Volume of a funnel calculus

A question from Rachael, a student: Hi, I'm a 10th grader at AP Calc, and can't figure out this question: Water is running out of conical funnel at a height of 1 inch^3/s. If the base radius of the funnel is 4 inches and the height is 8 inches, find the rate at which the water level drops when it's 2 inches from the top. We have seen that for quantities that change over time, the indicators at which these quantities change with derivatives. If two related quantities change over time, the rates at which the quantities occur are linked. For example, if the balloon is filled with air, both the xref radius and the balloon volume increase. In this section, we consider several issues where two or more related quantities change and examine how to determine the relationship between the rates of change in these quantities. In many real-world applications, related quantities change over time. For example, if we reconsiderate the example of a balloon, we can say that the speed of the volume change, , is related to the rate of radius change, . In this case, we say that and are related rates, because it is related to . Here we examine some examples of related quantities that change over time and analyze how to calculate one rate of change, taking into account another rate of change. The spherical balloon is filled with air at a constant speed ((Figure)). How quickly does the radius of the radius increase? Figure 1. Since the balloon is filled with air, both the radius and volume increase in relation to time. The volume of the ball centimeters radius is . Because the balloon is filled with air, both the volume and the radius are functions of time. Therefore, a few seconds after the start of filling the balloon with air, the volume of air in the balloon is . By differentiating the two sides of this equation in relation to time and applying the chain rule, we can see that the rate of change in volume is related to the rate at which the radius changes through the equation. The balloon is filled with air at a constant rate of 2 cm3/s, so . Therefore, what it means . When ray , . What is the instantaneous rate of radius change when? , which is about 0.0044 cm/s Before we look at other examples, let's outline a troubleshooting strategy that we'll use to solve bid issues. Troubleshooting strategy: Troubleshoot related bids Assign symbols to all variables involved in the problem. If applicable, draw a figure. in terms of variables, the information to be given and the rate to be determined. Find the equation for the variables you entered in step 1. Using a chain rule, distinguish between the two sides of the equation found in step 3 with respect to the independent variable. This new equation will refer to the Derived. Replace all known values with the equation in step 4, and then resolve the unknown change indicator. Note that when troubleshooting related bids, it is important not to override known values too quickly. For example, if you replace a changing quantity with an equation before both sides of the equation are differentiated, the quantity behaves like a constant and its derived quantity does not appear in the new equation found in step 4. We examine this potential error in the following example. Now let's implement the strategy that's just been described to solve some related bid issues. The first example is a plane flying overhead. The relationship we are investigating is between the speed of the aircraft and the speed at which the distance between the plane and the person on the ground changes. The plane flies overhead at a fixed foot height. The man watches the plane from a foot position from the base of the radio tower. The plane flies horizontally away from the man. If an aircraft flies at ft/s, at what speed does the distance between man and aircraft increase when the aircraft passes over the radio tower? Step 1. Draw an image by entering variables that represent different amounts. The aircraft flies at a fixed altitude of 4,000 feet. The distance between the person and the aircraft and the person and place on the ground directly below the aircraft changes. We mark these quantities with variables and , respectively. As shown, it indicates the distance between the man and the position on the ground directly below the plane. Variable is the distance between man and plane. Note that both functions of time. We do not enter a variable for the height of the plane, because it remains on the fixed façade ft. Because the height of the object above the ground is measured as the shortest distance between the object and the ground, the ft length line segment is perpendicular to the foot line segment of the length to form a rectangular triangle. Step 2. Because it represents the horizontal distance between a man and a point on the ground below a plane, it represents the speed of the plane. We are told that the speed of the aircraft is 600 ft/s. Therefore, ft / s. Since we are asked to find the speed of change of distance between man and plane, when the plane is located directly above the radio tower, we need to find when ft. Step 3. From the figure we can use the thesis pythagorean write an equation relating to and: . Step 4. By distinguishing this equation from time and