Trigonometric integration examples pdf

10.3 Integrating trigonometry functions Return to content move to problems and solutions 1. Major trigonometry integrals Recall from Section 10.1 Part 4 of that: For sec x: We group the main trigonometry integrals together here in the next field. Example 1.1 Rate: EOS's solution to go to problems and solutions to go back to the top of page 2. Trigonometry Replacement Score: Solution Let u and sin 3x. Then du 3 cos 3x dx, so cos 3x dx (1/3) du. Thus: EOS Replacement y and Sin 3x includes trigonometry function, and as a result, is called trigonometry replacement. Example 2.2 Find: Let's Solution and x2. Then du th 2x dx, so x dx (1/2) du. Thus: EOS Replacement u and x2 does not include any trigonometry function. There is no trigonometry replacement. Integrals, which include trigonometry functions, are not always processed with trigonometry replacement. Note that sin x2 is sin (x2), sinus x2, not (sin x)2, denoted sin2 x, square sin x. Return to the top of page 1. Rate: Solution Let u No. 1 and sin x. So du th ish x dx. So: Return to the top of page 2. Calculate: Solution Let y ln t. t so du y (1/t) dt. So: Return to the top of page 3. Calculate: Solution Let v No 2 and sin 3u. Then dv No 3 cos 3u du, so cos 3u du (1/3) dv. So: Return to the top of page 4. Find: Return solution at the top of page 5. a. Set the following identifiers: Solution a. We have cos (x - y) - cos x cos y - sin x sin y and cos (x- y) - cos x cos y - sin x sin y. So cos (x - y) - cos (x q y) 2 sin x sin y, which gives: Return to the top of the page return to content In this section we look like a variety of products. These integrals are called trigonometry integrals. They are an important part of an integration technique called trigonometry replacement, which is shown in trigonometry replacement. This method allows us to convert algebraic expressions that we cannot integrate into expressions related to trigonometry functions that we can integrate through the methods described in this section. In addition, these types of integrals often appear when we study polar, cylindrical and spherical coordinate systems later. Let's start our study with products, in that we (sin x) and (sic x.) The key idea of the strategy used to integrate combinations of products and powers (sin x) and (sic x) includes rewriting these expressions in the form of amounts and differences of integrals form (∫'sin'jx'cos x'dx') or (∫ cos'jx'sin'd'd'). After rewriting these integrals, we evaluate them using u-replacement. Before describing the overall process in detail, let's look at the following Example : Integration (PageIndex{1}): Integration ((display ∫ cos'j x'sin x'sin x',dx)) Rating Rating ∫'cos'3x'sin x',dx.)) Use the solution (u)-replacement and let (u'cos x). In this case, ∫'cos'3x'sin x,dx'∫u'3, du'frac{1}{4}u'4'C'frac{1}{4} cos'4x-C.onumber)Exercise (PageIndex{1}) Rate (display ∫ {1}) Sysin4x-cos,xx.) Hint Let (u'sin x.)) Answer (Displaystyle ∫'sin'4x'cos x', dx