

Section 1d: Whole Numbers | Factors and Multiples

TERM 1

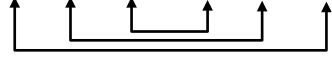
Question 1 | Factors and Rules of Divisibility

1. Study: 12 can be written in product form as

$$1 \times 12 = 12 \quad 2 \times 6 = 12 \quad 3 \times 4 = 12$$

factor
factor
factor
factor
factor
factor

1, 2, 3, 4, 6, 12 are the factors of 12.



We write factors in pairs, from the outside in.

2. Complete:
- The factors of 10 are
 - The factors of 18 are
 - The factors of 24 are
 - The factors of 36 are
 - The factors of 40 are

Always start with 1 and the number itself and then fill in the rest of the factor partners from the outside in.

3. Study: One number is divisible by another number if no remainder is obtained.

Examples:

a) $12 \div 3 = 4$ ← Quotient
 Dividend Divisor
 12 is divisible by 3.

b) $14 \div 3 = 4 \text{ r } 2$ ← Quotient
 Dividend Divisor Remainder
 14 is not divisible by 3.

4. Tick the correct numbers in each of the following:

- a) 24 is divisible by

2	3	4	5	6	7	8	9	10	11	12
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- b) 36 is divisible by

2	3	4	5	6	7	8	9	10	11	12
---	---	---	---	---	---	---	---	----	----	----
- c) 40 is divisible by

2	3	4	5	6	7	8	9	10	11	12
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5. A divisibility rule is a shorthand way of determining whether a given number is divisible by a divisor without performing the division. This is done by examining the digits of the dividend.

Study the rules of divisibility below:

2 – The last digit must be an even number.

3 – The sum of the digits must be divisible by 3.

4 – The last two digits must be divisible by 4.

5 – The last digit must be 0 or 5.

6 – The number must be divisible by 2 and 3.

8 – The last three digits must be divisible by 8.

9 – The sum of the digits must be divisible by 9.

10 – The last digit must be a 0.

12 – The number must be divisible by 3 and 4.

For 7, 11 and 13 etc. a division calculation must be done.

When writing the factors of larger numbers, it is useful to know the rules of divisibility.

6. Using the rules of divisibility, tick the correct numbers in each of the following:

- a) 72 is divisible by

2	3	4	5	6	7	8	9	10	11	12
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- b) 96 is divisible by

2	3	4	5	6	7	8	9	10	11	12
---	---	---	---	---	---	---	---	----	----	----
- c) 132 is divisible by

2	3	4	5	6	7	8	9	10	11	12
---	---	---	---	---	---	---	---	----	----	----

7. The factors of: a) 42 are
- b) 72 are
- c) 96 are
- d) 100 are
- e) 120 are
- f) 132 are

Use the
rules of
divisibility
to help you.

8. Using the rules of divisibility, tick the correct numbers in each.

a) 1 128 is divisible by

2	3	4	5	6	7	8	9	10	11	12
---	---	---	---	---	---	---	---	----	----	----

b) 2 490 is divisible by

2	3	4	5	6	7	8	9	10	11	12
---	---	---	---	---	---	---	---	----	----	----

c) 12 348 is divisible by

2	3	4	5	6	7	8	9	10	11	12
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Question 2 | Prime Numbers, Composite Numbers and Prime Factors

1. Study:

A prime number is a number that only has 2 factors, itself and 1.

Examples: 5 , 19 , 31

A composite number is a number that has more than 2 factors.

Examples: 6 , 15 , 40

1 only has one factor (itself). 1 is therefore neither prime nor composite.

2. Write down the factors of each number.

Mark the prime numbers with a "P" and the composite numbers with a "C".

- a) 1: b) 6: c) 11:
- 2: 7: 12:
- 3: 8: 13:
- 4: 9: 14:
- 5: 10: 15:

3. True or False? a) 39 is a prime number.
- b) Two is the only even prime number.
- c) All odd numbers are prime.
- d) 1 is a prime number.

4. List the prime numbers between:

- a) 1 and 10. b) 10 and 20.
- c) 20 and 40. *d) 90 and 100.