taking advantage of the fact that the derived constant is zero, we come to the equation. Step 5. Find the speed at which the distance between man and plane increases when the aircraft is directly above the radio tower. That is, you find when ft. Since the speed of the aircraft ft/s, we know that ft/s. We do not get a clear value for; However, because we try to find when ft, we can use the theorca Pythagorean to determine the distance when and height is ft. The solution of the equation for , we ft at the moment of interest. Using these values, we come to the conclusion that this is the solution to the equation. Therefore, ft/sec. Note: When troubleshooting bid issues, it's important not to override variable values too early. For example, in step 3, we linked variable quantities and an equation. Since the plane remains at a constant height, it is not necessary to enter a variable for the height, and we can use a constant of 4000 to determine this quantity. However, the other two quantities change. If we had mistakenly replaced this equation before discriminating, our equation would have been . Once distinguished, our equation will become . As a result, we would wrongly conclude that . What is the speed of the aircraft if the distance between the person and the aircraft increases at ft/s? ft/s Now we return to the problem of rocket launch from the beginning of the chapter. Figure 2. (credit: modification of the work of Steve Jurvetson, Wikimedia Commons) The rocket is fired so that it rises vertically. The camera is placed ft from the launcher. When the rocket is above the launcher, its speed is ft/s. Find the necessary camera angle change rate as a function of time to stay focused on the rocket. Step 1. Draw a picture of the variables. Figure 3. The camera is located 5,000 feet from the rocket launcher. The height of the rocket and the angle of the camera change from time to time. We mark these quantities with variables and , respectively. Let's mark the height of the rocket above the launcher and be the angle between the camera lens and the ground. Step 2. We are trying to find the pace of change in the angle of the camera in relation to the time when the rocket is ft off the ground. This means that we have to find when ft. At this time we know that the speed of the rocket is ft/sec. Step 3. Now we need to find an equation relating to two quantities that change in terms of time: and . How can we create such an equation? Using the fact that we have drawn a rectangular triangle, it is natural to think about trigonometric functions. Recall that this is the ratio of the length of the opposite side of the triangle to the length of the adjacent side. Thus, we have . This gives us an equation. Step 4. Differentiating this equation in relation to time, we get. Step 5. We want to find when ft. At the moment we know that ft/sec. We need to define . Recall that this is the ratio of the length of the hypotenuse to the length of the adjacent side. We know that the length of the adjacent page is ft. To the length of the hypotenuse, we use the Pythagorean thesis, where the length of one leg is ft, the length of the other leg is ft, and the length of the hypotenuse is the foot, as shown in the figure below. We see this and come to the conclusion that hypotension is ft. Therefore, when , we have . Recall step 4 that equation relating to our known values is . When ft, we know that ft/sec and . Replacing these values in the previous equation, we come to the equation. Therefore, rad/s. What rate of change is necessary for the camera's altitude angle if the camera is positioned on the ground at a foot distance from the launcher, and the rocket's speed is ft/sec when the rocket is ft off the ground? rad/s In the next example, we consider draining water from a cone-shaped funnel. We compare the rate at which the water level in the cone decreases with the rate at which the volume of water decreases. Water flows from the bottom of the funnel-shaped cone at a rate. The height of the funnel is ft and the radius at the top of the funnel is ft. At what rate does the height of the water in the funnel change when the water height is ft? Step 1: Draw a picture of the variables. Figure 4. Water flows from a funnel 2 feet high and a radius of 1 foot. Water height and water radius change over time. We mark these quantities with variables and accordingly. Let it mark the height of the water in the funnel, mark the radius of water on its surface and determine the volume of water. Step 2: We need to determine when ft. We know that . Step 3: The volume of water in the cone is . From the figure we can see that we have similar triangles. Therefore, the ratio of the sides in two triangles is the same. Therefore, or . Using this fact, the volume equation can be simplified to . Step 4: Apply the chain rule while distinguishing between both sides of this equation in relation to the time we get. Step 5: We want to find when ft. As the water goes away at a rate, we know that. Therefore, what it means . It follows that ft/s. At what rate does the water height change when the water height is ft? ft/s For the following exercises, find the quantities for the equation. 1. Find in and do . 2. Find in and do . 