frac{1}{5}sin5xC) Example (PageIndex{2}) : Preliminary example: Integration : Integration (∫-cos'jx'sin'kx,dx), where (c) is an odd score ((display ∫'cos'2x'sin'3x,3x,3x,dx.)) the decision to convert this integral into integral form ((displaystyle ∫'cos'jx'sin x',dx)) to rewrite (sin 3x'sin'2x'sin x)) and make a substitution (sin 2x'1'cos'2x.)) Thus, (display style (beginning) ∫ cos'2x'sin,dx ∫cos'2x (1'cos'2x)sin x', dx; (text, that dusin x',dx. 4pt) ∫u2 (1'u'2)∫, du 4'u'2), du 4pt frak{1}{5}u'frac{1}{3}u'3'C 4pt (frak{1}{5} Exercise 5x-frac{1}{3} cos3x-C. Exercise (PageIndex{2}) Rate (displaystyle ∫cos'3x) sin'2x,dx.)) Hint Write (Cos'3x'cos'2x'cos x'(1'sin'2x) In the following example ∫ we see a strategy{1}{3} that should be applied when there is only force (Sin x) in the following example we see a strategy{1}{5} that should be applied when there is only force (Sin x) and (cos x). For integrats of this type, the identity of sin2xfrac{1}{2}frac{1}{2} cos (2x)frac1'cos (2x){2} are invaluable and cos2x-frac{1}{2}frac{1}{2} cos (2x) frac 2xs 2x {2} . These identities are sometimes called identity identities, reduced power, and they can be derived from two-corner identity (cos (2x) and Pythagoran Identity (Cosa) Example: Integration{3} of The Oving Force (Sin x) Assessment ((display ∫'sin'2x, dx). Let's use trigonometry identity (sin2x'frac{1}{2}'frac{1}{2} cos (2x).) So Exercise (∫'sin'2x), dx-∫ (frak{1}{2}-frak{1}{2}'cos (2x) (right) , dx'frac{1}{2}x-frac{1}{4}'sin (2x) ((PageIndex{3})) (display style ∫'cos'2x,dx.)) Tip (Cos-2x'frac{1}{2}frac{1}{2}'cos (2x)) ∫ the following set{1}{4}{1}{2} guidelines summarize the overall process of product integration of powers: sin x and k (cos x) - general process of food integration. Use the following strategies: 1. If (c) strange, rewrite (sinkx'sin'k'1'x'sin x)) and use identification (sin'2x1'cos'2x) to rewrite (Sin'k'1'x) in terms of (cos x). Integrate with a replacement (u'cos x). This is does (do-sin x',dx.) 2. If (j)) strange, rewrite (cox'j'j'1'cos x' x' x) and use identification (cos'2x'1'sin'2x) rewrite (Cos'j'1'x) in terms of (x sin). Integrate with a replacement (u'sin x). This substitution makes q (du'cos x',dx.) (Note: If both (j) and q (k) are odd, either Strategy 1 or Strategy 2 can be used.) 3. If both (J) and (c) are oatmeal, use (Sin) (1/2) (1/2) cos (2x)) and (Kos-2x) (1/2) After applying these formulas simplify and re-apply strategies from 1 to 3 as needed. Example : Integration (PageIndex{4}): Integration (∫-cos'jx'sin'kx,dx), where (k) is the odd score (((display ∫'cos'8x'sin'5x,dx.)) Solution Since power on (sin x) is a strange, use strategy 1. Thus, (the start ∫cos'8x'sin,5x,dx ∫cos'8x'sin4x'sin x,dx 4pt (∫ cos'8x) 4pt (∫ cos'8x (8x(1 ∫' . . . . He said that I was the one who was the one who was the one who was the one who was not 4pt ∫ (8'8'2u'{10}'u'{12}) 4pt Frac{1}{9}u9'frac{2}{11}u'{11} Frak{1}{13}u{13}K Assess integral. 