Geometric distribution worksheet answer

There are three main characteristics of the geometric experiment. There are one or more Bernoulli trials with all the setbacks except the last, which is success. In other words, you keep repeating what you are doing until the first success. Then you stop. For example, you throw an arrow at a bullseye until it hits the bullseye. The first time you come across a bullseye is a success, so you stop throwing darts. It could take six tries to get the bullseye. You may think of exams as failure, failure, failure, failure, success, STOP. In theory, the number of trials could go on forever. There must be at least one attempt. The probability of p success and probability of failure q is the same for each trial. $p + q = 1$ and $q = 1 − p$. For example, the probability of rolling three when you throw one fair to die is $\dfrac{1}{6}$. That's right, no matter how many times you roll to die. Supposing you want to know the probability of getting the first three on the fifth roll. On rolls one to four, you don't get a face with three. The probability for each of the cylinders is $q =\dfrac{5}{6}$, probability of failure. The probability of getting three on the fifth roll is $\left(\dfrac{5}{6}\right)\left(\dfrac{5}{6}\right)\left(\dfrac{5}{6}\right)\left(\dfrac{5}{6}\right)\left(\dfrac{1}{6}\right) = 0.0804$. $X =$ number of independent attempts up to the first success. You play a game of chance that you can either win or lose (there are no other options) until you lose. Probability of loss is $p = 0.57$. What is the probability that it will take five games to lose? Leave $X =$ the number of games you play until you play (includes the game you lost). Then X takes over the values 1, 2, 3, ... (could continue indefinitely). The probability question is $P(x = 5)$. Example $\PageIndex{1}$ You play a game of chance that you can win or lose (there are no other options) until you lose. Probability of loss is $p = 0.57$. What is the probability that it will take five games to lose? Leave $X =$ the number of games you play until you play (includes the game you lost). Then X takes over the values 1, 2, 3, ... (could continue indefinitely). The probability question is $P(x = 5)$. Exercise $\PageIndex{1}$ You throw arrows on the board until you find the center area. The probability of hitting the central area is $p = 0.17$. You want to find the probability that it takes eight throws before you hit the middle. What values X takes over? Answer $1, 2, 3, 4, \dotsc, n$. It can go on indefinitely. Example $\PageIndex{2}$ A safety engineer believes that 35% of all industrial accidents at his plant are caused by employees not following the instructions. She decides to look at the reports of the accident (selected randomly and replaced in a pile after reading) until she finds which shows an accident caused by employees not following the instructions. On average, how many reports would a safety engineer expect to look at until he finds a report showing an accident caused by an employee failing to follow instructions? How likely is it that a safety engineer will have to review at least three reports until they find a report showing an accident caused by an employee's failure to follow instructions? Flight X = number of accidents that the safety engineer must examine until he finds a report showing the accident caused by the employee's failure to follow the instructions. X takes over 1, 2, 3, ... The first question will ask you to find the expected value or mean value. The second question will ask you to find $P(x \geq 3)$. (At least the symbol means greater than or equal). Exercise $\PageIndex{2}$ The instructor believes that 15% of students get a C on their final exam. She decides to look at the final exams (randomly selected and replaced in a pile after reading) until she finds one that shows a degree below C. We want to know the probability that the instructor will have to examine at least ten exams until she finds one with a class below C. What is the probability of the question listed mathematically? Answer $P(x \leq 10)$ Example $\PageIndex{3}$ Suppose you are looking for a student in your high school who lives within five miles of you. You know, 55% of the 25,000 students live within five miles of you. You randomly contact college students until one says they live within five miles of you. What is the probability that you will need to contact four people? This is a geometric problem because you may have a number of setbacks before you