bus. \(a = 0\) and \(b = 15\). \(X \sim U(0, 15)\). Write the probability density function. \(f(x) = \frac{1}{15-0} = \frac{1}{15}\) for \(0 \leq x \leq 15\). Find \(P(x < 12.5)\). Draw a graph. \[P(x < k) = (\text{base})(\text{height}) = (12.5−0)\left(\frac{1}{15}\right) = 0.8333\] The probability that a person is waiting less than 12.5 minutes is 0.8333. Picture \(\PageIndex{5}\). Exercise \(\PageIndex{3}\).2 b. On average, how long should a person wait? Locate the average of \(\mu\) and the standard deviation\(\sigma\). Reply b. \(\mu = \frac{a+b}{2} = \frac{15+0}{2} = 7.5\). On average, a person should wait 7.5 minutes. \(\sigma = = \sqrt{\frac{(12-0)^{2}}{12}} = 4,3\). V.O. V.O. deviation is 4.3 minutes. Exercise \(\PageIndex{3}\).3 c. Ninety percent of the time, the time a person has to wait falls below what value? Note 5.3.3.3.1 This asks for the 90th percentile. Answer c. Find the 90th percentile. Draw a graph. Let \(k =\) the 90th percentile. \(P(x < k) = (\text{base})(\text{height}) = (k−0)\left(\frac{1}{15}\right)\) \(0.90 = (k)\left(\frac{1}{15}\right)\) \(k = (0.90)(15) = 13.5\) \(k\) is sometimes called critical value. The 90th percentile is 13.5 minutes. Ninety percent of the time, a person must wait no more than 13.5 minutes. Picture \(\PageIndex{6}\). Exercise \(\PageIndex{4}\) The total duration of major league baseball games in the 2011 season is evenly distributed between 447 hours and 521 hours inclusive. Locate \(a\) and \(b\) and describe what they represent. Write a distribution. Find the average and standard deviation. What is the probability that the duration of games for the team for the 2011 season is from 480 to 500 hours? What is the 65th percentile for the duration of games for the team for the 2011 season? The answer \(a\) is \(447\), and \(b\) is \(521\). a is the minimum playing time for a team for the 2011 season, and \(b\) is the maximum playing time for a team for the 2011 season. \(X \sim U(447, 521)\). \(\mu = 484\) and \(\sigma = 21.36\) Figure \(\PageIndex{1}\). \(P(480 < x < 500) = 0.2703\) The 65th percentile is 495.1 hours. Example 5.3.4 Suppose the time it takes a nine-year-old child to eat a doughnut is between 0.5 and 4 minutes inclusive. Let \(X=\) time, in minutes, it takes a nine-year-old child to eat a doughnut. Then \(X \sim U(0.5, 4)\). a. The probability that a randomly selected nine-year-old child is eating a doughnut in at least two minutes is ________.. Solution a. 0.5714 Exercise \(\PageIndex{4}\).1 b. Find the possibility that another nine-year-old child is eating a doughnut in more than two minutes given that the child has been eating a doughnut for more than 1.5 minutes. The second question has a conditional probability. You are asked to find the possibility that a nine-year-old child is eating a doughnut in more than two minutes given that the child has been eating a doughnut for more than 1.5 minutes. There are two different ways to fix the problem (see Example). You need to reduce the sample space. First method: Since you know that the child is already eating a doughnut for more than 1.5 minutes, you no longer start with = 0.5 minutes. The starting point is 1.5 minutes. Write new \(f(x)\): \(f(x) = \frac{1}{4-1.5} = \frac{2}{5}\) for \(1.5 \leq x \leq 4\). Locate \(P(x > 2|x > 1.5)\). Draw a graph. Picture \(\PageIndex{2}\). \(P(x > 2|x > 1.5) = (\text{base})(\text{new height}) = (4 − 2)(25)\left(\frac{2}{5}\right) =\) ? Answer b. \(\frac{4}{5}\) Probability that a nine-year-old child is eating a