4pt frak{1}{9} cos9xfrac{2}{11}cos{11}xfrac{1}{13} cos'{13} ∫ example XC (PageIndex{5}): Integration (∫ cos'jx'sin'kx, dx), where even the (k) and (j) solution are even evaluated: Since the force on sin x) is even (k4x), and power on q ((j'0), and power on q (cos x) even (j'0), we should use strategy 3. Thus (∫-sin,dx-∫ (sin)2x (∫ (frak{1}{2}-frak{1}{2}{1}{4} ∫{1}{2}-kos (2x){1}{2}) frak{1}{2}co (2x)frak{1}{4}cos2 (2 (frak{1}{2}frak{1}{2}kos (2x)∫) {1}{4}-frak{1}{2}-kos (2x){1}{4} (Frak{1}{2}-frak{1}{2} cos (4x)) (2 frac{1}{2}'frac{1}{2} cos (4x)) : ∫ (frak{3}{8}frac{1}{2}'cos (2x) 'frac{1}{8} cos (4x)) (Frak{3}{8}x-frac{1}{4}'sin (2x) frak{1}{32} sin (4x)) Rate integral. Exercise ((PageIndex{4}) Assess the strategy ∫ use 2. Write (cos)2x'cos x) and replace (kos-2x1'sin'2x.)) Answer ((displaystyle ∫'cos'3x), dx sin x'frac{1}{3}'sin'3x-C) Exercise (PageIndex{5})) Rate ∫ strategy to use Hint 3. In some areas{1}{2} physics ∫{1}{2}{1}{2} such as mechanics, quantum mechanics{1}{12} and calculations of the Fourier series, it is often necessary to integrate products that include (sin (ax), sin (bx), cos (ax), and (cos (bx).)) These integrals are evaluated by applying trigonometry identities as stated in the following rule. Rule: Integration of Sines and Cosines products of different angles To integrate products that include (sin (ax), sin (bx), cos (ax), and s (cbs (bx))) use substitutions sin (ax) sin (bx) frac{1}{2}'cos (a'b){1}{2} (a'b) (a'b) ax) cbs (bx) frak{1}{2} sin ((a'b)x) frac{1}{2} sin (a'b)x{1}{2}) ({1}{2} Example{6}: Score (∫'sin (ax) cos (bx),dx)) Score (Displaystyle ∫'sin (5x) cos (3x)) Solution: Apply Identity (Sin (5x), Cos (3x) Frak{1}{2} Sin (2x) Frak{1}{2} Sin (8x ∫). sin (5x),cos(3x),dx-∫-frac{1}{2}'sin (2x{1}{2}) {1}{4}-cos (2x) frak{1}{16} cos (8x) C.) Exercise (PageIndex{6}) Rate display (∫ cos (6x) cos (5x) ,dx.)) Replacement Tip (Kos (6x) Cos (5x) frak{1}{2}'cos x'frac{1}{2} cos (11x).) Answer (displaystyle ∫ cos (6x)cos (5x),dx frac{1}{2} sin'x'frac{1}{22}'sin (11x) : 1. (display ∫ 2x,dx'tan x-C) 2. (display ∫ x'tan x',dx'sec x'C) 3. (display ∫ x,dx'ln'sec x'C) 4. ∫ For most integral products and abilities (x/t) and (sec x)) for most integral products and the authority to integrate in the form of the amount or difference of form integrats ((display style ∫'tan'jx'sec'2x,dx) or ((display ∫'sec'tan x'dx). we can evaluate these new integrals using u-replacement. Example: Score (PageIndex{7}): Score (∫-sec'jx'tan x',dx)) Score (display ∫'sec-5 x'tan x',dx.)) Solution: Start by rewriting (sec.5 x'tan x) as sec.4 x' sec x'tan x.)) (((the start style display is aligned ∫ sec.5x'tan x,dx ∫'sec4 x'sec x'tan x'tan x'tan x,dx , Due sec x x, dx. 4pt{1}{5} (∫u'4), text Rate integral. On this website you can read some interesting information on this website{1}{5} to learn about the common integral with the participation of the secant. Exercise (PageIndex{7})) ((display ∫'tan'5x-sec'2x,dx.)) Tip Let (u'tan x) and (du'sec) 2 x.)) The answer to the question of what we ∫ (display ∫'tan'5x-sec,dx, dx and tfrac{1}{6}6x-C) Now