have the one success you desire. Also, the probability of success remains the same every time you ask a student whether he or she lives within five miles of you. There is no certain number of attempts (how many times you ask a student). Whatever $X =$ number ______________ you need to ask _____________ one says yes. What values X takes over? What are p and q? The probability question is $P($_______$)$. Workaround Let $X =$ the number of students you must ask until one says yes. 1, 2, 3, ..., (total number of students) $p = 0.55, q = 0.45$ $P(x = 4)$ Tutorial $\PageIndex{3}$ You need to find storage that contains a special printer ink. You know that from stores that carry printer ink, 10% of them carry special ink. You randomly call each store until one has the ink you need. What are p and q? Answer $p = 0,1$ $q = 0,9$ Geometric Notation: $G =$ Geometric probability distribution function $X \sim G(p)$ Read as X is a random variable with geometric split. The parameter is p; $p =$ probability of success for each trial. Example $\PageIndex{4}$ Assume probability the defective computer component is 0,02. Components are randomly selected. Find the probability that the first error is caused by the seventh component tested. How many components do you expect to test until it came out that one is faulty? Flight X = number of computer components tested until the first error was found. X takes over values 1, 2, 3, ... where $p = 0.02$. $X \sim G(0.02)$ Find $P(x =7)$. $P(x = 7) = 0.0177$. To determine the probability that $x = 7$, Type 2nd, DISTR Scroll down and select geometpdf(Press ENTER Enter 0.02, 7); press ENTER to display the result: $P(x = 7) = 0.0177$ To determine the probability of $x \leq 7$, follow the same instructions BESIDES to select E: geometcdf as the distribution function. The probability that the seventh component is the first error is 0.0177. The chart $X \sim G(0.02)$ is: Picture $\PageIndex{1}$ The y-axis contains the probability x where $X =$ the number of computer components tested. The number of components you would expect to test until you find the first defect is $\mu = 50$. The formula for the mean is $\mu =\dfrac{1}{\text{p}} = \dfrac{1}{0.02} = 50$ The variance formula is $\sigma^{2} =\left(\dfrac {1}{p}\right)\left(\dfrac{1}{p} - 1 \right) = \left(\dfrac{1}{0.02}\right)\left(\dfrac{1}{0.02} - 1 \right) = 2,450$ Standard deviation is $\sigma= \sqrt{\left(\dfrac{1}{p}\right)\left(\dfrac{1}{p} - 1\right)} = \sq {\left(\1dfrac{1}{0.02}\right)\left(\dfrac{1}{0.02} - 1\right)} = 49.5$ Exercise $\PageIndex{4}$ The probability of a faulty steel rod is 0,01. Steel bars shall be selected at random. Find the probability of the first defect occurring on the ninth steel rod. For an answer, use the TI-83+ or TI-84 calculator. Answer $P(x = 9) = 0.0092$ Example $\PageIndex{5}$ The lifetime risk of pancreatic cancer is about one in 78 (1.28%). Let $X =$ the number of people you ask while one says he or she has pancreatic cancer. Then X is a discrete random variable with geometric distribution: $X \sim G\left(\dfrac{1}{78}\right)$ or $X \sim G(0.0128)$. What is the probability that you ask ten people how one says he or she has pancreatic cancer? What is the probability that you need to ask 20 people? Find (i) medium and (ii) standard deviation X. Answer $P(x = 10) = \text{geometpdf}(0.0128, 10) = 0.0114$ $P(x =20) = \text{geometpdf}(0.0128, 20) = 0.01$ Mean $= \mu = \dfrac{1}{p} = \dfrac{1}{0.0128} = 78$ Standard deviation $= \sigma = \sqrt{\dfrac{1-p}{p^{2}}} = \sqrt{\dfrac{1-0.0128}{0.0128^{2}}} \approx. 77.6234$ Exercise $\PageIndex{5}$ The nation's literacy rate measures the proportion of people aged 15 and over. who can read and write. The literacy rate for women in Afghanistan is 12%. Let $X =$ the number of Afghan women you ask while one says she is literate. What probability distribution X? How likely are you to ask five women before one says she's literate? How likely are you to ask ten women? Find (i) medium and (ii) standard deviation X. Answer $X \sim G(0.12)$ $P(x = 5) = \text{geometpdf}(0.12, 5) = 0.0720$ $P(x = 10) = \text{geometpdf}(0,12, 10) = 0.0380$ Mean $= \mu = \dfrac{1}{p} = \dfrac{1}{0.12} \approx. 3333$ Standard deviation $= \sigma = \dfrac{12} \approx. 3333$ Standard deviation $= \sigma = \dfrac{12} 1-p}{p^{2}} = \dfrac{1-0.12}{0.12^{2}} \approx. 7.8174$ Millennium References: Next Generation Portrait, PewResearchCenter. Available online www.pewsocialtrends.org/files...