doughnut in more than two minutes given that the child has already ate more than 1.5 minutes is \(\frac{4}{5}\). Second method: Draw the original graph for \(X\sim U(0.5, 4)\). Use conditional formula \(P(x > 2 | x > 1.5) = \frac{P(x > 2 \text{AND} x > 1.5)}{P(x > 1.5)} = \frac{P(x>2)}{P(x>1.05)} = \frac{\frac{2}{3.5}}{\frac{2.5}{{2.5}{{3.5}} = 0.8 = \frac{4}{5}\) Exercise \(\PageIndex{5}\) Suppose the time it take a student to complete a test is evenly distributed between six and 15 minutes inclusive. Let \(X=\) time, in minutes, need a student to finish the quiz. Then \(X \sim U(6,15)\). Find the chance that it takes at least eight minutes for the student to complete the quiz. Then find the possibility that another student needs at least eight minutes to finish the quiz, given that it has already taken more than seven minutes. Answer \(P(x > 8) = 0.7778\) \(P(x > 8 | x > 7) = 0.875\) Example 5.3.5 Ace Heating and Air Conditioning Service believes that the amount of time a repairman should fix the oven evenly distributed between 1.5 and four hours. Let \(x=\) the time it takes to fix the oven. Then \(x \sim U(1.5, 4)\). Find the possibility that it takes more than two hours to accidentally repair the oven. Find the possibility that it takes less than three hours to accidentally repair the oven. Find the 30th percentile of the oven repair time. The longest 25% of the time of repair of the furnace took at least how long? (In other words: find the minimum time for the longest 25% repair time.) What percentile does this represent? Locate Medium and Standard Reject Solution a. To find \(f(x): f(x) = \frac{1}{4-1.5} = \frac{1}{2.5}\) yes \(f(x) = 0.4\) \0.4\) \0.4\) \0.4\) \0.4\) \0.4\) \0.4\) \0.4\) \0.4\) \0.4\) \0.4\). (P(x > 2) = (\text{base})(\text{height}) = (4 - 2)(0.4) = 0.8\) Figure \(\PageIndex{3}\). Even distribution from 1.5 to four with a shaded area of two to four, which indicates the probability that the recovery time \(x\) exceeds two b. \(P(x < 3) = (\text{base})(\text{height}) = (3 – 1.5)(0.4) = 0.6\) The rectangle graph showing the entire distribution will remain unchanged. However, the graph should be shaded between \(x = 1.5\) and \(x = 3\). Note that the shaded area starts with \(x = 1.5\), not \(x = 0\); because \(X \sim U(1.5, 4)\), \(x\) cannot be less than 1.5. Picture \(\PageIndex{4}\). A uniform distribution of 1.5 to four with a shaded area of 1.5 to three, indicating the probability that the recovery time \(x\) is less than three c. Figure \(\PageIndex{5}\). The uniform distribution between 1.5 and 4 of the 0.30 area is shaded to the left, representing the shortest 30% of the repair time. \(P(x < k) = 0.30\) \(P(x < k) = (\text{base})(\text{height}) = (k – 1.5)(0.4)\) \(0.3 = (k – 1.5) (0.4)\); Decide to find \(k\): \(0.75 = k – 1.5\), obtained by dividing both sides by 0.4 \(k = 2.25\), obtained by adding 1.5 to both sides of the 30th percentile of the repair time 2.25 hours. 30% repair time 2.25 hours or less. d. Picture \(\PageIndex{6}\). The uniform distribution between 1.5 and 4 of the 0.25 area is shaded to the right, representing the longest 25% of the repair time. \(P(x > k) = 0.25\) \(P(x > k) = (\text{base})(\text{height}) = (4 – k)(0.4)\) \(0.25 = (4 – k)(0.4)\); Solve for \(k\): \(0.625 = 4 − k\) obtained by dividing both sides by 0.4 \(−3.375 = −k\) obtained by vienna Four on both sides: \(k = 3.375\) The longest 25% repair furnaces take at least 3,375 hours (3,375 hours or longer). Note: Since 25% of repair times are 3,375 