Reconciling Flow Measurement Errors of a Gas Compression Unit

By Mohit Narain, Jatin Ponda & GNV Sridhar, KOC

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- What is Data Reconciliation & its significance
- Errors in measurement
- Probability Density Function - Bell Curve
- Basic concept of data reconciliation
- Effect of standard deviation on data reconciliation
- Flow reconciliation applied to a compressor unit
- Observations & Conclusions
Objective

- Objective: To carry out the data reconciliation on a compressor system
- Reconciliation literally means a process through which people with different opinions agree in a friendly manner
Data reconciliation (DR) is a process of rectifying erroneous data

System Constraints are satisfied

Uses mathematical modelling

Works best with Random Errors
Data Reconciliation

- Data Reconciliation is basically a constrained optimization problem
- The objective function is optimized (error minimized)
- The constraints are satisfied (A+B=C)

Red Colour- Measured Values
Brown Colour- Reconciled Values

A=10
A=11.2
B=15
B=14.3
C=27
C=25.5
Need for Data Reconciliation

- Measured Data may contain errors
- Such data is often non-repeatable
- Some data may be missing or has not been measured
- Data may not satisfy Process Constraints
Data Reconciliation (DR) for Compressor

- Differences between true and measured values need to be minimized
- Constraints of mass and energy balances need to be fulfilled
Most critical requirement of Data Reconciliation is for custody transfer application.

It may benefit and mutually satisfy both parties involved as custody transfer often is linked to fiscal transactions.
Repeatability & Accuracy

Target

Random Values

Repeetable but not precise
Errors in Measurement

Gross Errors: Caused by errors in reading, remembering 87.46 as 78.46

Measurement Errors

Random Instrument Errors
Cause is not fully clear

Systematic Errors

Require Corr. Factor

Env. Factors Temp, Humidity

Parallax errors
Random Errors

- Random errors are the Instrument indicated error as no Instrument is perfect
- Expected to be scattered in a Normal or Gaussian Distribution
- DR is best applicable to random errors
Instrument Accuracy

Accuracy of a meter is often defined to be +/- 1-3% (avg +/-2%) as a percentage of full scale reading with a confidence level of 95%

If the full scale reading which the Instrument can read is 1500 kg, and its error is +/-1%, then it can be said with a 95% confidence that the true value shall lie within +/- 15 kgs of the measured value.

The accuracy may sometimes also be specified in terms of the actual reading.
• Horizontal axis is the range of possible numeric outcomes

• Vertical axis is the probability of outcome

Probability Density Function

\[ F(X) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(\frac{-(X-\mu)^2}{2\sigma^2}\right) \]

Measured variable

Probability

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Probability Density Function & Confidence Levels

- 95% Confidence Level: 95% probability for which $P(X_1<X<X_2)=0.95$
- $X=\text{measured value}$

In gaussian distribution curve:
- $X_1=\mu-1.96\sigma\ (\sim 2\sigma)$
- $X_2=\mu+1.96\sigma\ (\sim 2\sigma)$

$F(X)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(X-\mu)^2}{2\sigma^2}\right)$
# Standard Deviation

The Weight of a block is measured as follows:

<table>
<thead>
<tr>
<th>S.No</th>
<th>Weight, in gms, ( X_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.8</td>
</tr>
<tr>
<td>2</td>
<td>51.4</td>
</tr>
<tr>
<td>3</td>
<td>49.6</td>
</tr>
<tr>
<td>4</td>
<td>48.7</td>
</tr>
<tr>
<td>5</td>
<td>50.2</td>
</tr>
<tr>
<td>6</td>
<td>49.4</td>
</tr>
<tr>
<td>7</td>
<td>49.9</td>
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</tbody>
</table>

\[
\text{Sum} = \sum X_i = 350
\]

\[
\text{Mean} = \mu = \frac{\sum X_i}{n} = 50
\]

\[
\text{Std. Dev} = \sigma = \sqrt{\frac{\sum (X_i-\mu)^2}{(n-1)}} = 0.9000
\]
The Probability Function Values

\[ F(X) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(X - \mu)^2}{2\sigma^2} \right) \]

<table>
<thead>
<tr>
<th>( \chi )</th>
<th>( \mu )</th>
<th>( \sigma )</th>
<th>( F(X) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>50</td>
<td>0.9</td>
<td>0.05398</td>
</tr>
<tr>
<td>48.5</td>
<td>50</td>
<td>0.9</td>
<td>0.129492</td>
</tr>
<tr>
<td>49</td>
<td>50</td>
<td>0.9</td>
<td>0.241922</td>
</tr>
<tr>
<td>49.5</td>
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<td>0.398862</td>
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<td>50.5</td>
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<td>52</td>
<td>50</td>
<td>0.9</td>
<td>0.05398</td>
</tr>
</tbody>
</table>
Our interest: Probability of finding measured variables in a range (between an $X_1$ & $X_2$)

$$P (X_1 < X < X_2) = \frac{1}{\sigma \sqrt{2\pi}} \int_{x_1}^{x_2} \exp\left(-\frac{(X-\mu)^2}{2\sigma^2}\right) dX$$
If Mean of X measured values =μ=50, σ =0.9