we look at different to integrate the products and capabilities of Sek x and (Tan x.) Problem-solving strategy: Integration (displaystyle ∫tan'kx'sec'jx,dx)) For integration (displaystyle ∫'tan'cx'sec'jx,dx,) Use the following strategies: 1. If (j)) is an ovedia and (j≥2), rewrite (sec'j'sec'j'j'2'x'2x) and use (sec.2x'tan'2x'1) to rewrite (sec'j'2'x) in terms of (xx). Let (waton x) and (du-sec 2x.) 2. If (c) is strange and (j≥1), rewrite (tan-kksex-sex-yukanak-1'x'sec'j'1'x'sec x'x'tan) and use (2x2x2x2x1) to rewrite ((Taneka) in terms of sec x). Let (uex x) and (do-sec x'tan x'tan x',dx.) (Note: If (j) is even and (to) strange, either Strategy 1 or Strategy 2 can be used.) 3. If (to) strange, where (k≥3) and (j'0), rewrite (tan-k-tan'k-2'x'tan'2x'tan'k'2'x (Sec-2x-1) tan-2'x'x'sec-2x-tan'k-2'x.x.) may need to repeat this process in the term tan2. If (c) is edit and th (j) strange, then use for the expression (tan)) in terms of sec x). Use part-to-part integration to integrate the odd powers in the example (PageIndex{8}): Integration (∫-tan'kx'sec'jx,dx) when (j)) is even an estimate ((displaystyle ∫'tan'6x'sec'4x.,,4x.,,dx.) Decision From the moment you place power to rewrite (sec.4x'sec.2x-sec)) and use (sec.2x'tan'2x'1) to rewrite the first (sec.2x) in terms of (tan x.)) Thus - (∫-tan-6x-sec., dx-∫-tan-6x (2x2x1)-sec.2x,dx)Let (hue x) and (duex-2x.∫) (∫ (u'8'u'6),) Appreciate integral. (Frak{1}{9}-9frac{1}{7}u'7'C) Replacement (Tan Heu). (Frak{1}{9}-Tan{9}9x-frak{1}{7}-tan-7x-C.) Example :(PageIndex{9}): Integration (∫'tan'kx'sec'jx,dx) when (k) is the odd score ((display)style ∫'tan'5x-sec-3x,dx.)) Solution Since power on q (tan x)) is strange, start by rewriting ('tan'5x'sec'3x'tan'4x'sec'2x'sec x'tan x'tan x.) (Tan-5x-sec.3x'tan-4x-2x'sec x'tan x.) (tan-4x) (∫-5x-sec-3x,dx-∫ (zagar-2x) 2 sec.2 sec tan x'dx)) Use (2x2x2x22 sec.) 1.)) (∫ Sec-2x2x1) 2 sec.2 x2 sec x-tan x,ds) Let (uex x) and (dua sec xu tan x', dks). (∫ (u-2)2u-2du) Expand. (∫ (u'6'2u'4'u'2)du') Integration. (Frak{1}{7}u'7'frac{2}{5}u'5'frac{1}{3}u'3'C) Replacement (sec xu.) c{1}{7}'sec-7x-frac{2}{5}'sec-5x-frac{1}{3}'sec-3x-C) Example (PageIndex{10}):Integration (∫tankx,dx), where (c)) is odd and (k≥3)) Rate (displaystyle ∫tan3x,dx.) starts with rewriting (3x-tan x'tan-2x'tan x (Sec-2x-1) (∫-tan-3x,dx-∫ (Tan h-sec/2x-tan x ∫) 2x,dx-∫'tan x'dx)) For the first integral{1}{2}, Use a replacement H.K.) Use a formula for the second integral. Example : Integration (PageIndex{11}): Integration ((displaystyle ∫'sec'3x,dx)) Integration (displaystyle ∫'3x,dx.)) This integral solution requires integration piece by piece. For starters, let (u'sec x) and (dv'sec'2x). These options do (dua sec x'tan x) and q (v'tan x). Thus, simplification: (∫-sec x'tan x'tan x'∫'tan x'sec x'tan x'tan',dx') (sec x'tan x'tan'∫ tan-2x'sec x'dx) Simplification. (Sec x-tan x-∫ (Sec 2x1) sec x, ds) Replacement (2x2x2x2x1.) (sec x'tan x'∫'sec x',dx-∫-sec'3x)... Overwrite. (Sec x-tan x'l'sec x'tan x'tan