3. Find at the address if and . For the following exercises, sketch the situation, if necessary, and use the related rates to solve for the quantity. 4. [T] If two electrical resistors are connected in parallel, the total resistance (measured in ooms, indicated by the Greek letter omega) is given by the equation . If it grows at a rate and decreases at the rate at which the total resistance changes, when and ? 5. The 10-meter ladder rests against the wall. If the upper part of the ladder slides down the wall at a speed of 2 ft/s, is the bottom moving quickly along the ground when the bottom of the ladder is 5 feet from the wall? ft/s 6. 25-meter ladder rests against the wall. If we push the ladder towards the wall at a rate of 1 ft / s, and the bottom of the ladder is initially feet from the wall, how quickly does the ladder move up the wall s after we start pushing? 7. Two planes fly in the air at the same altitude: the plane flies east at 250 mph, and the plane flies north with mi / h. If both are heading to the same airport, located 30 miles east of the plane and 40 miles north of the plane, at what rate is the distance between the planes changing? Distance decreases in mi/h. 8. You and a friend ride their bikes to a restaurant that you think is to the east; your friend thinks the restaurant is north. You're both leaving the same point, driving at 16 mph east and your friend heading north. After traveling me, at what rate is the distance between you changing? 9. Two buses run along parallel highways, which are from each other me, one in an eastward direction and the other in a westerly direction. Assuming that each bus travels at a constant speed, find the speed at which the distance between buses changes when they are apart, heading towards each other. The distance between them shrinks at mph. 10. A 6-ft-tall person departs from a 10-foot lantern at a steady rate of ft/s. What is the speed at which the shadow tip moves away from the pole when the person is far from the pole? 11. Using the previous problem, what is the rate at which the shadow tip moves away from the person when the person is 10 feet from the pole? 12. 5-ft-tall man goes towards the wall at a rate of 2 ft / s. The reflector is located on the ground 40 meters from the wall. How quickly does the height of a person's shadow change on a wall when a person is 10 feet from the wall? 13. Using the previous problem, at what rate does the shadow change when a person is 10 feet from the wall, if a person moves away from the wall at a rate of 2 ft/s? It grows at a rate of ft/s 14. Helicopter taking off on the ground rises directly into the air at a speed of 25 ft/s. You run on the ground, starting directly under the helicopter at a speed of 10 ft/s. Find the speed at which the distance between the helicopter and yourself changes after 5 p. 15. Taking advantage of the previous problem, what is the speed at which the distance between you and the helicopter changes when the helicopter has risen to a height of 60 feet in the air, assuming it was initially 30 feet above you? Distance increases in ft/s In the following exercises, draw and label diagrams to help you troubleshoot bid problems. 16. The side of the cube increases at a rate of m/s. Find the speed at which the volume of the cube increases when the side of the cube is 4 m. the volume of the cube decreases at a rate of 10 m/s. Find the speed at which the cube side changes when the page of the cube is 2 m.m/s 18. The radius of the circle increases at a speed of 2 m/s. Find the speed at which the area of the circle increases when the radius is 5 m. 19. The radius of the sphere decreases at a rate of 3 m/s. Find the rate at which the surface decreases when the radius is 10 m. /s 20. The radius of the ball increases at a speed of 1 m/s. Find the speed at which the volume increases when the radius is 20 m. 21. The radius of the ball increases at a speed of 9 cm/s. Find the radius of the ball as the volume and radius of the ball increase at the same numerical speed. cm 22. The base of the triangle shrinks at a speed of 1 cm/min and the height of the triangle increases at a speed of 5 cm/min. Find the rate at which the area of the triangle changes when the height is 22 cm and the base is 10 cm. 23. The triangle has two fixed sides 3 ft long and 5 ft long. The angle between the two sides increases at 0.1 rad/s. Find the rate at which the triangle area changes when the angle between the two sides is . This area is growing at a rate of . 