The probability that a measured value will fall between +/-1σ of the mean i.e between 49.1 & 50.9

\[
\frac{1}{\sigma \sqrt{2\pi}} \int \exp \left( -\frac{(X-\mu)^2}{2\sigma^2} \right) = 0.6827 = 68.27\%
\]

Likewise, Probability that the measured value will fall between +/- 2σ i.e. between 48.2 and 51.8 = 95.45%

Likewise, Probability that the measured value will fall between +/- 3σ i.e. between 47.3 and 52.7 = 99.7%
Data Reconciliation – Basic Concept

Let reconciled flow rates $= F_1^*, F_2^*, F_3^*, F_4^*, F_5^*$

Minimize 

$$
(\frac{(100-F_1^*)}{5})^2 + (\frac{(150-F_2^*)}{8})^2 + (\frac{(350-F_3^*)}{6})^2 + (\frac{(291-F_4^*)}{9})^2 + (\frac{(293-F_5^*)}{10})^2
$$

Subject to the constraints: 

$$(F_1^*+F_2^*+F_3^*-F_4^*-F_5^*)^2 < (1*10^{-6})$$
Let reconciled flow rates = $F_1^*, F_2^*, F_3^*, F_4^*, F_5^*$

Minimize $(\frac{(100-F_1^*)}{10})^2 + (\frac{(150-F_2^*)}{9})^2 + (\frac{(350-F_3^*)}{8})^2 + (\frac{(291-F_4^*)}{5})^2 + (\frac{(293-F_5^*)}{6})^2$

Subject to the constraints: $(F_1^*+F_2^*+F_3^*-F_4^*-F_5^*)^2 < (1*10^{-6})$
Weighted Least squares method

- Monitors the gradient or slope of the objective function as the input values (or decision variables) change

- An optimum solution is obtained when the partial derivatives are equal to zero.

- Input values (or decision variables) change to final reconciled values

- Can get trapped into a local minima-depending on initial conditions

- Global Minima is achieved only when epsilon < 10^{-6}
- The mass and energy balances were ensured
- Achieves convergence (global minima) with any random initial conditions
Compressor System-Blind data used

Material Balance:
F1 + F2 - F3 - F4 = 0
F3 - F2 + F7 - F6 - F5 = 0
F6 - F7 - F8 - F9 = 0
The weights are used as the inverse of variances (w=1/σ²)

Minimize: Σ ((Measured value-Reconciled Value)/σ)²

Constraints:
(F1+F2-F3-F4)² < Epsilon (typically 10⁻⁶)
(F3-F2+F7-F6-F5)² < Epsilon
(F6-F7-F8-F9)² < Epsilon
## Comparison of Excel Solver results vs Software

<table>
<thead>
<tr>
<th>Parameters Reconciled</th>
<th>Excel Solver</th>
<th>Process Software</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1 Feed</td>
<td>5142</td>
<td>5176</td>
</tr>
<tr>
<td>F2 Recycle-1</td>
<td>569</td>
<td>569</td>
</tr>
<tr>
<td>F3 Feed to 1st Comp</td>
<td>5711</td>
<td>5745</td>
</tr>
<tr>
<td>F4 1st Stg.Suc Scrub Liq</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>F5 1st Stg Disch Scrub Liq</td>
<td>77.3</td>
<td>76.1</td>
</tr>
<tr>
<td>F6 Feed to 2nd Comp</td>
<td>5626</td>
<td>5660</td>
</tr>
<tr>
<td>F7 2nd stg. Recycle</td>
<td>561</td>
<td>561</td>
</tr>
<tr>
<td>F8 2nd Stg Scrubber Liq</td>
<td>101</td>
<td>130</td>
</tr>
<tr>
<td>F9 Export Gas</td>
<td>4963</td>
<td>4970</td>
</tr>
<tr>
<td>Constraint</td>
<td>$&lt;10^{-8}$</td>
<td>$&lt;10^{-10}$</td>
</tr>
</tbody>
</table>
Observations & Conclusions

- It can be seen that the results of weighted least squares method are based only on material balances and ignores the energy balances (i.e. purely on measured flow data).

- The Process Solver carries out the reconciliation with both mass and energy Balances.

- Though the weighted least squares method results are based only on measured flow rates, measured data is based on heat & material balance.

- Hence the difference between the weighted least squares method and the Process Software reconciliation software is not significant (within 2-3 standard deviations) of any measured variable.
Thank You