5. Study: A prime factor of a number: a) divides into the number exactly i.e. is a factor.
b) is a prime number.

6. The factors of:

- a) 12 are 1, 2, 3, 4, 6, 12.
- b) 20 are
- c) 36 are
- d) 45 are
- e) 66 are

7. The prime factors of:

- a) 12 are 2 and 3.
- b) 20 are
- c) 36 are
- d) 45 are
- e) 66 are

8. The prime factors of: *First write out all of the factors and then find the prime factors.*

- a) 18 are 2 and 3.
- b) 24 are
- c) 42 are
- d) 52 are
- e) 60 are
- f) 78 are

Question 3 | Highest Common Factor (HCF)

1. Study: a) The factors of 8 are: $F_8 = \{1; 2; 4; 8\}$
The factors of 12 are: $F_{12} = \{1; 2; 3; 4; 6; 12\}$
b) The common factors of 8 and 12 are 2 and 4.
c) The highest common factor (HCF) of 8 and 12 is 4.

In maths the word common means "belonging to both".

2. Find the highest common factor of each pair of numbers.

Think: What is the biggest number that can divide (fit) into both numbers exactly?

- a) 4 and 8 HCF = 4 b) 9 and 3 HCF =
- c) 4 and 12 HCF =
- d) 15 and 5 HCF =
- e) 20 and 10 HCF =
- f) 9 and 45 HCF =

What do you notice about your answers?

3. Write down the factors of the given numbers and then find the highest common factor.

- | | | |
|---|-----------------------------|------------------------------|
| a) 12 and 18 HCF = 6
$F_{12} = \{1; 2; 3; 4; \underline{6}; 12\}$
$F_{18} = \{1; 2; 3; \underline{6}; 9; 18\}$ | b) 18 and 24 HCF = | c) 20 and 36 HCF = |
| d) 24 and 36 HCF = | e) 12 and 60 HCF = | *f) 42 and 56 HCF = |

4. Find the highest common factor of each pair of numbers, by doing a mental calculation.

Think: What is the biggest number that can divide (fit) into both numbers exactly?

- a) 4 and 6 HCF = 2 b) 8 and 10 HCF =
- c) 9 and 12 HCF =
- d) 15 and 6 HCF =
- e) 15 and 20 HCF =
- f) 8 and 24 HCF =

5. Write down the factors of the given numbers and then find the highest common factor.

a) 6, 9 and 12. HCF = b) 8, 10 and 12. HCF = c) 12, 16 and 20 HCF =

$$F_6 = \{1; 2; \underline{3}; 6\}$$

$$F_9 = \{1; \underline{3}; 9\}$$

$$F_{12} = \{1; 2; \underline{3}; 4; 6; 12\}$$

d) 12; 18 and 24. HCF = e) 12; 36 and 48. HCF = f) 24; 42 and 60 HCF =

*6. Complete: a) What is the HCF of 45, 60 and 120?

b) What is the HCF of 72, 90 and 180?

Question 4 | Lowest Common Multiple (LCM)

1. The multiples of 3 are: $M_3 = \{3; 6; 9; 12; 15; 18; \dots\}$

“Multiples think multiply”
 $1 \times 3 = 3$ $2 \times 3 = 6$ $3 \times 3 = 9$
 $4 \times 3 = 12$ $5 \times 3 = 15$ $6 \times 3 = 18$

2. Write down: a) the first 6 multiples of 4.

b) the first 7 multiples of 9 using set-notation. $M_9 = \{9; \dots; \dots; \dots; \dots; \dots; \dots\}$

c) the multiples of 6 between 10 and 50.

d) the multiples of 8 between 30 and 80.

3. Study: a) $M_{12} = \{\underline{12}; 24; 36; \dots\}$

b) $M_{20} = \{\underline{20}; 40; 60; \dots\}$

$M_4 = \{4; 8; \underline{12}; 16; \dots\}$

$M_5 = \{5; 10; 15; \underline{20}; 25; \dots\}$

The lowest common multiple (LCM) of 4 and 12 is **12**.

The lowest common multiple (LCM) of 5 and 20 is **20**.