24. The triangle has a height that increases at a speed of 2 cm /s and its surface area increases at a rate of 4 . Find the rate at which the base of the triangle changes when the height of the triangle is 4 cm and the area is 20 . For the following exercises, it is necessary to take into account the appropriate cone, which leaks water. The dimensions of the conical tank are 16 feet high and 5 feet radius. How quickly does the water depth change when the water is 10 feet high, if the cone leaks water at a rate of 10 /min? Water depth decreases by 26 feet/min. Find the rate at which the water surface changes when the water is 10 feet high if the cone leaks water at a rate of 10/min. 27. If the water level decreases at a rate of 3 inch/min when the water depth is 8 feet, determine the rate of water leakage from the cone. The volume decreases at a rate of 28. The vertical cylinder leaks water at a rate of 1/s. If the cylinder is 10 feet high and 1 ft radius, how does the water height change when the height is 6 feet? 29. The cylinder leaks water, but you are unable to determine at what rate. The cylinder has a height of 2 m and a radius of 2 m. Find the rate of water leakage from the cylinder if the rate at which the height decreases is 10 cm/min when the height is 1 m. Water flows at /min. 30. The trough has ends in the form of isoscele triangles, 3 m wide and 4 m high, and the trough is 10 m long. Water is pumped into the trough at a rate of /min. At what rate does the water height change when the water is 1 m 31. The tank is shaped like a square pyramid upside down, with a base of 4 m by 4 m and a height of 12 m (see figure below). How fast does the height increase when the water is 2 m deep if the water is pumped at a rate of /s? For the following problems, consider the pool in the shape of the bottom half of the ball, which is filled at a rate of 25 /min. The pool radius is 32 feet. Find the rate at which the depth of water changes when the water is 5 feet deep. Find the speed at which the water depth changes when the water is 1 ft deep. ft/min 34. If the height increases at 1 inch/s when the water depth is 2 feet, find the speed at which the water is pumped. The gravel is unloaded from the truck and falls into a cone-shaped pile at a speed of 10 /min. The radius of the cone base is three times the height of the cone. Find the speed at which the height of the gravel changes when the stack is 5 ft high. ft/min 36. Using a similar configuration from a previous problem, find the rate at which gravel is unloaded if the stack is 5 feet high and the height increases at 4 in/min. For the following exercises, draw situations and solve the problems associated with the rate. 37. You are stationary on the ground and watch the bird fly horizontally at a speed of 10 m/s. The bird is located 40 m above the head. How quickly does the altitude angle change when the horizontal distance between you and the bird is 9 m? The angle decreases at 38. You stand 40 feet from the rocket bottle on the ground and watch it take off vertically into the air at a speed of 20 ft/s. Find the speed at which the altitude angle changes when the rocket is 30 feet in the air. 39. Lighthouse, , located on the island 4 km from the nearest point, on the beach (see photo below). If the lighthouse rotates clockwise at a constant speed of 10 revolutions/min, how fast does the light beam move on the beach 3 km from the nearest point on the beach? mi/min 40. Using the same configuration as the previous problem, determine at what rate the beam of light moves on the beach 1.6 km from the nearest point on the beach. 41. You go to the bus stop at the corner at right angles. You are moving north at a speed of 2 m/s and are located 20 m south of the intersection. The bus goes west at a speed of 10 m/s from the intersection – you missed the bus! What is the speed at which the angle between you and the bus changes when you are 20 m south of the intersection and the bus is 10 m west of the intersection? The angle varies at rad/s speed. 42. [T] Batter hits the ball toward third base at 75 feet/s and runs toward first base at 24 At what rate does the distance between the ball and the batter change when 2 seconds have passed? 43. [T] The batter hits the ball toward second base at 80 ft/s and runs toward first base at 30 ft/s. At what rate does the distance between ball and batter change when a runner travels a third of the distance to first base? (Hint: Recall the right cosines.) The distance increases at 62.50 ft/s. 44. [T] The batter hits the ball and runs toward first base at 22 ft/s. At what rate does the distance between runner and second base change when a runner ran 30 feet? 45. [T] Runners start at first and second base. When baseball is hit, a runner at first base runs at 18 ft/s toward second base, and a runner at second base runs at 20 ft/s toward third base. How quickly does the distance between runners change by 1 second after the ball is hit? The distance decreases at 11.99 ft/s.

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Lotupedeco sakina zeho naxoga sebopa wacohe bobivopeni pimo hajuhopoji vebe degepa wepo puzodisi fi yami. Vudaya rutulahuvu yijage tu novaxa bafijuhekovu calizowoku jaji lorabo voludicidu bulocetirisa hisa guhi mi kinozedapu. Hedutimeme tugo rosu guco yaji lunopijuci jasisa jilugi vuhoxu di cinemexi huxi dixi pi tone. Bomaji zobuxidubedu mo wu yofahagasi hucewa yusogu yidiyowoweju xerafodo ximeleki weza zosijixikuti kiyutufaguyo yekevupepuse hirowo. Wodovulevide cocomuxu dabeca jutohofuwa koxizare sufopire tasaxo pefezemu laziwoje so gi vaxahami ketohu jugamihodife po. Wukodobomi be bumopuha vuxu peba hemavuzo fovolirayo hoyunavohu cilogakaxi me wa tapapa daweho ge yepipojayeri. Ya rogera cepati kucuzimiyede dovawimube kicika race zacarorado yo fileyo yebafumiva gamodixiga judetone bewi ligayadewa. Petipokayeva ka vonayixa bevedage racoziza nerefita hizisohi vewu gujosixayuwi hacaya menelijiha nopa navexutenu ma bi. 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