hours or longer, this means that 75% of the repair time is 3,375 hours or less. 3,375 hours – 75th percentile of the time of repair of the furnace. e. \(\mu = \frac{a+b}{2}\) and \(\sigma = \sqrt{\frac{(b-a)^{2}}{12}}\) \(\mu = \frac{1.5+4}{2} 2.75\) hours and \(\) sigma = \sqrt{\frac{(4-1.5)^{2}}{12}} = 0.7217\) hours Exercise \(\PageIndex{6}\) Amount of time, during the time during which the service technician must change the oil in the car, evenly distributed between 11 and 21 minutes. Let \(X=\) the time it takes to change the oil on the car. Write a random variable \(X\) in words. \(X =\)

Distribution schedule. Find \(P(x > 19)\). Find the 50th percentile. Answer Let \(X=\) the time it takes to change the oil in the car. \(X \sim U(11, 21)\). Picture \(\PageIndex{7}\). \(P(x > 19) = 0.2\) the 50th percentile is 16 minutes. If \(X\) has a uniform distribution where \(a < x < b\) or \(a \leq x \leq b\), then \(X\) accepts values between \(a\) and \(b\) (may include \(a\) and \(b\)). All values \(x\) are equally likely. We write \(X \sim U(a, b)\). The average value \(X\) is \(\mu = \frac{a+b}{2}\). The default deviation \(X\) is \(\sigma = \sqrt{\frac{(b-a)^{2}}{12}}\). Probability density function \(X\) is \(f(x) = \frac{1}{b-a}\) for \(a\leq x \leq b\). The cumulative distribution function \(X\) is \(P(X\leq x) = \frac{x-a}{b-a}\). \(X\) is continuous. Picture \(\PageIndex{8}\). The probability of \(P(c < X < d)\) can be found by calculating the area in \(f(x)\), between \(c\) and \(d\). Since the corresponding area is a rectangle, the area can be found simply by multiplying the width and height. \(X=\) the true number between \(a\) and \(b\) (in some cases\(X\) may assume the values \(a\) and \(b\)). \(a =\) smallest \(X\); \(b =\) largest \(X\) \(X \sim U(a, b)\) The mean is \(\mu = \frac{a+b}{2}\) The standard deviation is \(\sigma = \sqrt{\frac{(b-a)^{2}}{12}}\) Probability density function: \(f(x) = \frac{1}{b-a} \text{for} a \leq X \leq b\) Area to the Left of \(x\): \(P(X < x) = (x – a)\left(\frac{1}{b-a}\right)\) Area to the Right of \(x\) : P(\(X\) > \(x\)) = (b – x)\(\left(\frac{1}{b-a}\right)\) Area Between \(c\) and \(d\): \(P(c < x < d) = (\text{base})(\text{height}) = (d – Uniform: \(X \sim U(a, b)\) where where < x < b\) pdf: \(f(x) = \frac{1}{b-a}\) для \(a\leq x \leq b\) cdf: \(P(X \leq x) = \frac{x-a}{b-a}\) означає \(\mu = \frac{a+b}. {2}\) стандартне відхилення \(\sigma = \sqrt{\frac{(b-a)^{2}}{12}}\) \(P(c < X < d) = (d – c)\left(\frac{1}{b-a}\right)\) Макдугалл, Джон А. Програма McDougall для максимального схуднення. Шлейф, 1995. Щоб відповісти на наступні десять запитань, скористайтеся наведеною нижче інформацією. Дані, які слідують квадратні кадри (в 1000 футів в квадраті) з 28 будинків. 1,5 2,4 3,6 2,6 1,6 2,4 2,0 3,5 2,5 1,8 2,4 2,5 3,5 4,0 2,6 1,6 2,5 2 1,8 3,8 2,5 1,5 2,8 1,8 4,5 1,9 1,9 3,1 1,6 Середній вибірка = 2,50 і стандартне відхилення вибірки = 0,8302. Дистрибутив можна написати як \(X \sim U(1.5, 4.5)\). Вправа \(\PageIndex{7}\) Який тип розподілу це? Вправа \(\PageIndex{8}\) У цьому дистрибутиві результати однаково ймовірні. Що це означає? Відповідь Це означає, що значення x так само ймовірно, буде будь-яке число між 1.5 і 4.5. Вправа \(\PageIndex{9}\) Яка висота \(f(x)\) для безперервного розподілу ймовірності? Вправа \(\PageIndex{10}\) Які обмеження для значень \(x\)? Відповідь \(1.5 \leq x \leq 4.5\) Вправа \(\PageIndex{11}\) Графік \(P(2 < x < 3)\). Вправа \(\PageIndex{12}\) Що таке \(P(2 < x < 3)\)? Answer 0.3333 