Write the multiples of the larger number first.

In this way, the LCM usually “jumps out” at you.

4.1. Find the lowest common multiple of the following pairs of numbers.

In other words, what is the smallest number that both numbers can divide (fit) into exactly.

a) 2 and 4 LCM = 4 b) 6 and 3 LCM = 6 c) 12 and 3 LCM =

d) 5 and 20 LCM = e) 24 and 6 LCM = f) 10 and 30 LCM =

g) 12 and 24 LCM = h) 8 and 56 LCM = i) 45 and 15 LCM =

4.2. What do you notice from your answers?

5. Study: a) $M_4 = \{ 4; 8; \mathbf{12}; 16; \dots \}$ b) $M_5 = \{ 5; 10; \mathbf{15}; 20; \dots \}$ c) $M_6 = \{ 6; \mathbf{12}; 18; 24; \dots \}$
 $M_3 = \{ 3; 6; 9; \mathbf{12}; 15; \dots \}$ $M_3 = \{ 3; 6; 9; 12; \mathbf{15}; \dots \}$ $M_4 = \{ 4; 8; \mathbf{12}; 16; \dots \}$
 The lowest common multiple of 3 and 4 is **12**. The lowest common multiple of 3 and 5 is **15**. The lowest common multiple of 4 and 6 is **12**.
 Notice that $3 \times 4 = 12$ Notice that $3 \times 5 = 15$ Even though $4 \times 6 = 24$, the LCM is not 24.

6. Find the lowest common multiple by doing a mental calculation.

- a) 2 and 3 LCM = 6 b) 3 and 4 LCM = c) 5 and 3 LCM =
 d) 4 and 8 LCM = e) 9 and 3 LCM = f) 6 and 4 LCM =
 g) 6 and 18 LCM = h) 4 and 5 LCM = i) 10 and 20 LCM =

7. Find the lowest common multiple (LCM) of:

Hint: Write the multiples of the larger number first. In this way, the LCM usually "jumps out" at you.

- | | | |
|--|------------------------------|-------------------------------|
| a) 6 and 15. LCM = 30
$M_{15} = \{ 15; \mathbf{30}; 45; \dots \}$
$M_6 = \{ 6; 12; 18; 24; \mathbf{30}; 36; \dots \}$ | b) 8 and 12. LCM = | c) 9 and 15. LCM = |
| d) 12 and 15. LCM = | e) 20 and 25. LCM = | *f) 15 and 18. LCM = |

8. Find the lowest common multiple (LCM) of:

- | | | |
|--------------------------------|-------------------------------|--------------------------------|
| a) 4, 6 and 12. LCM = | b) 3, 4 and 6. LCM = | *c) 3, 4 and 5. LCM = |
|--------------------------------|-------------------------------|--------------------------------|

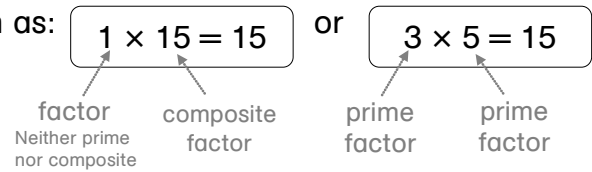
Question 5 | Mixed Questions [Factors, Prime Numbers, HCF and LCM]

1. The factors of: a) 60 are
 b) 180 are
2. Find the HCF and the LCM of:
- | | | |
|---|--|---|
| a) 6 and 9.
HCF = LCM = | b) 8 and 12.
HCF = LCM = | c) 30 and 45.
HCF = LCM = |
|---|--|---|
3. List the prime numbers between 40 and 60.
4. The prime factors of 56 are

Question 6 | Factorisation [By Inspection]

1. Study: To factorise a number means to write it as a product of its prime factors.

We know that 15 can be written in product form as:



NB: To factorise 15 means to write it as a product of its prime factors.