x'∫'sec-3x,dx.) ∫) Grade (∫ sec x',dx.)) Now we have since ∫ on the right side re-emerging integral style ∫'sec'3x, dx, dx, we can solve it for q (displaystyle ∫'sec'3x, dx) by adding it to both sides. By doing so, we get 2∫ sec.3x, dx'sec x'tan x'l'sec x'tan x'tan x'tan x'tan x.tan x. Onumber' Division 2, we come to ∫ sec3x,dx'frac{1}{2}'sec x'tan x'frac{1}{2}'ln' sec x'tan x'Conumber Exercise (PageIndex{8}) Assess ∫ as a guide to using a hint (PageIndex{9}) as a guide. Answer to this question : (display ∫'tan-3x-sec'7x,dx{1}{9} (frac{1}{9}'sec-9x'frac{1}{7}'sec'7x-C) Score ((display ∫'sec'nx), dx) for values (n) in the place where In addition, we also need to know the value of q (displaystyle ∫'sec'n'2'x',dx) for evaluation (displaystyle ∫ ∫'sec'nx,dx ∫). ,dx). To facilitate the process, we can get and apply the following power reduction formulas. These rules allow us to replace the power integral (sec x) or (Tan x) with a lower power integral (sec x) or (qten x.) Rule: Formula Reduction for (∫-sec,dx) and (∫ nx'dx)) (∫ sec.) , dx-frak{1} n-1'sec'n'2'x'x'2'x'c'n'2'n'1'∫'sec'n'2'x, dx'∫ n'1'tan'n x'n x'n'frac{1}'n'1'tan'1'x'∫'tan'n'2'x'x'x', dx', the first power reduction rule can be tested by the application of the integration. The second can be tested by following the strategy outlined for the integration of the odd powers (tan x.) Example (PageIndex{12}): Review (∫ sec.,dx)) Apply a reduction formula for evaluation ((displaystyle ∫'3x,dx.) Solution: Applying the first shortening formula, we get (∫-sec.3x,dx'frac{1}{2}'sec x'tan x'frac{1}{2}∫'sec x x,dx) (Frak{1}{2} sec x-tan x-frak{1}{2}xen x-tan h's-S.) Example (PageIndex{13} In): Using The Cut-Up (display ∫'tan'4x,4x.) Solution: Application of the shortening formula for (∫-tan'4x, dx) we have (Frak{1}{3}-tan-3h (Tan x-∫-tan,0h,0x)) Apply the reduction formula to (∫ 2x,dx.) (frak{1}{3}-tan 3x-tan x-∫1',dx) Simplification. (Frak{1}{3}-tan 3x'tan x'x'C) Score (∫1,dx) Exercise (PageIndex{9})) Apply the reduction formula to q ((displaystyle ∫'sec'5x,dx.) Hint Use Formula 1 and let (n'5 .)) Answer (Displaystyle ∫'sec'5x,dx'frac{1}{4}'sec'3x'tan x'frac{3}{4}∫'sec'3x) To integrate products involving (sin (ax), , Kos (axe), and (cos (bx),) use replacements. (Sin (ax) Sin (bx) frak{1}{2}'kos ((a'b)x)-frak{1}{2}'cos ((a'b)))) Kos (bx) frak{1}{2} sin (a'b)x) frak{1}{2} sin ((a'b)x) (Kos (ax)) Frak{1}{2}co ((a'b)x) frak{1}{2}'kos (a'b)x) displaystyle ∫ sec'nx, dx frak{1}-n-1'sec.2'x 'tan x-fraq'2'n1∫ sec.h,dx)) (display ∫-tan-n{1} dx'frac{1} n'1 ∫'tan'∫ nx, dx'frac{1}'n-1'tan'1'1'1'1'1'x-∫'n'2'x' ,dx') trigonometric integration examples and solutions. trigonometric integration examples pdf. basic trigonometric integration examples. solved examples integration of trigonometric functions. integration by trigonometric substitution examples and solutions. integration of inverse trigonometric functions examples. integration by trigonometric substitution examples and solutions pdf. integration of trigonometric functions examples