-to-change.pdf (accessible May 15, 2013). Millennials: Confident. Connected. Open the Change app. Summary of PewResearch Social & Demographic Trends, 2013. Available online at (accessible May 15, 2013). HIV prevalence, total (% of population aged 15-49), World Bank, 2013. Available online amp;amp;sort=desc (accessible May 15, 2013). Pryor, John H., Linda DeAngelo, Laura Palucki Blake, Sylvia Hurtado, Serge Tran. American Freshman: National Standards Fall 2011. Los Angeles: Cooperative Institutional Research Program at the Institute for Higher Education Research at UCLA, 2011. Available online (accessible 15.5.2013). Summary of the National Risk and Vulnerability Assessment 2007/8: Afghanistan Profile, European Union and ICON-Institute. Available online at ec.europa.eu/europeaid/where/ ... summary_en.pdf (accessed May 15, 2013). World Fact Book, Central Intelligence Agency. Available online at (accessible May 15, 2013). UNICEF reports on women's literacy centers in Afghanistan based to teach women and girls basic reseading [sic] and writing skills, UNICEF Television reported. The video is available online (accessible May 15, 2013). There are three characteristics of a geometric experiment: There are one or more Bernoulli trials with all the failures except the last, which is a success. In theory, the number of trials could go on forever. There must be at least one attempt. The probability of p success and probability of failure q is the same for each trial. In a geometric experiment, define a separate random variable X as the number of independent experiments until the first success. We say that X has geometric distribution and write $X \sim G(p)$, where p is the probability of success in one trial. The mean geometric distribution $X \sim G(p)$ is $\mu = \dfrac{1-p}{p^{2}} = \sqrt{\dfrac{1}{p}\left(\dfrac{1}{p} - 1\right)}$. Contributors and attribution Barbara a Susan Susan (De Anza College) with many other contributing authors. Content created by OpenStax College is licensed under a Creative Commons Attribution License 4.0. Download for free at 18.114. $X \sim G(p)$ means that the discrete random variable X has a geometric probability distribution with a probability of success in one test value p. $X =$ number of independent tests, until the first success X takes over the values $x = 1, 2, 3, \dotsc$ $p =$ probability of success for any test $q =$ probability of failure for any test $p + q = 1$ $q = 1 – p$ The mean is $\mu = \dfrac{1}{p}$. The standard deviation is $\sigma = \dfrac{1-p}{p^{2}} = \sqrt{\dfrac{1}{p}\left(\dfrac{1}{p} - 1\right)}$. Use the following information to answer the other six exercises: The College Research Institute at UCLA collected data from 203,967 incoming first-time, full-time freshmen from 270 four-year colleges and U.S. universities 71.3% of those students replied that yes, they believe same-sex couples should have the right to legal marital status. Suppose you randomly select a freshman from a study until you find the one who answers yes. You're interested in the number of freshmans you need to ask about. Exercise 4.5.6 Use words to define the random variable X. Answer $X =$ number of freshmans selected from the study, while one answered yes that same-sex couples should have the right to legal marital status. Exercise 4.5.7 $X \sim$ _____(_____,_____) Exercise 4.5.8 What values does the random variable X download? Answer 1,2,... Exercise 4.5.9 Builds a probability distribution (PDF) function. Stop at $x = 6$. x $P(x)$ 1 2 3 4 5 6 Exercise 4.5.10 On average (μ), how many freshman would you expect to be asked until you find the one who answers yes? Answer 1.4 Exercise 4.5.11 What is the probability that you will need to ask fewer than three freshman? Geometric distribution of discrete random variable (RV) resulting from bernoulli tests; tests are repeated until the first success. The geometric variable X is defined as the number of attempts until the first success. Notation: $X \sim G(p)$. The mean is $\mu = \dfrac{1}{p}$ and the standard deviation is $\sigma =$ $\sqrt{\dfrac{1}{p}\left(\dfrac{1}{p} - 1\right)}$. The probability of failure exactly x before the first success is determined by the formula: $P(X = x) = p(1 -p)^{x-1}$. Geometric experiment statistical experiment with the following characteristics: There are one or more Bernoulli trials with all the failures except the last, which is a success. In theory, the number of trials could go on forever. There must be at least one attempt. The probability of p success and probability of failure q does not change from