Therefore, $15 = 3 \times 5$ in factorised form. *It is not 1×15 because neither 1 nor 15 are prime numbers.*

2. Complete: To factorise a number means to write it as a of its factors.

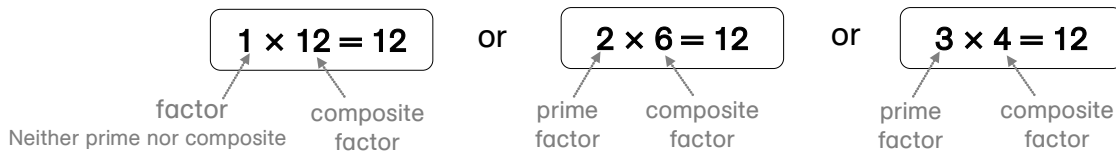
3. Circle the correct answer in each.

a) 6 written in factorised form is: **A** $6 = 1 \times 6$ **B** $6 = 3 + 3$ **C** $6 = 2 \times 3$

b) 10 written in factorised form is: **A** $10 = 7 + 3$ **B** $10 = 2 \times 5$ **C** $10 = 1 \times 10$

c) 14 written in factorised form is: **A** $14 = 1 \times 14$ **B** $14 = 7 + 7$ **C** $14 = 2 \times 7$

4. Study: We know that 12 can be written in product form as:



To factorise 12 we need to write it as a product of **only** prime factors.

Therefore, $12 = 2 \times 6$ *6 is not a prime number so we replace it with 2×3 .* or $12 = 3 \times 4$ *4 is not a prime number so we replace it with 2×2 .*
 $12 = 2 \times 2 \times 3$ $12 = 2 \times 2 \times 3$

We always write the factors in ascending order of size i.e. $12 = 2 \times 2 \times 3$

5. Circle the correct answer in each.

a) 8 written in factorised form is: **A** $8 = 2 \times 4$ **B** $8 = 2 \times 2 \times 2$ **C** $8 = 4 + 4$

b) 18 written in factorised form is: **A** $18 = 9 + 9$ **B** $18 = 2 \times 9$ **C** $18 = 2 \times 3 \times 3$

c) 20 written in factorised form is: **A** $20 = 4 \times 5$ **B** $20 = 2 + 0$ **C** $20 = 2 \times 2 \times 5$

6. Factorise each number. *In other words, write each number as a product of its prime factors.*

a) $8 = 2 \times 4 = 2 \times 2 \times 2$ b) $10 = \dots\dots\dots$ c) $12 = \dots\dots\dots$

d) $18 = \dots\dots\dots$ e) $20 = \dots\dots\dots$ f) $28 = \dots\dots\dots$

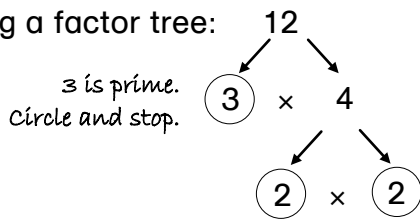
g) $30 = \dots\dots\dots$ h) $35 = \dots\dots\dots$ i) $38 = \dots\dots\dots$

j) $42 = \dots\dots\dots$ k) $50 = \dots\dots\dots$ l) $65 = \dots\dots\dots$

Question 7 | Factorisation [Factor Trees]

1. Study: Drawing a factor tree is another way to complete the factorising process.

Factorise 12 using a factor tree:



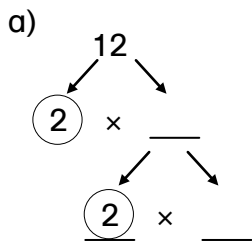
Answer:

$12 = 2 \times 2 \times 3$

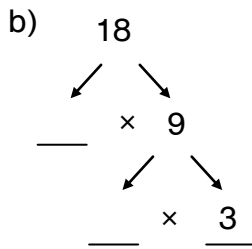
- 1 Plant the number 12.
- 2 Grow the 1st branch of factors. We have started with "3 x 4".
- 3 4 is not prime. Use it to grow the 2nd branch of factors.
- 4 Repeat until you only have primes.
- 5 Write out the prime factors in product form in ascending order of size.

"2 x 6" can also be used to start and the answer will be the same.

