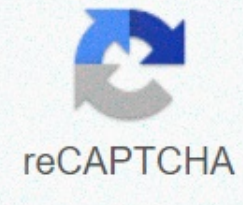




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Volume of a funnel calculus

A question from Rachael, a student: Hi, I'm a 10th grader at AP Calc, and can't figure out this question: Water is running out of conical funnel at a height of $1 \text{ inch}^3/\text{s}$. If the base radius of the funnel is 4 inches and the height is 8 inches, find the rate at which the water level drops when it's 2 inches from the top. We have seen that for quantities that change over time, the indicators at which these quantities change with derivatives. If two related quantities change over time, the rates at which the quantities occur are linked. For example, if the balloon is filled with air, both the xref radius and the balloon volume increase. In this section, we consider several issues where two or more related quantities change and examine how to determine the relationship between the rates of change in these quantities. In many real-world applications, related quantities change over time. For example, if we reconsiderate the example of a balloon, we can say that the speed of the volume change, $\frac{dV}{dt}$, is related to the rate of radius change, $\frac{dr}{dt}$. In this case, we say that $\frac{dV}{dt}$ and $\frac{dr}{dt}$ are related rates, because it is related to $\frac{dr}{dt}$. Here we examine some examples of related quantities that change over time and analyze how to calculate one rate of change, taking into account another rate of change. The spherical balloon is filled with air at a constant speed (Figure). How quickly does the radius of the radius increase? Figure 1. Since the balloon is filled with air, both the radius and volume increase in relation to time. The volume of the ball centimeters radius is $V = \frac{4}{3}\pi r^3$. Because the balloon is filled with air, both the volume and the radius are functions of time. Therefore, a few seconds after the start of filling the balloon with air, the volume of air in the balloon is $V = \frac{4}{3}\pi r^3$. By differentiating the two sides of this equation in relation to time and applying the chain rule, we can see that the rate of change in volume is related to the rate at which the radius changes through the equation. The balloon is filled with air at a constant rate of $2 \text{ cm}^3/\text{s}$, so $\frac{dV}{dt} = 2$. Therefore, what it means $\frac{dV}{dt} = 2$. When ray $r = 2$. What is the instantaneous rate of radius change when? $\frac{dr}{dt}$, which is about 0.0044 cm/s Before we look at other examples, let's outline a troubleshooting strategy that we'll use to solve bid issues. Troubleshooting strategy: Troubleshoot related bids Assign symbols to all variables involved in the problem. If applicable, draw a figure. in terms of variables, the information to be given and the rate to be determined. Find the equation for the variables you entered in step 1. Using a chain rule, distinguish between the two sides of the equation found in step 3 with respect to the independent variable. This new equation will refer to the Derived. Replace all known values with the equation in step 4, and then resolve the unknown change indicator. Note that when troubleshooting related bids, it is important not to override known values too quickly. For example, if you replace a changing quantity with an equation before both sides of the equation are differentiated, the quantity behaves like a constant and its derived quantity does not appear in the new equation found in step 4. We examine this potential error in the following example. Now let's implement the strategy that's just been described to solve some related bid issues. The first example is a plane flying overhead. The relationship we are investigating is between the speed of the aircraft and the speed at which the distance between the plane and the person on the ground changes. The plane flies overhead at a fixed foot height. The man watches the plane from a foot position from the base of the radio tower. The plane flies horizontally away from the man. If an aircraft flies at 600 ft/s , at what speed does the distance between man and aircraft increase when the aircraft passes over the radio tower? Step 1. Draw an image by entering variables that represent different amounts. The aircraft flies at a fixed altitude of 4,000 feet. The distance between the person and the aircraft and the person and place on the ground directly below the aircraft changes. We mark these quantities with variables x and y , respectively. As shown, it indicates the distance between the man and the position on the ground directly below the plane. Variable x is the distance between man and plane. Note that both functions of time. We do not enter a variable for the height of the plane, because it remains on the fixed façade 4000 ft . Because the height of the object above the ground is measured as the shortest distance between the object and the ground, the 4000 ft length line segment is perpendicular to the foot line segment of the length to form a rectangular triangle. Step 2. Because it represents the horizontal distance between a man and a point on the ground below a plane, it represents the speed of the plane. We are told that the speed of the aircraft is 600 ft/s . Therefore, $\frac{dx}{dt} = 600$. Since we are asked to find the speed of change of distance between man and plane, when the plane is located directly above the radio tower, we need to find when $\frac{dy}{dt}$. Step 3. From the figure we can use the thesis pythagorean write an equation relating to and: $y^2 = x^2 + 4000^2$. Step 4. By distinguishing this equation from time and taking advantage of the fact that the derived constant is zero, we come to the equation. Step 5. Find the speed at which the distance between man and plane increases when the aircraft is directly above the radio tower. That is, you find when $\frac{dy}{dt}$. Since the speed of the aircraft 600 ft/s , we know that $\frac{dx}{dt} = 600$. We do not get a clear value for; However, because we try to find when $\frac{dy}{dt}$, we can use the theorca Pythagorean to determine the distance when and height is 4000 ft . The solution of the equation for x we 4000 ft at the moment of interest. Using these values, we come to the conclusion that this is the solution to the equation. Therefore, $\frac{dy}{dt} = 600$. Note: When troubleshooting bid issues, it's important not to override variable values too early. For example, in step 3, we linked variable quantities and an equation. Since the plane remains at a constant height, it is not necessary to enter a variable for the height, and we can use a constant of 4000 to determine this quantity. However, the other two quantities change. If we had mistakenly replaced this equation before discriminating, our equation would have been $y^2 = x^2 + 4000^2$. Once distinguished, our equation will become $2y \frac{dy}{dt} = 2x \frac{dx}{dt}$. As a result, we would wrongly conclude that $\frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt}$. What is the speed of the aircraft if the