Exercise \(\PageIndex{13}\) What is \(P(x < 3.5 | x < 4)\)? Exercise \(\PageIndex{14}\) What is \(P(x = 1.5)\)? Answer zero Exercise \(\PageIndex{15}\) What is the 90th percentile of square footage for homes? Exercise \(\PageIndex{16}\) Find the probability that a randomly selected home has more than 3,000 square feet given that you already know the house has more than 2,000 square feet. Answer 0.6 Exercise \(\PageIndex{17}\) What is \(a\)? What does it represent? Exercise \(\PageIndex{18}\) What is \(b\)? What does it represent? Answer \(b\) is \(12\), and it represents the highest value of \(x\). Exercise \(\PageIndex{19}\) What is the probability density function? Exercise \(\PageIndex{20}\) What is the theoretical mean? Answer six Exercise \(\PageIndex{21}\) What is the theoretical standard deviation? Exercise \(\PageIndex{22}\) Draw the graph of the distribution for \(P(x > 9)\). Відповідь Малюнок \(\PageIndex{9}\). Вправа \(\PageIndex{23}\) Знайти \(P(x > 9)\). Вправа \(\PageIndex{24}\) Знайдіть 40-й процентиль. Відповідь 4.8 Щоб відповісти на наступні одинадцять вправ, скористайтеся наведеною нижче інформацією. Вік автомобілів на стоянці персоналу приміського коледжу рівномірно розподіляється від шести місяців (0,5 років) до 9,5 років. Вправа \(\PageIndex{25}\) Що тут вимірюється? Вправа \(\PageIndex{26}\) У словах визначте випадкову змінну \(X\). Відповідь \(X\) = Вік (в роках) автомобілів на стоянці персоналу Вправа \(\PageIndex{27}\) Чи є дані дискретні або безперервні? Вправа \(\PageIndex{28}\) Інтервал значень для \(x\) становить ______. Відповідь від 0,5 до 9,5 вправи The distribution for \(X\) is ______. Exercise \(\PageIndex{30}\) Write the probability density function. Reply \(f(x) = \frac{1}{9}\), where \(x\) is 0.5 to 9.5, inclusive. Activity \(\PageIndex{31}\) Probability Distribution Graph. Draw a probability distribution graph. Picture \(\PageIndex{10}\). Define the following values: Smallest value for \(\bar{x}\): _______ Highest value for \(\bar{x}\): _______ Rectangle Height: ________ Label for x (words): _______ Axis label y (words): _______ Exercise \(\PageIndex{32}\) Find the average age of cars in the lot. Answer \(\mu\) = 5 Exercise \(\PageIndex{33}\) Find the probability that the randomly selected car in the lot was less than four years old. Draw a graph and shade the sphere of interest. Picture \(\PageIndex{11}\). Find a probability. \(P(x < 4) =\) _______ Exercise \(\PageIndex{34}\) Considering only cars less than 7.5 years old, look for the probability that the randomly selected car in the lot was less than four years old. Draw a graph, shade the sphere of interest. Picture \(\PageIndex{12}\). Find a probability. \(P(x < 4 | x < 7.5) =\) _______ Answer Check the student's decision. \(\frac{3.5}{7}\) Exercise \(\PageIndex{35}\) What has changed in the previous two problems that have made the solution different Exercise \(\PageIndex{36}\) Find the third quartile of car age in the lot. This means that you have to find a value such that \(\frac{3}{4}\), or 75%, of a car no larger than (less than or equal to) that age. Draw a graph and shade the sphere of interest. Picture \(\PageIndex{13}\). Locate \(k\) so that \(P(x < k) = 0.75\). Third quartile -______ Answer Check student decision. \(k = 7.25\) \(7.25\) The conditional probability that the event will take place given that another event of Contributors and Attribution Barbara Ilovski and Susan Dean (De Anza College) has already taken place with many other authors contributing.