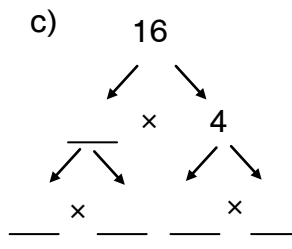
2. Complete: 2.1. Fill in the missing factors in each factor tree. Circle the prime numbers.
2.2. Write each number in factorised form.



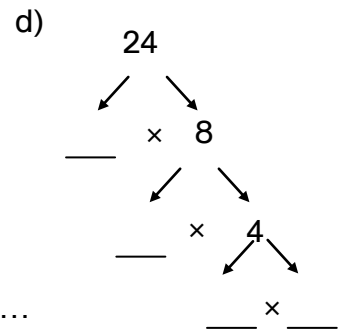
$12 = 2 \times 2 \times 3$



$18 = \dots\dots\dots$



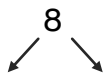
$16 = \dots\dots\dots$



$24 = \dots\dots\dots$

3. Draw a factor tree for each number below. Use it to write each number in factorised form.

- a) $8 = \dots\dots\dots$ b) $20 = \dots\dots\dots$ c) $24 = \dots\dots\dots$ d) $27 = \dots\dots\dots$



- e) $30 = \dots\dots\dots$ f) $45 = \dots\dots\dots$ g) $54 = \dots\dots\dots$ h) $60 = \dots\dots\dots$

Question 8 | Factorisation [The Ladder Method]

1. Study: Another method used to factorise numbers is the “ladder method”. It is especially useful when large numbers need to be factorised.

Study the following examples:

a) Factorise 30.

Step 1: Write the number to be factorised at the top of the ladder.

Step 2: What is the smallest prime factor of 30? *Answer: 2*
 $30 \div 2 = 15$. Write 2 next to 30 and 15 beneath 30.

Step 3: What is the smallest prime factor of 15? *Answer: 3*
 $15 \div 3 = 5$. Write 3 next to 15 and 5 beneath 15.

Step 4: What is the smallest prime factor of 5? *Answer: 5*
 $5 \div 5 = 1$. Write 5 next to 5 and 1 beneath 5. Your ladder is complete 😊.

Step 5: Thus $30 = 2 \times 3 \times 5$ in factorised form.

2	30
3	15
5	5
	1

b) Factorise 48.

2	48	The smallest prime factor of 48 is 2.
2	24	The smallest prime factor of 24 is 2.
2	12	The smallest prime factor of 12 is 2.
2	6	The smallest prime factor of 6 is 2.
3	3	Divide 3 by 3 to end on 1.
	1	

Thus $48 = 2 \times 2 \times 2 \times 2 \times 3$ in factorised form.

Take Note: 1. The first six prime numbers are 2, 3, 5, 7, 11, 13.
 2. Revise the rules of divisibility.

c) Factorise 75.

*The sum of the digits of 75 is 12.
 75 is divisible by 3.

3	75	The smallest prime factor of 75 is 3.*
5	25	The smallest prime factor of 25 is 5.
5	5	Divide 5 by 5 to end on 1.
	1	

Thus $75 = 3 \times 5 \times 5$ in factorised form.

2. Factorise each 2-digit number using the “ladder method”.

a) $18 = 2 \times 3 \times 3$

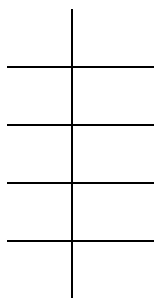
2	18
3	9
3	3
	1

b) 24

c) 27

d) 36

e) 40



f) 45

g) 56

h) 64

i) 72

j) 78

k) 84

l) 98

*2. Factorise each 3-digit number using the “ladder method”.

a) 120

b) 136

c) 175

d) 248

Question 9 | Mixed Questions

1. What is the: a) HCF of 20, 36 and 80? b) LCM of 5, 6 and 12?

2. Factorise each number.

a) 20

b) 76

c) 180

3. Using the rules of divisibility, tick the correct numbers.

3024 is divisible by

2

3

4

5

6

7

8

9

10

11

12

4. True or False? a) The LCM of 4 and 6 is 24.

b) 47 is a prime number.

c) All even numbers are composite.